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An Alternative Approach to the Sensitivity Analysis of Efficient and Inefficient DMUs with respect to Changes in the Reference Set in DEA

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1 Introduction

Data Envelopment Analysis(DEA) was developed in order to measure the relative efficiency of a decision making unit(DMU) with multiple inputs and outputs. Sensitivity analysis should be routinely discussed in DEA, because data are not always deterministic and are subject to several factors of disturbance. Some techniques related to sensitivity analysis have been studied. In this paper, we investigate the stability of efficient and inefficient DMUs, more specifically with respect to changes in efficiency classification, efficiency score, and slacks in the reference set. Few studies of sensitivity analysis have focused on inefficient DMUs. A notable exception is the discussion by Charnes et al.[3]. Moreover, to the best of the author's knowledge, changes in efficiency with respect to changes in the reference set have not been studied. Prior studies have focused on data variation, but changes in the reference set itself may be at least as critical. We propose an alternative approach to sensitivity analysis for establishing the stability of both efficient and inefficient DMUs with respect to changes in the reference set. Applying the idea of cross-evaluation(efficiency) to the EDM, we propose a new concept called the "cross-sensitivity analysis", and two new measures called the "DEA cross-reference efficiency measure(DCR)" and the "DEA cross-sensitivity measure(DCS)".

The proposed approach has the following advantages over conventional methods.

1. Sensitivity of both efficient and inefficient DMUs can be(cross-)evaluated at the same time.
2. When a particular DMU is evaluated, conventional DEA does not use the data for inefficient DMUs completely, whereas the new approach incorporates them in the sensitivity evaluations.
3. Elimination of an extreme-efficient DMU affects the spanning of the set of reference points for inefficient DMUs. This approach investigates the effects of eliminating an extreme-efficient DMU and the efficiency change for an inefficient DMU.
4. DCR measure provides the way of ranking efficient DMUs perfectly, whereas the procedure using the extended DEA measure by Andersen et al.[1] often fails in ranking efficient DMUs because of the infeasible problem.
5. The stability of the score of an inefficient DMU is examined with respect to changes in the reference set.

A.Charnes, W.W.Cooper and R.M.Thrall[2] suggest the way of classifying DMUs by reference to the dimensions of their multiplier sets, and partitions the set of all DMUs into the six classes ; E, E', F, NE, NE', NF. We call $E \cup E'$ the set of all DEA-efficient DMUs, and $N \cup F$ the set of all DEA-inefficient DMUs. This classification can be applied to the new efficiency measure.

2 Evaluating DEA-efficiency with respect to changes in the reference set

We propose the DEA cross-reference efficiency measure (DCR) as a means for considering the sensitivity relationship between two DMUs. When DMU a is under evaluation, the new measure $\delta_{a,b}$ can be calculated after DMU b is excluded from the reference set R where $R = \{1, \ldots, n\}$ is the set of DMU a , as in **Program 1** for the input-oriented BCC model.

[Program 1]

$$\begin{aligned}
\textbf{min} \quad & \delta_{a,b} \\
\textbf{s.t.} \quad & \delta_{a,b} \cdot X_{i,a} = \sum_{j \in R-\{b\}} X_{i,j} \cdot \lambda_j + t_i^- \\
& Y_{r,a} = \sum_{j \in R-\{b\}} Y_{r,j} \cdot \lambda_j - t_r^+ \\
& \sum_{j \in R-\{b\}} \lambda_j = 1 \\
& \lambda_j,\ t_i^-,\ t_r^+ \geq 0 \\
& (j \in R - \{b\},\ i = 1, \ldots, m,\ r = 1, \ldots, k)
\end{aligned}$$

where m, k and n are the number of inputs, outputs, and DMUs, $X_{i,j}$ is the amount of input i to DMU j and $Y_{r,j}$ is the amount of output r from DMU j.

Efficiency score and slacks for input-oriented BCC model are not always equivalent to them for output-oriented BCC model. We should pay attention to those differences.

In sensitivity analysis in the radial efficiency model, not only efficiency measure but also slacks play an important role. In **Program 1**, cross-reference slacks t_i^- and t_r^+ are also examined as an aid in interpreting importance of the cross-reference model.

We rewrite the input-oriented BCC model in more general form as **Program 2**. Let $\theta_a(R^*)$ be a general efficiency measure for the DMU a under evaluation where R^* is the reference set.

[Program 2]

$$\begin{aligned}
\min \quad & \theta_a(R^*) \\
\text{s.t.} \quad & \theta_a(R^*) \cdot X_{i,a} = \sum_{j \in R^*} X_{i,j} \cdot \lambda_j + s_i^- \\
& Y_{r,a} = \sum_{j \in R^*} Y_{r,j} \cdot \lambda_j - s_r^+ \\
& \sum_{j \in R^*} \lambda_j = 1 \\
& \lambda_j,\ s_i^-,\ s_r^+ \geq 0 \\
& (j \in R^*,\ i = 1, \ldots, m,\ r = 1, \ldots, k)
\end{aligned}$$

When $R^* = R$, $\theta_a(R^*) = \theta_a^*$ where θ_a^* is the conventional (input-oriented) BCC efficiency measure. When $R^* = R - \{a\}$, $\theta_a(R^*) = \delta_a^*$ where δ_a^* is the extended DEA measure of Andersen et al. If DMU a is evaluated DEA-efficient, $\delta_a^* \geq 1$. If DMU a is evaluated DEA-inefficient, $\delta_a^* = \theta_a^*$. When $R^* = R - \{b\}$, $\theta_a(R^*) = \delta_{a,b}^*$ where $\delta_{a,b}^*$ is the DEA cross-reference efficiency measure.

We show the properties of the DCR for the input-oriented model as follows, in which DMU a is the unit under evaluation and DMU b is the excluded unit.

Property 1. $\delta_{a,b}^* \geq \theta_a^*$. However, DCR slacks are not always smaller than DEA slacks.

Property 2. If DMU a is the same as DMU b, then $\delta_{a,a}^* = \delta_a^*$. In particular, $\delta_{a,a}^* \geq 1$ or **Program 1** is infeasible if and only if DMU a is DEA-efficient.

Property 3. If DMU a is not the same as DMU b, then **Program 1** is feasible.

Property 4. If DMU b is not DEA-extreme-efficient, then $\delta_{a,b}^* = \theta_a^*$, and $t_i^{-*} = s_i^{-*}$ and $t_r^{+*} = s_r^{+*}$.

Property 5. If DMU b is not included in the set of reference points, E_a, for DMU a, then $\delta_{a,b}^* = \theta_a^*$, and $t_i^{-*} = s_i^{-*}$ and $t_r^{+*} = s_r^{+*}$.

Property 6. If DMU a is DEA-efficient and different from DMU b, then $\delta_{a,b}^* = 1$, and $t_i^{-*} = t_r^{+*} = 0$.

3 Cross-sensitivity Analysis

We use both the DCR (as defined in Section 2) and the DEA-efficiency score to define the sensitivity of a DMU's efficiency rating with respect to the reference set. We let $D_{a,b}^*(\%)$ denote the DEA cross-sensitivity measure(DCS), and define it as follows [1].

$$D_{a,b}^*(\%) = \delta_{a,b}^* - \theta_a^* \tag{1}$$

Furthermore, we define the normalized change of the DCR slack to the DEA slack as the DCS slack measure, $d_i^{-*} \in R^m$ and $d_r^{+*} \in R^k$.

$$d_i^{-*} = g_i^-(s_i^{-*} - t_i^{-*}),\ \text{and}\ d_r^{+*} = g_r^+(s_r^{+*} - t_r^{+*}), \tag{2}$$

where g_i^- and g_r^+ are normalizing coefficients such as positive weights introduced by R.M.Thrall [4]. The reason the change is normalized is that we can use these measures with DCS radial measure. The DCS $D_{a,b}^*(\%)$ is the change in the efficiency rating of DMU a when DMU b is excluded from the reference set. When a and b refer to the same DMU, we call this sensitivity measure the "DEA self-sensitivity measure(DSS)" of DMU a and denote it by $D_{a,a}^*(\%)$.

4 Numerical examples

We test our procedures with the numerical examples using economic data of the developing countries in Asia-Pacific area.

We will show some results in the presentation.

5 Concluding remarks

We propose the DEA cross-reference efficiency measure (DCR), applying the idea of cross-evaluation into the extended DEA measure (EDM). Moreover, we propose the cross-sensitivity analysis, using the DCR and the DEA cross-sensitivity (DCS). We can find the useful information about the sensitivity relationship between DMUs through the proposed analysis.

References

[1] P.Andersen and N.C.Petersen : A Procedure for Ranking Efficient Units in Data Envelopment Analysis, *Mgmt. Sci.* , 39(1993), 1261–1264.

[2] A.Charnes, W.W.Cooper and R.M.Thrall : A Structure for Classifying and Characterizing Efficiency and Inefficiency in Data Envelopment Analysis, *J. Prod. Anal.* , 2(1991), 197–237.

[3] A.Charnes, J.J.Rousseau and J.H.Semple : Sensitivity and Stability of Efficieny Classifications in Data Envelopment Analysis, *J. Prod. Anal.* , 7(1996), 5–18.

[4] R.M.Thrall : Duality, Classification and Slacks in DEA, *Annals of O.R.* , 66(1996), 109–138.

[1] The proportional change in efficiency rating of DMU a, that is the ratio of the DCR radial measure to the DEA-efficiency measure, can also defined as follows.

$$S_{a,b}^*(\%) = \left(\frac{\delta_{a,b}^*}{\theta_a^*} - 1 \right) \times 100(\%)$$