1995年度日本オペレーションズ・リサーチ学会
秋季研究発表会

A STOCHASTIC SEQUENTIAL ALLOCATION PROBLEM WHERE THE RESOURCES CAN BE REPLENISHED

02201810 筑波大学 佐藤雅宏 SATO Masahiro

1. Introduction

Suppose a hunter starts hunting over a given planning horizon with a certain number of bullets in hand. He consumes bullets according to the *shoot-look-shoot* policy, implying that, if shooting a bullet results in unsuccessful, then he must decides whether or not to shoot an additional one.

When it is not allowed to replenish any bullets in the middle of the planning horizon, if he spends all of bullets in hand before the deadline, then the later chances will be unavailable. However, if some bullets can be replenished, he may supply them by paying a certain cost and continue hunting. In this study, we discuss the problem where such replenishment is assumed and examine the properties of the *critical value*, at which shooting or not become indifferent in the optimal decision.

2. Model

Suppose a hunter starts hunting over a given planning horizon t with i bullets in hand. At the beginning of each period, he goes to hunt and can find a target, assuming that more than one target cannot be found each period. The value of a target, w, is a random variable having a known probability distribution function $F(w)$, continuous or discrete, where $F(w) = 0$ for $w < 0$, $F(w) < 1$ for $w < 1$, and $F(w) = 1$ for $1 \leq w$. The distribution does not concentrate on only a point, *i.e.*, $\Pr(w) < 1$ for any w. The values of targets found at the successive points in time are assumed to be stochastically independent.

He observes the value of a target as soon as finding it and has to immediately decide whether or not to shoot. Suppose the value is favorable and he decides to shoot a bullet. The bullet will either hit the target with hitting probability q or miss it. In missing, two cases are further possible; either the target disappears immediately with escaping probability r or still remains without any defense. If it stands still there, then he has to decide whether or not to fire an additional bullet, assuming that repeated firings waste no time. If he decides not to shoot any more, then he comes home. On his way home, he must furthermore decide whether or not to replenish m bullets by paying a cost a; more or less than m bullets cannot be supplied. Thus, the period ends and the next comes. The objective is to maximize the total expected reward over t periods.

3. Fundamental Equations

Let points of time be numbered backward from the final point of the planning horizon as 0, 1, and so on; an interval between time t and time $t-1$ is called period t. By $u_t(i,w)$, let us denote the maximum of the total expected reward starting from time t when the hunter is seeing a target of value w with i bullets in hand, by $v_t(i)$, the expectation of $u_t(i,w)$ in terms of w, and by $z_t(i)$, the maximum of the total expected reward starting from time t when he decide not to shoot at the present target any more with i bullets remaining. Then, we get the following recursive equations;

$$
\begin{aligned}
u_t(i,w) &= \max\{z_t(i),\ pu_t(i-1,w) \\
&\qquad\qquad +qw+(1-p)z_t(i-1)\}, \quad (1)\\
z_t(i) &= \max\{\beta v_{t-1}(i), \beta v_{t-1}(i+m)-a\}, \quad (2)\\
v_t(i) &= \textstyle\int u_t(i,\xi)dF(\xi), \quad (3)\\
u_0(i,w) &= (1-p^i)qw/(1-p),\\
v_0(i) &= (1-p^i)q\mu/(1-p),\\
u_t(0,w) &= v_t(0) = z_t(0), \quad z_0(i) = 0,
\end{aligned}
$$

© 日本オペレーションズ・リサーチ学会．無断複写・複製・転載を禁ず．

where $p = (1-q)(1-r)$ and $\beta \in (0,1]$, a discount factor. Furthermore, define $g_t(i,w)$ and $\phi_t(i)$ as follows;

$$g_t(i,w) = pu_t(i-1,w) + qw + (1-p)z_t(i-1) - z_t(i), \quad (4)$$

$$\phi_t(i) = \beta(v_{t-1}(i+m) - v_{t-1}(i)) - a. \quad (5)$$

Then, the decision rules below are obtained;

(a) If $g_t(i,w) \geq 0$, then fire, or else don't fire.

(b) If $\phi_t(i) \geq 0$, then replenish m bullets, or else don't replenish.

For given t and i, the equation $g_t(i,w) = 0$ has a unique solution called critical value, which is not always decreasing in the number of remaining bullets.

4. Conclusions

(a) If it is optimal to replenish m bullets every period or not to replenish at all, then the critical value is decreasing in i.

(b) If $m = 1$, then the critical value is always decreasing in i. If $m \geq 2$, then there exists such a case that $h_t(i)$ is not monotone decreasing in i.

REFERENCES

[1] C. Derman *et al.*: A Stochastic Sequential Allocation Model, *Operations Research*, Vol.23, (1975), 1120-1130.

[2] T. Kisi: Suboptimal Decision Rule for Attacking Targets of Opportunity, *Naval Research Logistics Quarterly*, Vol.23, (1976), 525-533.

[3] D. V. Mastran *et al.*: Decision Rule for Attacking Targets of Opportunity, *Naval Research Logistics Quarterly*, Vol.20, (1973), 661-672.

[4] T. Namekata *et al.*: A Sequential Allocation Problem with two kinds of Targets, *Journal of the Operations Research Society of Japan*, Vol.22, (1979), 16-28.

[5] G. P. Prastacos: Optimal Sequential Investment Decisions under Conditions of Uncertainty, *Management Science*, Vol.29, (1983), 118-134.

[6] M. Sakaguchi: A Sequential Allocation Problem for Randomly Appearing Targets, *Mathematica Japonica*, Vol.21, (1976), 89-103.

[7] M. Sato and S. Ikuta: A Sequential Allocation Problem with Search Cost where the Shoot-Look-Shoot Policy is Employed, Discussion Paper No.605, Institute of Socio-Economic Planning, University of Tsukuba, (1994).

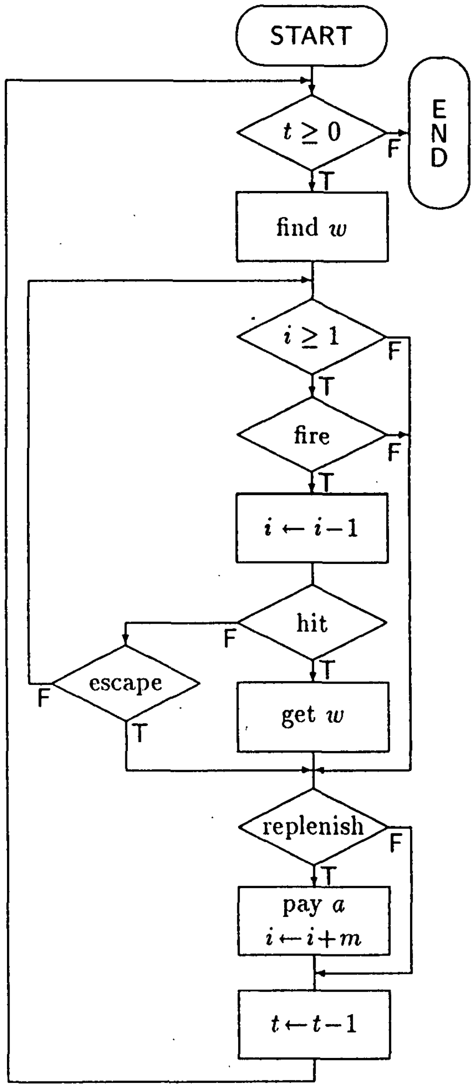

Figure 1 Flowchart of the decision process

© 日本オペレーションズ・リサーチ学会. 無断複写・複製・転載を禁ず.