TWO-PERSON HI-LO POKER—— STUD AND DRAW, II

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ABSTRACT This paper analyses a continuous version of a class of two person Hi-Lo poker. Stud-poker and draw-poker versions are discussed in each of which simultaneous-move and bilateral-move one-round games are formulated and explicit solutions are derived. It is shown that in bilateral-move games the first-mover innevitably gives his opponent some information about his true hand, and so the second-mover is able to utilize this information in deciding his best response in the optimal play. A connection between Hi-Lo poker and simple exchange games is mentioned.

Abstract Continuing the work in the previous paper, Part I [1] , we derive the solution to a bilateral-move Hi-Lo poker for the case $0<A\leqq B$. Parameter A (parameters B and 1) represents the weight of the situation where the competitors' intentions are opposite (consistent). It is surprizing that the problem is so simple,but the solution is very much complicated.

§1. Introduction —— A Bilateral-Move Hi-Lo Draw Poker

Player I(II) privately observes $x(y) \sim \Pi_{[0,1]}$, and chooses Bet/Pass.

$\begin{Bmatrix}\text{Bet}\\ \text{Pass}\end{Bmatrix}$ means $\begin{Bmatrix}\text{drawing another card } z(w) \text{ and using } x\vee z(y\vee w) \text{ for I(II)}\\ \text{not-drawing a new card and using } x(y) \text{ for I(II)}\end{Bmatrix}$

Players' hands	1st move	2nd move	Player I's payoff
I: x	Bet	Bet	$a_{11}(x,y)$
		Pass	$a_{10}(x,y)$
II: y	Pass	Bet	$a_{01}(x,y)$
		Pass	$a_{00}(x,y)$

where
$$a_{11}(x,y) \equiv E_{z,w}\{B\,\mathrm{sgn}(x\vee z - y\vee w)\} = B(x\vee y)^2\,\mathrm{sgn}(x-y),$$
$$a_{10}(x,y) \equiv E_z\{A\,\mathrm{sgn}(x\vee z - y)\} = A(\bar{y} + y\,\mathrm{sgn}(x-y)),$$
$$a_{01}(x,y) \equiv E_w\{A\,\mathrm{sgn}(x - y\vee w)\} = -A(\bar{x} - x\,\mathrm{sgn}(x-y)),$$
$$a_{00}(x,y) \equiv \mathrm{sgn}(y-x).$$

The exp. payoff to I under the strategy-triple $(\alpha(\cdot), \beta(\cdot), \gamma(\cdot))$ is

(1.3) $$M(\alpha,\beta,\gamma) = E_{x,y}\left[\alpha(x)\left(a_{11}\beta(y) + a_{10}\bar{\beta}(y)\right) + \bar{\alpha}(x)\left(a_{01}\gamma(y) + a_{00}\bar{\gamma}(y)\right)\right],$$

The case $0=A\leqq B$ was solved in [1].

§2 Results.

Theorem 1. The solution to (13) when $0<A\leqq B$ is:

Case 1. $1<B/A\leqq 1+\sqrt{3}\doteqdot 2.732$, Bet-Bet is opt. Value is 0.

Case 2. $1+\sqrt{3}<B/A\leqq 3$, Bet-(Bet iff $y\in[0,b_1]\cup[b_2,1]$) is opt., where $b_1<b_2$ are two roots of $-(4/3)Bb^3+2Ab^2-A+B/3=0$.

Case 3. $3<B/A\leqq \hat{\nu}^{-1}\doteqdot 3.401$, Bet-(Bet iff $y\geqq b_1$) is opt., where $(b_1,\hat{\nu})\in[0,1]\times[0,1/3]$ is the root of
$$-2(\nu+\tfrac{1}{2}b_1)^3+3\nu(b_1+1)-1=0,\quad -(4/3)b_1^3+2\nu b_1^2-\nu+\tfrac{1}{3}=0.$$

Case 4. $\hat{\nu}^{-1}<B/A\leqq\lambda(A)$. Bet-(Bet iff $y\geqq b_1$) is opt., where $(b_1,\lambda)\in[0,1]\times[1,\infty)$ is the root of (24). Value is $-b_1^3(Bb_1-\frac{4}{3}A)$

Case 5. $B/A>\lambda(A)$. (Bet iff $x\in[a_1,a_2]\cup[a_3,1]$)-(Bet iff $y\geqq\begin{Bmatrix}b_1\\ b_0\end{Bmatrix}$, if I chooses $\begin{Bmatrix}\text{Bet}\\ \text{Pass}\end{Bmatrix}$) is opt., where $0<a_1<b_0<a_2<b_1<a_3<1$ satisfy (26).

Cor. 1.1 (Asymptotic analysis)

In Case 4, $\lambda(A)\xrightarrow[(A\to\infty)]{}\hat{\lambda}\doteqdot 3.495$

In Case 5, if A is fixed and $B\to\infty$, then Pass-(Bet iff $y\geqq b_0$) is opt. and the value is $-(A/3)(1-b_0^3)-b_0(1-b_0)$, where $b_0=(\sqrt{A+1}-1)/A$.

Table 2. Solutions of poker in Cases 4 and 5.

A	B	B/A	a_1	b_0	a_2	b_1	a_3	V
1	3.5	3.5		0.136		0.547		−0.095
2	7	3.5		0.181		0.547		−0.191
2	7.2	3.6		0.166		0.552		−0.222
1	4	4	0.031	0.121	0.561	0.586	0.615	−0.170
2	8	4	0.048	0.165	0.543	0.602	0.647	−0.334
0.2	1	5	0.048	0.073	0.578	0.612	0.634	−0.062
1	5	5	0.134	0.182	0.515	0.681	0.731	−0.271
2	10	5	0.146	0.200	0.470	0.708	0.768	−0.501
0.5	3	6	0.159	0.186	0.527	0.699	0.738	−0.190
1	6	6	0.196	0.223	0.480	0.744	0.789	−0.332
3	20	6.667	0.206	0.228	0.382	0.813	0.860	−0.894
0.1	1	10	0.126	0.135	0.564	0.683	0.700	−0.091
1	10	10	0.298	0.304	0.432	0.862	0.885	−0.435
10	100	10	0.174	0.184	0.256	0.893	0.929	−3.066
2	100	50	0.350	0.350	0.366	0.980	0.984	−0.842
2	1000	500	0.364	0.364	0.366	0.998	0.998	−0.864
0.1	100	1000	0.482	0.482	0.488	0.994	0.994	−0.276

§3. Remarks (略)

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