A Sequential Internet Auction with Reserve

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Abstract

A given amount of items is auctioned by several sequential auctions one after another. The seller sets a quantity of items to be offered for each auction, while the buyers arrive according to a Poisson process. We consider this problem as a Markov decision process model and then show the monotone properties of the optimal policy. Finally, a numerical example is given.

1 Introduction

Researches on Internet auctions are rich. Based on eBay, Wilcox [1] focused on the impact of buyers' experience on their bidding behavior. Ockenfels and Roth [2] discussed several bidding strategies in Internet auctions, such as late bidding and incremental bidding, and pointed out that the late bidding is the best response in many environments.

Bean et al. [3] studied a sequential auction, where a seller holds a given amount of items and wants to sell them in the auctions sequentially. The buyers arrive according to a Poisson process and bid honestly. While the seller should determine the quantities of items to be offered for each auction. But they assumed that all the items offered to each auction will be auctioned off. So they formulate this problem as a finite horizon deterministic dynamic programming and compute its solution for an example based on eBay. Vulcano et al. [4] studied a similar problem but more general on the auction mechanism based on an assumption that the number of items auctioned at each auction will be determined after all buyers' bids. Vulcano et al. [5] studied the same problem for infinite horizon case.

In this paper, we study the same problem as that in [3] without the assumption that all the items offered to each auction will be auctioned off. The problem is treated by Markov decision processes, and the monotone properties of the optimal policy are shown.

2 Auction Model

The seller intends to sell K items by W auctions within T days. For simplicity, we assume that the length of each auction period is same, and so the duration for each auction is $t_0 = T/W$.

Suppose that at the beginning of the auction, a holding cost with per item per time period being h is incurred, while the quantity s_n offered to the n-th auction is determined by the seller. The problem for the seller is to choose $\{s_n^*\}$ to maximize his total expected revenue. We make the following assumptions:

(1) The buyers arrive according to a Poisson Process with rate λ.

(2) The buyers are all risk-neutral.

(3) The buyers are symmetric, and their bids are independent and identically distributed uniformly on the interval $[a, b]$, with the distribution function $F(x)$.

(4) Each buyer purchases at most one item.

(5) The seller has a reserve v for each item.

3 The Markov Decision Process Model

The Markov decision process (MDP) model is as follows. The state is the number of remained items at the beginning of the auction, while the action is the quantity of items offered to this auction. Let $\delta = \lambda t_0/(b-a)$ and

$$\begin{aligned} q_k &= e^{-\delta(b-v)}\frac{[\delta(b-v)]^k}{k!}, \ k = 0, 1, 2\cdots, \\ r(s) &= sb - \frac{s(s+1)}{2\delta} \\ &\quad + \sum_{k=0}^{s-1}\left(\frac{(s+1-k)}{2\delta} - v\right)(s-k)q_k \quad (1) \end{aligned}$$

where q_k is the probability that there are k buyers arrived in an auction duration, whose bids are not less than the reserve of the seller.

Then the state transition probability and the reward function are respectively,

$$
\begin{aligned}
p_{ij}(s) &= \begin{cases} q_{i-j}, & i-s<j\le i \\ \sum_{k=s}^{\infty} q_k, & j=i-s \\ 0, & \text{otherwise}, \end{cases} \\
r(i,s) &= r(s)-ih. \qquad (2)
\end{aligned}
$$

So the optimality equation is given by

$$
\begin{aligned}
V_n(i) &= \max_{s=0,1,\cdots,i} \{r(s)+\sum_{k=0}^{s-1} q_k V_{n-1}(i-k) \\
&\quad + \sum_{k=s}^{\infty} q_k V_{n-1}(i-s)\} - ih \qquad (3)
\end{aligned}
$$

with the boundary conditions:

$$V_0(i) = V_n(0)=0,\ \forall\, n,i. \qquad (4)$$

4 Monotone Properties

Let

$$
\begin{aligned}
V_n(i,s) &= r(s)-ih+\sum_{k=0}^{s-1} q_k V_{n-1}(i-k) \\
&\quad + \sum_{k=s}^{\infty} q_k V_{n-1}(i-s), \\
s_n^*(i) &= \max\{s \mid V_n(i,s)-V_n(i,s-1)\ge 0, \\
&\qquad s=1,2,\cdots,i\}. \qquad (5)
\end{aligned}
$$

Lemma 1 *$r(s)$ is increasing and concave.*

Lemma 2 *$V_n(i,\ s)$ is concave in s if $V_{n-1}(i)$ is concave in i.*

Lemma 3 *If $V_{n-1}(i)$ is concave in i, then $V_n(i,\ s)$ is supermodular in (i,s) and so $s_n^*(i)$ is increasing in i.*

Theorem *$V_n(i)$ is concave in i for each $n\ge 0$, and so $s_n^*(i)$ is increasing in i for each $n\ge 0$. $s_n^*(i)$ is the optimal quantity of items offered to the n-th auction when there are i items remained.*

The increasingness of $s_n^*(i)$ in i means that the more the items remained, the more the items auctioned in each auction.

5 A Numerical Example

Using the data in Bean et al. [3], we compute the optimality equation. The data in [3] includes five consecutive auctions of Item 2050, the CD Receiver and these parameters are estimated.

These are as follows: (1) The seller opens at most 5 auctions for each shipment, and the length of each auction period is 3.5 days. Initially, the total amount of the items is 35. That is, $W = 5, t_0 = 3.5, K = 35$. (2) The holding cost per item per auction period is 0.13, i.e., $h = 0.13$. (3) The arrival rate of buyers is $\lambda = 13.6$. (4) The bids are distributed uniformly on $[75,150]$, so $a = 75, b = 150$. (5) The reserve price is zero, i.e., $v = 0$.

The maximal total revenue is $V_5(35) = 5016.67$, while the maximal total revenue is $V_5(35) = 4454.35$ obtained in [3]. So it is not reasonable to assume in [3] that all the items can be auctioned at each auction. The optimal policy $s_n^*(i)$ can be computed, e.g., when $n = 5$,

$$
\begin{aligned}
s_5^*(i) &= 1, \quad i=1,\cdots,5, \\
s_5^*(i) &= 2, \quad i=6,\cdots,10, \\
s_5^*(i) &= 3, \quad i=11,\cdots,15, \\
s_5^*(i) &= 4, \quad i=16,\cdots,20, \\
s_5^*(i) &= 5, \quad i=21,\cdots,25, \\
s_5^*(i) &= 6, \quad i=26,\cdots,30, \\
s_5^*(i) &= 7, \quad i=31,\cdots,35.
\end{aligned}
$$

References

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