2000年度日本オペレーションズ・リサーチ学会
秋季研究発表会

A Non-parametric Hard Limiter Neural Network For Convex Quadratic Programming

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June 26, 2000

Abstract

Quadratic programming (QP) problems have fundamental importance in the theory and practice of optimization. Especially Convex QP problems play an important role in the algorithm design of nonlinear programming.

Since Hopfield and Tank initiated the application of neural network (NN) in the field of optimization, a variety of feedback continuous NN models similar to the Hopfield model have been proposed to solve QP problems.

In this paper, we discuss a new non-parametric hard limiter neural network. It is also a feedback continuous neural network and can be used to solve convex QP problems.

Consider a QP problem with inequality constraints:

$$\begin{array}{ll} \text{minimize} & f(\boldsymbol{x}) = \frac{1}{2}\boldsymbol{x}^T\boldsymbol{Q}\boldsymbol{x} + \boldsymbol{q}^T\boldsymbol{x} \\ \text{subject to} & \boldsymbol{A}\boldsymbol{x} - \boldsymbol{b} \leq 0, \end{array} \tag{1}$$

where $\boldsymbol{x} \in \mathbb{R}^n$, $\boldsymbol{A}$ is an $m \times n$ matrix with $\text{rank}(\boldsymbol{A}) = m$. The corresponding Lagrangian function of (1) is as follows:

$$\mathcal{L}(\boldsymbol{x}, \boldsymbol{\lambda}) = f(\boldsymbol{x}) + \boldsymbol{\lambda}^T(\boldsymbol{A}\boldsymbol{x} - \boldsymbol{b}), \tag{2}$$

where $\boldsymbol{\lambda} \in \mathbb{R}^m$, $\boldsymbol{\lambda} \geq 0$, is the Lagrange multiplier vector. From the nonlinear programming theory, a local minimum for (1) is a stationary point of $\mathcal{L}(\boldsymbol{x}, \boldsymbol{\lambda})$ with respect to $\boldsymbol{x}$ and with other conditions. That is, let $(\boldsymbol{x}^*, \boldsymbol{\lambda}^*)$ be a local minimum. It satisfies,

$$\nabla_{\boldsymbol{x}}\mathcal{L}(\boldsymbol{x}^*, \boldsymbol{\lambda}^*) = \boldsymbol{Q}\boldsymbol{x}^* + \boldsymbol{q} + \boldsymbol{A}^T\boldsymbol{x}^* = 0, \tag{3}$$

$$\nabla_{\boldsymbol{\lambda}}\mathcal{L}(\boldsymbol{x}^*, \boldsymbol{\lambda}^*) = \boldsymbol{A}\boldsymbol{x}^* - \boldsymbol{b} \leq 0, \tag{4}$$

$$\nabla_{\boldsymbol{\lambda}}\mathcal{L}(\boldsymbol{x}^*, \boldsymbol{\lambda}^*)^T\boldsymbol{\lambda}^* = \boldsymbol{\lambda}^{*T}(\boldsymbol{A}\boldsymbol{x}^* - \boldsymbol{b}) = 0, \tag{5}$$

$$\boldsymbol{\lambda}^* \geq 0. \tag{6}$$

From this basic equations and the methodology initiated by Zhang and Constantinides, we can design a hard limiter network as a QP solver. The dynamic equations are given below.

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= -\boldsymbol{Q}\boldsymbol{x} - \boldsymbol{q} - \boldsymbol{A}^T\boldsymbol{\lambda}, & (7)\\ \dot{\lambda}_i(t) &= [1 - \mathrm{sgn}^*(\boldsymbol{A}_i\boldsymbol{x} - b_i)\mathrm{sgn}^*(\lambda_i)](\boldsymbol{A}_i\boldsymbol{x} - b_i), \qquad i = 1, \cdots, m & (8) \end{aligned}$$

where $\mathrm{sgn}^*(\cdot)$ is defined as

$$\mathrm{sgn}^*(x) = \begin{cases} 0 & \text{for } x > 0 \\ -1 & \text{for } x \le 0 \end{cases} \tag{9}$$

which is referred to as *semi-signum function* or *quasi-hard limiter*. This system can be expressed in a shorter form:

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= -\boldsymbol{Q}\boldsymbol{x} - \boldsymbol{q} - \boldsymbol{A}^T\boldsymbol{\lambda}, & (10)\\ \dot{\boldsymbol{\lambda}}(t) &= \boldsymbol{B}(\boldsymbol{A}\boldsymbol{x} - \boldsymbol{b}), & (11) \end{aligned}$$

where

$$\boldsymbol{B} = \mathrm{diag}\{[1 - \mathrm{sgn}^*(\boldsymbol{A}_1\boldsymbol{x} - b_1)\mathrm{sgn}^*(\lambda_1)], \cdots, [1 - \mathrm{sgn}^*(\boldsymbol{A}_m\boldsymbol{x} - b_m)\mathrm{sgn}^*(\lambda_m)]\}. \tag{12}$$

A convergence discuss is presented in paper. We shown that the network is convergent if Q is positive. Furthermore, when the network converge to an equilibrium point, the equilibrium point is a K-T point.