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ENTROPIC DECISION CRITERIA APPLIED TO BIMATRIX GAMES

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ABSTRACT "Three non-zero-sum games, based on the concept of Shannon's entropy and its variant are considered, in which the two players compete in a bimatrix game. In the first, each player wants to maximize Shannon entropy contained in his strategy plus his expected payoff resulted from his own and rival's strategy; in the second the aim is to maximize the ratio of Shannon entropy to the expected payoff; in the third, Shannon entropy in the first game is replaced by Havrda-Charvat entropy with parameter β=2. In each case the game is solved and examples are given。

1. Introduction & Summary

2. An Entropic Decision Criterion.

Let us now consider a competitive choice problem with two players I and II. They want to choose an object from their own set of n objects. If I(II) chooses the i-th (the j-th) object, they obtain the benefit a_{ij} for I and b_{ij} for II. The benefit may be positive, negative or zero, since a choice of any object may have positive, negative or zero, incentive to choose. (The "benefit" is the terminology borrowed from Guiasu [1: Chapter 22]).

Let $x=\langle x_1, \ldots, x_n\rangle$ and $y=\langle y_1, \cdots, y_n\rangle$ be any mixed strategies for I and II, respectively. We define the expected benefit for player I by

$$B_1(x,y) \equiv -\sum_{i=1}^{n} x_i \log x_i + \sum_{i=1}^{n} x_i M_1(i,y) \;; \tag{2.1}$$

and, for II, by

$$B_2(x,y) \equiv -\sum_{j=1}^{n} y_j \log y_j + \sum_{j=1}^{n} y_j M_2(x,j). \tag{2.2}$$

Here we use the notations for the expected payoffs in game theory

$$M_1(x,y)=\sum_{j=1}^{n}\sum_{i=1}^{n} a_{ij}x_i y_j, \qquad M_2(x,y)=\sum_{j=1}^{n}\sum_{i=1}^{n} b_{ij}x_i y_j$$

$$M_1(i,y)=\sum_{j=1}^{n} a_{ij} y_j, \qquad M_2(x,j)=\sum_{i=1}^{n} b_{ij} x_i .$$

Theorem 2. For the non-zero-sum game with payoff-functions (2. 1)-(2. 2), *if the system of simultaneous equation.*

$$x_i = e^{M_1(i,y)} \Big/ \sum_{i=1}^{n} e^{M_1(i,y)}, \qquad i=1,\cdots,n \tag{2.3a}$$

$$y_j = e^{M_2(x,j)} \Big/ \sum_{j=1}^{n} e^{M_2(x,j)}, \qquad j=1,\cdots,n \tag{2.3b}$$

has a unique root (x^*, y^*), *then the game has equilibrium strategies* x^* *for* I, *and* y^* *for* II. *The equilibrium values are* $\log \sum_{i=1}^{n} e^{M_1(i,y^*)}$ *for* I, *and* $\log \sum_{j=1}^{n} e^{M_2(x^*,j)}$ for II.

3. Another Entropic Decision Criterion

Kunisawa[4] considered an entropy model in which he discussed on the problem

$$-\sum_i x_i \log x_i \Big/ \sum_i w_i x_i \longrightarrow \max_x$$

and its application to management science.

Theorem 4. Let A(B) be any $n \times n$ matrix having no zero-rows (zero-columns). *For the non-zero-sum game with payoff functions*

$$(3.1) \quad R_1(x,y) \equiv -\sum_{i=1}^n x_i \log x_i \Big/ \sum_{i=1}^n x_i M_1(i,y),$$

$$(3.2) \quad R_2(x,y) \equiv -\sum_{j=1}^n y_j \log y_j \Big/ \sum_{j=1}^n y_j M_2(x,j),$$

if the system of simultaneous equations

$$(3.3a) \quad x_i = e^{-\gamma_1 M_1(i,y)}, \qquad i=1,\cdots,n$$

$$(3.3b) \quad y_j = e^{-\gamma_2 M_2(x,j)}, \qquad j=1,\cdots,n$$

$$(3.3c) \quad \sum_{i=1}^n x_i = \sum_{j=1}^n y_j = 1$$

has a unique root ($\mathbf{x}^*$, $\mathbf{y}^*$, γ_1, γ_2), *then the game has an equilibrium strategy-pair* $\mathbf{x}^*$-$\mathbf{y}^*$, and equilibrium values γ_1-γ_2, provided that mixed strategies giving positive $M_1(i,y)$ and $M_2(x,j)$ for all i,j are considered.

4. Entropic Decision Criterion Based on Havrda-Charvat Entropy.

Charvat entropy (See, for example, Kumar [3] and Sakaguchi [12].)

$$H^\beta(p) \equiv (1-\beta)^{-1}\left(\sum_{i=1}^n p_i^\beta - 1\right), \qquad \beta > 0$$

is more general than Shannon entropy.

Consider the non-zero-sum game with payoff functions

$$(4.1) \quad H_1(x,y) \equiv 1 - \sum_{i=1}^n x_i^2 + \sum_{i=1}^n x_i M_1(i,y),$$

$$(4.2) \quad H_2(x,y) \equiv 1 - \sum_{j=1}^n y_j^2 + \sum_{j=1}^n y_j M_2(x,j),$$

Theorem 5. *For the non-zero-sum game with payoff functions* (4.1)-(4.2), *if the system of linear simulaneous equation*

$$(4.3a) \quad x_i = \frac{1}{n} + \frac{1}{2}\left(M_1(i,y) - M_1(\bar{\bar{x}},y)\right), \qquad i=1,\cdots,n$$

$$(4.3b) \quad y_j = \frac{1}{n} + \frac{1}{2}\left(M_2(x,j) - M_2(x,\bar{\bar{y}})\right), \qquad j=1,\cdots,n.$$

where $\bar{\bar{x}} = \bar{\bar{y}} = \langle \frac{1}{n},\cdots,\frac{1}{n} \rangle$, *has a unique root* ($\mathbf{x}^*$, $\mathbf{y}^*$), *then the game has an equilibrium mixed-strategy pair* $\mathbf{x}^*$-$\mathbf{y}^*$, *and the equilibrium values are*

$$v_1^* = 1 - \frac{1}{n} + M_1(\bar{\bar{x}},y^*) + \frac{1}{4}\sum_{i=1}^n \left(M_1(i,y^*) - M_1(\bar{\bar{x}},y^*)\right)^2,$$

$$v_2^* = 1 - \frac{1}{n} + M_2(x^*,\bar{\bar{y}}) + \frac{1}{4}\sum_{j=1}^n \left(M_2(x^*,j) - M_2(x^*,\bar{\bar{y}})\right)^2.$$

5 Conclusion (略).