2000年度日本オペレーションズ・リサーチ学会
秋季研究発表会

An Approach to Coalition Formation in Cooperative Decision Situations

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Abstract

As an approach to examine coalition formation in a group, we propose a way to transform the expression of cooperation in the group with games in characteristic function form into a model with which not only cooperation but also competition in the group can be analyzed. By using the way, once we have a game in characteristic function form with the set of all possible coalition structures and the rules of benefit allocation in possible coalitions, we can transform it into the model that can be analyzed in the framework of conflict analysis.

1 Introduction

In this paper, we deal with coalition formation in a group. Usually, cooperative relation among the members of a group is seen as a cooperative decision situation. There is, however, not only an aspect of cooperation, but also that of competition, in the behavior of members of a group. In particular, the coalition formation in a group is related to both of these aspects. In order to examine the coalition formation appropriately, therefore, it is required to treat both of the cooperation and the competition in a group at the same time.

The procedure proposed in this paper for the analysis of coalition formation in a group are base on the framework of cooperative games [1] and that of conflict analysis [2]. Cooperative relation among the members of a group is often expressed by a game in characteristic function form. Thus, we think of the procedure of analysis that starts with the expression of cooperation in a group within the framework of cooperative games. In the framework of cooperative games, however, it is difficult to treat satisfactory both of the cooperation and competition. So, in the procedure, we transform the expression within the framework of cooperative games into another framework, in which the analysis of the aspect of competition is possible. In this paper, we adopt the framework of conflict analysis to treat the competitive aspect. Since it is possible in the framework to treat competitive behavior of decision makers in cooperative decision situations, we can deal with cooperative and competitive behavior at the same time, and we can specify the coalitions that are likely to be formed as a consequence of competitive behavior of decision makers in cooperative decision situations.

the way to transform gives a way to specify the elements required to identify the decision situation in the framework of conflict analysis, that is, the set of decision makers, the set of possible options, the set of possible strategies, the set of possible outcomes, and the preferences of the decision makers, beginning with the elements to describe a cooperative game with the set of all possible coalition structures, that is, the set of decision makers, the characteristic function, the set of possible coalitions, and the set of possible coalition structures. Moreover, we need a rule of benefit allocation in possible coalitions for the transformation, in particular, for specifying the preferences of the decision makers for the possible outcomes in the framework of conflict analysis. In this paper, we adopt the Shapley value and the concept of nucleolus as the rules of benefit allocation, because both of them satisfy several normatively desirable properties and always uniquely exists for any game with any coalition structure. The preferences of decision makers are defined reflecting the benefit allocated to decision maker in the coalition to which the decision maker belongs.

The way to transform the cooperative games into the models that can be analyzed in the framework of conflict analysis is defined so as to treat many person situations, that is, n-person situations, but the examples examined in this paper are of the decision situations that three decision

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makers are involved. In spite of the simplicity of the example, some great insights about coalition formation can be obtained. One of the most important properties is that it is not always true in a cooperative game that the grand coalition, that is, the coalition that all of the decision makers participate, is formed. In the case that the example in this paper shows, several coalition can be formed in a group in the consequence of the cooperation and the competition among decision makers.

In the next section, we briefly see the framework of cooperative games and that of conflict analysis. Subsequently, we give the explanation of the procedure of the transformation, followed by the examinations of examples.

2 Models

In this section, we see the concepts used in the framework of cooperative games and that of conflict analysis. The expression of cooperation in a group with games in characteristic function form is (N, v, C, B), and the expression of a decision situation in the framework of conflict analysis is (N, O, T, U, P).

2.1 Cooperative Games [1]

In the framework of cooperative games, a coalition that can be formed in a group and the benefit that the coalition can make are respectively expressed by a subset S of the set N of all decision makers and the value $v(S)$ of a characteristic function v. N, S, v satisfy that

- N is a finite set, and
- v is a function that corresponds a real value $v(S)$ to each subset S of N, where $v(\emptyset) = 0$.

It is required for a coalition to be formed that the decision makers who would like to belong to the coalition have to agree. It is not always true in some reasons, however, that any coalition can be formed if the members agree. There can be political, time, and monetary restrictions. Moreover, there can be some coalitions that cannot be consistent with each other by the interactions among them. In order to concentrate only on the coalitions that are possible to be formed, we use the concept of coalition formations. The set of all coalitions that can be formed is denoted by C, where

- C is a set of non-empty subsets of N, and
- for any $i \in N$, $\{i\} \in C$.

A coalition structure is a subset of C that forms a partition of N. A coalition structure expresses a set of all coalitions that exist in a group at the same time. The set of all feasible coalition structures are denoted by B, where

- for any $S \in C$, there exists a $\beta \in B$ such that $S \in \beta$, and
- for any $\beta = \{S_1, S_2, \dots, S_m\} \in B$, and any $j = 1, 2, \dots, m$,

  $$\{S_1, S_2, \dots, S_{j-1}, (\{i\})_{i \in S_j}, S_{j+1}, \dots, S_m\}$$

  is also an element of B.

The former condition expresses that for any possible coalition, there exists a coalition structure that the coalition is realized, and the latter condition means that for any coalition, it is possible to dissolve the coalition.

In the framework of cooperative games, a decision situation in which a group is involved is expressed by a 4-tuple (N, v, C, B):

Definition 1 (Cooperative Games)
A cooperative game is a 4-tuple (N, v, C, B). □

2.2 Conflict Analysis [2]

In the framework of conflict analysis, the situations are treated, the situations that each decision maker has some strategies as the alternatives of actions, and the benefit of the decision makers depends on the selection of strategy by each decision maker.

The set of all decision makers are denoted by N, and a strategy that each decision maker can utilize is expressed by a combination of options. An option is an element of behavior of a decision maker. For each option, a decision maker can select it, or not. Since a strategy is a combination of options, if the set of all options for decision maker $i \in N$ is O_i, then a strategy of this decision maker is a subset of O_i. Because it is not always true that all possible combinations of options are available, the set of all strategies of decision maker i is a subset of 2^{O_i}, and it is denoted by T_i.

The benefit of the decision makers depends on the selection of strategy by each decision maker. We call a combination of selections of strategies by each decision makers, an outcome. Again, all combinations of selections of strategies by each decision makers are not always feasible, the set of all possible outcomes is a subset of $\prod_{i \in N} T_i$, denoted by U. An outcome $u = (u_i)_{i \in N} \in U$ is often expressed, with some subset S of N, by (u_S, u_{-S}), where $u_S = (u_i)_{i \in S}$ and $u_{-S} = (u_j)_{j \in N \setminus S}$. The set of all u_S, and the set of all u_{-S} are respectively denoted by U_S and U_{-S}.

For any $i \in N$, the benefit of decision maker i in outcome $u \in U$ is expressed by a benefit function P_i. P_i is a function from U to the set of all real numbers, and for any $u \in U$, $P_i(u)$ means decision maker i's benefit in outcome u.

$(O_i)_{i \in N}$, $(T_i)_{i \in N}$, $(P_i)_{i \in N}$ are denoted by O, T, P, respectively, and 5-tuple (N, O, T, U, P) gives an expression of a decision situation.

Definition 2 (Conflict Situations)
A conflict situation is a 5-tuple (N, O, T, U, P). □

3 Transformation

The way of transformation of a cooperative game, that is, (N, v, C, B), into a conflict situation, that is, (N, O, T, U, P), is proposed in this section. Consider a cooperative game (N, v, C, B) is given.

First, we need to specify the set N in the conflict situation. Here, we simply use the set N in the cooperative game.

Next, it is required to specify the set O_i of options of decision maker i for any $i \in N$. By using the set C of all coalitions in the cooperative game, we define O_i as the set of all possible coalitions that the decision maker i can belong.

For the set T_i of all strategies of decision maker $i \in N$, we assign the set of all possible combinations of the options O_i of decision makers. In this case, since we assumed that each decision maker can belong just one coalition at the same time, O_i and T_i can be identified.

Moreover, the set U is the possible combination of the strategies for each decision makers. Since it is assumed that only the possible coalition structures, that is, the elements β of B, can be realized, U can be identified with a subset of B.

Determining the benefit function P_i of decision maker $i \in N$, we use the Shapley value and the concept of nucleolus. In both cases, given a combination of selections of strategies by each decision maker, we can specify the benefit of each decision maker.

After the transformation, we analyze the coalition formation in a group by using the framework of conflict analysis [2].

4 Example

As an example, we examine the following cooperative game:

- $N = \{1, 2, 3\}$
- v:

$$v(\emptyset) = v(\{1\}) = v(\{2\}) = v(\{3\}) = 0,$$

$$v(\{1,2\}) = 0.2, v(\{2,3\}) = 0.8,$$

$$v(\{3,1\}) = 0.9, v(\{1,2,3\}) = 1$$

- $C = \{\{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{3,1\}, \{1,2,3\}\}$
- $B = \{\{\{1\}, \{2\}, \{3\}\}, \{\{1,2\}, \{3\}\}, \{\{1\}, \{2,3\}\}, \{\{2\}, \{1,3\}\}, \{\{1,2,3\}\}\}$

This game consists of three decision makers, and any coalition and any coalition structures are possible. By using the way of transformation given in the preceding section with the concept of Shapley value, we have the following conflict situation.

- $N = \{1, 2, 3\}$
- $O = (O_i)_{i \in N}$:
  - $O_1 = \{\{1\}, \{1,2\}, \{1,3\}, \{1,2,3\}\}$
  - $O_2 = \{\{2\}, \{1,2\}, \{2,3\}, \{1,2,3\}\}$
  - $O_3 = \{\{3\}, \{2,3\}, \{3,1\}, \{1,2,3\}\}$
- $T = (T_i)_{i \in N}$:
  - $T_1 = \{\{\{1\}\}, \{\{1,2\}\}, \{\{1,3\}\}, \{\{1,2,3\}\}\}$
  - $T_2 = \{\{\{2\}\}, \{\{1,2\}\}, \{\{2,3\}\}, \{\{1,2,3\}\}\}$

– $T_3 = \{\{\{3\}\}, \{\{2,3\}\}, \{\{3,1\}\}, \{\{1,2,3\}\}\}$

- $U = \prod_{i \in N} T_i$:
- $P = (P_i)_{i \in N}$:

For any $i \in N$ and $u \in U$, $P_i(u)$ is defined by the following equation:

$$P_i(u) = \varphi_i(v, g \circ f(u)),$$

where $\varphi_i(v, \beta)$ is the Shapley value assigned to decision maker $i \in N$ under the coalition structure β, and the function f is defined for any $u \in U$ by $f(u) = u' = (t'_i)_{i \in N} \in U$, where for any $i \in N$,

$$t'_i = \begin{cases} \{\{i\}\} & \text{if } \exists j \in O_i \in t_i \text{s.t.} t_j \neq t_i \\ t_i & \text{if } \forall j \in O_i \in t_i, t_j = t_i, \end{cases}$$

and the function g is an identification mapping from U to B.

For example, if

$$u = (\{\{1,2\}\}, \{\{2,3\}\}, \{\{2,3\}\}),$$

then

$$f(u) = (\{\{1\}\}, \{\{2,3\}\}, \{\{2,3\}\})$$

and

$$g \circ f(u) = \{\{1\}, \{\{2,3\}\}\}.$$

By using these definitions, we have the values of benefit function of each decision maker and each possible outcome as follows.

Table: Values of Benefit Function

$U \backslash N$	1	2	3
$\{\{1\},\{2\},\{3\}\}$	0	0	0
$\{\{1\},\{2,3\}\}$	0	0.213	0.587
$\{\{2\},\{3,1\}\}$	0.281	0	0.619
$\{\{1,2\},\{3\}\}$	0.111	0.089	0
$\{\{1,2,3\}\}$	0.25	0.2	0.55

5 Conclusion

By examining the situation given in the preceding section by using the analysis procedure in the framework of conflict analysis, we have that the coalition structure $\{\{2\}, \{3,1\}\}$ is stable. Therefore, we see that in this case the coalition $\{3,1\}$ seems to be formed in this group. This example showed that it is not always true in a cooperative game that the grand coalition, that is, the coalition that all of the decision makers participate, is formed, and the procedure proposed in this paper is useful to analyze coalition formation in a cooperative decision situation.

There are some topics that are related to the contents of this paper, and should be investigated in the future researches. One of them is about the computational complexity that would be involved in the procedure of analysis for coalition formation given in this paper when the number of decision makers increase. Another can be the arbitrarity of the determination of the benefit function. Moreover, we should investigate the issue of change of coalitions in a group depending on the changes of the environment of the group.

References

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