Newsboy Problem with Pricing Policy

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1 Objective

The objective of the paper is to find the optimal decision rule to maximize the total expected present discounted net profit over the planning horizon, i.e., the total expected present discounted revenue *minus* the total expected present discounted costs paid to purchase items at the start of the process *minus* the total expected present discounted holding cost *plus* the total expected present discounted salvage value.

2 Model

Consider the following discrete-time sequential stochastic decision problem of purchasing a certain quantity of items at a certain point in time and then selling them at certain points in time that follows. The points in time are numbered backward from the final point in time of the planning horizon, time 0. In the model we assume that a buyer who requests an item appears with a probability λ $(0 < \lambda < 1)$ and then the seller offers a selling price z to the buyer. Let us denote the reservation price of a buyer by w, implying that the buyer is willing to buy an item if and only if the selling price offered for the item by the seller is lower than or equal to w, i.e., $z \le w$. Here, assume that subsequent buyers' reservation price $w,\ w', \cdots$ are i.i.d. random variables having a known continuous distribution function $F(w)$ with a finite expectation μ. Also, let $f(w)$ denote its probability density function, which is truncated on both sides. In addition, the probability of an arriving buyer buying the item, provided that a price z is offered by the seller, is given by $p(z) = \Pr\{z \le w\}$, where $0 \le p(z) \le 1$.

Furthermore, by β $(0 < \beta \le 1)$ let us denote the discount factor, by $c > 0$ the purchasing price per item, by $h \ge 0$ the inventory holding cost per item remaining unsold for a period, and by ρ the salvage price that an item remaining unsold at time 0 can be sold. Here, $\rho < 0$ implies the disposal cost per item to discard the unsold item. The decision rule of the model consists of the following two rules:

1. *Ordering rule* prescribing how many items to be ordered at the time when the process starts.
2. *Pricing rule* prescribing what price to offer to a buyer at each point in time.

3 Related Functions

For any real number x define $T(x) = \max_z p(z)(z-x)$. By $z(x)$ let us designate the z attaining the maximum of the right hand side of the above equation if it exists. For any x define

$$K(x) = \lambda\beta T(x) - (1-\beta)x, \qquad N(x) = K(x) + x - c - h,$$

$$\nu(x) = (1-\beta)x + h, \qquad G_t(\rho) = \beta^t\rho - h\sum_{k=0}^{t-1}\beta^k - c, \quad t \ge 1.$$

By x_1^*, $x^\diamond$, $x^\bullet$, and ρ_t^* let us denote the solutions of $K(x) = h$, $N(x) = 0$, $\nu(x) = 0$, and $G_t(\rho) = 0$ if they exist. Hence, we get $x^\bullet = -h/(1-\beta)$ for $\beta < 1$ and $\rho_t^* = (h\sum_{k=0}^{t-1}\beta^k + c)/\beta^t$ for $t \ge 1$.

4 Optimal Equation

Suppose that a certain amount of a product had been purchased at a certain past point in time and that i items remain unsold at a time t after that. Let $u_t(i,\phi)$ and $u_t(i,1)$ be the maximum total expected present discounted profits, respectively, with no buyer and with a buyer. Then, clearly $u_0(i,\phi) = \rho i$ for $i \ge 0$, and

$$\begin{aligned} u_t(i,\phi) &= \beta\big(\lambda u_{t-1}(i,1) + (1-\lambda)u_{t-1}(i,\phi)\big) - hi, \qquad t \ge 1, \quad i \ge 0, \\ u_t(i,1) &= \max_z \big\{p(z)\big(z + u_t(i-1,\phi)\big) + (1-p(z))u_t(i,\phi)\big\} = T(U_t(i)) + u_t(i,\phi), \quad t \ge 0, \quad i \ge 1. \end{aligned}$$

where $U_t(i) = u_t(i,\phi) - u_t(i-1,\phi)$ for $t \ge 0$ and $i \ge 1$. Then the *optimal selling price* of time $t \ge 0$ with $i \ge 1$ items remaining unsold is given by the z attaining the maximum in the definition of the function $T(U_t(i))$, denoted by $z_t(i)$.

Next, by $v_t(i)$ let us define the maximum of the total expected present discounted net profit, provided that i items are ordered at time t with no buyer and that they are sold at the optimal selling price at each point in time up to time 0, the deadline. Here, we assume there is no lead time between ordering and receiving a product. Then we have $v_t(i) = u_t(i, \phi) - ci$ for $t \geq 0$ and $i \geq 0$. Hence the *optimal ordering quantity* when the process starts from time t is given by the smallest i maximizing $v_t(i)$ on $i \geq 0$ if it exists, denoted by i_t^*, that is, $v_t(i_t^*) = \max_{i \geq 0} v_t(i)$.

5 Summary of Decision Rules

The optimal decision rule can be summarized as follows.

A. Optimal selling price

1. $z_t(i)$ is nonincreasing in $i \geq 1$ for all $t \geq 0$.
2. Monotonicity of $z_t(i)$ in t can be summarized as in Table 5.1.

Table 5.1: Conditions for the monotonicity of $z_t(i)$

Case	$\beta = 1$	$\beta < 1$	$z_t(i)$
C1	$h = 0$	$\rho \leq x^\bullet$	$z_t(i)$ is nondecreasing in $t \geq 0$ for $i \geq 0$.
C2	$h > 0$, $x_1^* > \rho$	$x^\bullet < \rho < x_1^*$	$z_t(1)$ is nondecreasing in $t \geq 0$. $z_t(i)$ does not oscillate in $t \geq 0$ for $i \geq 2$.
C3	$h > 0$, $x_1^* \leq \rho$	$x_1^* \leq \rho$	$z_t(i)$ is nonincreasing in $t \geq 0$ for $i \geq 0$.

B. Optimal ordering quantity

Here, if $1 \leq i_t^* \leq t$, let a business deal to order a product be said to be *viable* (V), if $i_t^* = 0$, *nonviable* (N), or if $i_t^* = \infty$, *unpractical* (U). Then, the optimal ordering quantity i_t^* for $t > 0$ can be prescribed as follows:

Table 5.2: Optimal ordering quantity

$(1-\beta)^2 + h^2 = 0$		Q1 — $x^\diamond$ — Q3 — c — Q5 — ρ
$(1-\beta)^2 + h^2 \neq 0$	$x_1^* > c$	Q1 — $x^\diamond$ — Q3 (c) — ρ_t^* — Q5 — ρ
	$x_1^* = c$	Q6 — $x^\diamond = c$ — Q3 — ρ_t^* — Q5 — ρ
	$x_1^* < c$	Q6 (c) — $x^\diamond$ — Q2 — ρ_t^* — Q4 — ρ

Q1: 1 — N: $i_t^* = 0$ — t^* — V: $1 \leq i_t^* \leq t$ — t

Q2: 1 — V: $1 \leq i_t^* \leq t$ — t^* — N: $i_t^* = 0$ — t

Q3: 1 — V: $1 \leq i_t^* \leq t$ — t

Q4: 1 — U: $i_t^* = \infty$ — t^* — N: $i_t^* = 0$ — t

Q5: 1 — U: $i_t^* = \infty$ — t

Q6: 1 — N: $i_t^* = 0$ — t

6 Conclusions

1. The monotonicity in t for the optimal selling price $z_t(i)$ is partially against our intuitive conjecture.
2. All the model's parameters interrelate in determining the viability and practicality of a deal to order a product.
3. The length of the planning horizon also plays an important role in determining whether a deal is viable, unpractical or nonviable.

References

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