1996年度日本オペレーションズ・リサーチ学会
秋季研究発表会

A Project Network Scheduling Problems with Fuzzy Precedence Relation

02991654 大阪国際大学 *朴 泰 根 PARK Tae-Geun
01991244 大阪国際大学 韓 尚 秀 HAN Sang-Su
01005194 大 阪 大 学 石井 博昭 ISHII Hiroaki

1. INTRODUCTION

In this research, we investigate a biobjective project scheduling problem with fuzzy precedence relations. In real-world systems, there exist many situations that a single objective function is not sufficient to characterize the problem and optimal decisions are usually made under some uncertainties. This indicates the necessity to study project scheduling problems with multi objectives in a fuzzy situations. From these viewpoints, we concentrate on two factors, namely multi objectivity and fuzziness. In order to consider the problem, we introduce a concept of of scheduling vector for treating biobjective and satisfaction level for formulating fuzzy precedence relations. Our objective is to find a non dominated schedule maximizing minimum satisfaction with respect to precedence relations between tasks while simultaneously minimizing maximum lateness. An efficient algorithm based on PERT technique is proposed and its validity and computational complexity are demonstrated.

2. PROBLEM FORMULATION

We consider a project scheduling problem with fuzzy precedence constraints given as follows.

(1) A project is composed of a large number of tasks $T_j(j=1,2,\cdots,n)$ and each task's duration p_j is associated with each task T_j.

(2) Fuzzy precedence relation is given between all two tasks as precedence relation matrix. This relation is denoted with a satisfaction level μ_{ij} for all two tasks T_i and T_j which denotes degree of desirability when task T_i is processed before T_j. Assume that if $\mu_{ij}>0$ then $\mu_{ij}=1$, and that $\mu_{ij}=\mu_{ji}=\lambda$ means that T_j, T_j are independent.

(3) Each task T_j has a due-date d_j to be completed. Assume that $L_j(\pi)$ denotes difference between task's completion time $C_j(\pi)$ and the due-date d_j for a project schedule π, and that $L_{max}(\pi)$ denotes maximum of $L_j(\pi)$ for all tasks. Then, for π, minimum desirable degree of task's precedence relation is denoted with $\mu_{min}(\pi)=\min\{\mu_{ik}(\pi) | 1,k=1,2,\cdots,n,i\neq k\}$.

(4) Under the above setting (1)-(4), we formulate a following bicriteria project scheduling problem (**FPRP**;**F**uzzy **P**recedence **R**elation **P**roblem) to find schedule(s) maximizing the minimum satisfaction(desirability) with respect to precedence relation and minimizing maximum lateness, simultaneously.

FPRP : | Minimize $L_{max}(\pi)$ Maximize $\mu_{min}(\pi)$
| subject to $\pi\in\Pi$(A set of feasible schedules)

3. SOLUTION PROCEDURE

We define schedule vector $(L_{max}(\pi),\mu_{min}(\pi))$ consisting of two objectives as elements. Next, to define a non dominated schedule, assume two vectors $v(\pi)=(v_1(\pi),v_2(\pi))$ and $v(\rho)=(v_1(\rho),v_2(\rho))$. We call $v(\pi)$ dominates $v(\rho)$ and denote it by $v(\pi)\ll v(\rho)$ when $v_1(\pi)\leqq v_1(\rho)$, $v_2(\pi)\geqq v_2(\rho)$ and $v(\pi)$

$\neq v(\rho)$.

Definition: *A feasible schedule π is called to be a non dominated schedule if and only if there exists no feasible schedule which dominates π..*

Now we propose a solution procedure **FPRA(Fuzzy Precedence Relation Algorithm)** for solving the problem **FPRP**.

Solution Procedure **FPRA** for solving **FPRP**

Step0: By sorting $0<\mu_{ij}<1$, set $\mu^0\equiv 1>\mu^1>\mu^2>\cdots>\mu^\kappa>0$ where k is the number of different μ_{ij}.

Step1: Construct PERT-Network PN(V,A) as Flow Diagram; V is the set of tasks and A is the set of arcs (T_i,T_j) which means that T_i precedes T_j. When T_i and T_j are independent, there is no arc. First, for $\mu^0=1$, construct $PN^0(V,A^0)$ consists of V and $A^0=\{(T_i,T_j)|\mu_{ij}=\mu^0$ and $\mu_{ji}\neq\mu^0\}$. Further let $\mathring{A}^\alpha \equiv\{(T_i,T_j)|\mu_{ij}=\mu^0$ and $\mu_{ji}\neq\mu^\alpha\}$, $\alpha=1,2,\cdots,k$ and let *DS* the current set of schedules corresponding to each vector of *DV*.

Step2: Transform PERT-Network PN(V,A) to Arrow Diagram $\dot{P}N(V,A)$; Solve Ordinary PERT problem and calculate L_{max}. Let optimal L_{max} value be $L_{max}{}^0$ and optimal schedule $\pi^0=\pi$. Set $DV=\{(L_{max}{}^0,1)\}$, $DS=\{\pi^0\}$ and $\alpha=1$. Go to Step3.

Step3: Set $A^\alpha=A^{\alpha-1}-\mathring{A}^\alpha$ and construct $\dot{P}N^\alpha(V,A^\alpha)$. Solve Ordinary PERT problem and calculate L_{max}. Let optimal value L_{max} be $L_{max}{}^\alpha$ and optimal schedule $\pi^0=\pi^\alpha$. Construct corresponding schedule vector $v^\alpha=(L_{max}{}^\alpha,\mu_{min}{}^\alpha)$. If v^α is dominated by some vector of *DV*, then go to Step4. Otherwise, set $DV=DV\cup\{v^\alpha\}$ and $DS=DS\cup\{\pi^\alpha\}$. Go to Step4.

Step4: Set $\alpha=\alpha+1$. If =k+1, terminate. Otherwise, return to Step3.

Theorem: *FPRA finds set of non dominated schedules in $O(n^3)$ polynomial times.*

Since there exist, probably, many non dominated schedules which have the same values with respect to both criteria, we concentrate our attention on finding at least one feasible schedule corresponding to each non dominated schedule vector. More details are introduced in presentation.

4. CONCLUSION

It is hopeful to apply resource constrained problem with another multi criterion.

REFERENCES

[1] S.S. Han, H. Ishii and S. Fujii: One Machine Scheduling Problem with Fuzzy Due dates, European Journal of Operational Research, (1994).