A Decomposition of Cost Efficiency and its Application to US-Japan Electric Utilities Comparisons

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1. Introduction

Technology and cost are the pair of wheels which drive modern enterprises. Some have advantages in technology and others in cost. So, management is eager to know how and to what extent their resources are being effectively and efficiently utilized, compared with other similar enterprises in the same field. Under multiple input-output correspondences, data envelopment analysis (DEA) has created a new route map for this purpose.

Given the quantities of input resources and output products, the representative DEA models can evaluate the relative technical efficiency of the concerned enterprise, called DMU in the DEA terminology. If, furthermore, unit prices of input resources are known, the cost efficiency model can be utilized to explore the optimal input-mix that produces the observed outputs at the minimum cost. Based on this optimal solution, the cost and allocative efficiencies are obtained.

However, these traditional cost and allocative efficiencies, which assume given uniform input prices, suffer from a critical shortcoming, as pointed out by Tone (2002), if the unit prices of inputs are not identical among DMUs in the actual economy. To cite a case, if two DMUs have the same amount of inputs and outputs, and the unit price for one DMU is twice that of the others, then the traditional cost efficiency model assigns the same cost and allocative efficiencies. This sounds very strange and impractical in actual economic activity. After pointing out this shortcoming, Tone proposed a new scheme that is free from such inconsistencies.

This paper can be positioned as an extension of Tone (2002). We decompose actual total cost into global optimal cost and losses due to technical inefficiency, input price differences and inefficient cost mixture. Technical efficiency is obtained using the traditional CCR model within the supposed technical production possibility set. Then, using the optimal input value thus obtained, we construct a cost-based production possibility set and solve the New Tech and New Cost models on this set. Then we can obtain two efficiency indices, i.e., the price efficiency and the global allocative efficiency index.

2. Methodology

In this section we develop our scheme and discuss its rationality.

2.1 Data

Throughout this paper, we deal with n DMUs, each having m inputs for producing s outputs. For each DMU$_o$ ($o=1,\cdots,n$), we denote respectively the input/output vectors by $x_o \in R^m$ and $y_o \in R^s$. The input/output matrices are defined by $X=(x_1,\cdots,x_n)\in R^{m\times n}$ and $Y=(y_1,\cdots,y_n)\in R^{s\times n}$. For each DMU$_o$ ($o=1,\cdots,n$), the input factor price vector for input x_o is denoted by $w_o \in R^m$ and the input factor price matrix is defined by $W=(w_1,\cdots,w_n)\in R^{m\times n}$. For DMU$_o$, the actual total input cost C_o is calculated by

$$C_o = \sum_{i=1}^{m} w_{io}x_{io}. \tag{1}$$

We assume that the elements $w_{1o}x_{1o},\cdots,w_{mo}x_{mo}$ are denominated in homogenous units, viz., dollars, so that adding up them has a meaning.

2.2 Technical Efficiency

The production possibility set P is defined as

$$P=\{(x,y)\mid x\ge X\lambda,\ \ y\le Y\lambda,\ \ \lambda\ge 0\}. \tag{2}$$

The *technical efficiency* θ^* of DMU$_o$ is measured in the traditional CCR Model. The projection is given by

$$[CCR-projection]\ \ x_o^*=\theta^* x_o,\ \ y_o^*=y_o. \tag{3}$$

x_o^* indicates the amount of technical efficient inputs for DMU$_o$ and cannot be radially reduced any more by eliminating input inefficiency for producing y_o.

The technically efficient total input cost for DMU$_o$ is calculated by

$$C_o^* = \sum_{i=1}^{m} w_{io}x_{io}^* = \theta^*\sum_{i=1}^{m} w_{io}x_{io} = \theta^* C_o, \tag{4}$$

and the technical efficiency θ^* can be redefined as

$$\theta^* = \frac{C_o^*}{C_o} = \frac{w_o x_o^*}{w_o x_o}. \tag{5}$$

The loss due to this technical inefficiency is described as follows,

$$L_o^* = C_o - C_o^*(\geq 0)\,. \qquad (6)$$

2.3 Price Efficiency

The above (x_o^*, y_o) is the production pair in the production possibility set P, and its input cost wx_o^* cannot be reduced any more through reducing the input x_o radially. However, the cost can be reduced through changing the input mixture, or eliminating the differences of input prices under the situation that unit prices may differ from DMU to DMU.

In order to fathom the minimum cost, we observe the cost-based production possibility set P_c in the spirit of Tone (2002) as follows:

$$P_c = \{(\bar{x}, y) \mid \bar{x} \geq \bar{X}\mu,\ \ y \leq Y\mu,\ \ \mu \geq 0\}\,, \qquad (7)$$

where $\bar{X} = (\bar{x}_1, \cdots, \bar{x}_n) \in R^{m \times n}$, $\bar{x}_o = (\bar{x}_{1o}, \cdots, \bar{x}_{mo})$ and $\bar{x}_{io} = w_{io}x_{io}^*$. Notice that x_o^* represents the technically efficient input for producing y_o and hence we utilize $w_{io}x_{io}^*$ instead of $w_{io}x_{io}$ so as to eliminate the technical inefficiency at the maximum. Then we solve the CCR model on P_c corresponding to the [NTec] in Tone (2002) as follows,

$$\begin{aligned} &[\text{NTec}] && \rho^* = \min \rho \\ &\text{subject to} && \rho\bar{x}_o \geq \bar{X}\mu \\ & && y_o \leq Y\mu \\ & && \mu \geq 0. \end{aligned} \qquad (8)$$

Because $\bar{x}_o = w_o x_o^*$ cannot be reduced by cutting down the amount of input x_o^* any more, the optimal solution ρ^* radially reduces input price w_o to the supposed optimal input price, say w_o^*, mathematically. In the context of economics, ρ^* might indicate the radial difference of input price under the imperfect market compared to the least input price in the same cost mixture. In this study, this input price difference is labeled as *price efficiency*.

The NTec projection is given by

$$[NTec - projection]\ \ \bar{x}_o^* = \rho^*\bar{x}_o,\ \ y_o^* = y_o.$$

We define the radial efficient cost C_o^{**}, which is the technical and price efficient, and the loss L_o^{**} due to the difference of the input price as

$$C_o^{**} = \textstyle\sum_{i=1}^{m} \bar{x}_{io}^* = \rho^* \sum_{i=1}^{m} \bar{x}_{io} = \rho^* C_o^* \leq C_o^*\,, \qquad (9)$$

$$L_o^{**} = C_o^* - C_o^{**}(\geq 0). \qquad (10)$$

2.4 Allocative Efficiency

Furthermore we solve NCost model on P_c as

$$\begin{aligned} &[\text{NCost}] && C^{***} = \min e\bar{x} \\ &\text{subject to} && e\bar{x} \geq e\bar{X}\mu \\ & && y_o \leq Y\mu \\ & && \mu \geq 0, \end{aligned} \qquad (11)$$

where $e \in R^m$ is a row vector with all elements equal to 1. Let an optimal solution be $(\bar{x}_o^{**}, \mu^*)$. Then the cost-based pair $(\bar{x}_o^{**}, y_o)$ is the cost minimum production in the supposed production possibility set P_c, which substantially differs from P if the unit prices of inputs vary from DMU to DMU. The (global) allocative efficiency α^* of DMU$_o$ is defined by

$$\alpha^* = \frac{C_o^{***}}{C_o^{**}}(\leq 1)\,. \qquad (12)$$

Compared to the traditional (local) allocative efficiency, which indicates the adjustment of the input mixture based on the given input price ratio, α^* represents the optimal cost mixture, viz., the combination of the optimal input amount and input price mixture. We also define the loss L_o^{***} due to the suboptimal cost mixture as

$$L_o^{***} = C_o^{**} - C_o^{***}(\geq 0). \qquad (13)$$

2.5 Decomposition of Actual Cost

Based on the relationship among the optimal cost and losses, we can decompose the actual total cost into:

$$C_o = L_o^* + L_o^{**} + L_o^{***} + C_o^{***}\,. \qquad (14)$$

If the DMU$_o$ is fully efficient, all losses are zero and we have $C_o = C_o^{***}$. The above decomposition indicates the effect of the existing technical inefficiency and price difference on the total cost.

3. Application to electric power companies

We applied this framework to a comparison of US-Japan electric utilities and clarified the differences in the inefficiency structure regarding technology and cost between the two countries. Our empirical results will be shown at the conference.

Reference

Tone, K. (2002) "A strange case of the cost and allocative efficiencies in DEA," Journal of the Operational Research Society, 53, 1225-1231.