A Markovian Availability Modeling for Software Systems

鳥取大学 *得能貢一 TOKUNO Koichi
01702425 鳥取大学 山田茂 YAMADA Shigeru

1 Introduction

We develop a software availability model incorporating software failure-occurrence and fault-correction times, under the assumption that the hazard rate for software-failure occurrence reduces geometrically with the progress in fault-removal process. Taking into consideration that the software system alternates the operational state that the system is operating and the maintenance state that the system is fixed, we model the time-dependent behavior of the system with a Markov process. Expressions for several quantities of software system performance are derived from this model. Finally, numerical examples are presented for illustration of software availability measurement.

2 Model Description

The following assumptions are made for software availability modeling:

1. The hazard rate is constant between software failures caused by faults in the software system, and geometrically decreases whenever each fault is corrected and removed (see Moranda [1]).
2. All faults that have caused software failures are corrected and removed, and no new faults are introduced during the correction activity.
3. The time to remove a fault follows an exponential distribution with parameter μ. The parameter μ is called the fault correction rate.

Now, we consider a stochastic process $\{X(t),\ t \geq 0\}$ with the state space $(\boldsymbol{W}, \boldsymbol{R})$ where up state vector $\boldsymbol{W}=\{W_n;\ n=0,\ 1,\ 2,\ \cdots\}$ and down state vector $\boldsymbol{R}=\{R_n;\ n=0,\ 1,\ 2,\ \cdots\}$. Then, the events $\{X(t)=W_n\}$ and $\{X(t)=R_n\}$ mean that the system is operating and fixed at time point t when n faults have been removed, respectively. From assumption 1, when n faults have been removed, the hazard rate for the next software failure occurrence is given by

$$z_n(t) = Dk^n \ (n=0,\ 1,\ 2,\ \cdots;\ D>0,\ 0<k<1), \tag{1}$$

where D and k are the initial hazard rate and the decreasing ratio, respectively. The sample state transition of $X(t)$ is illustrated in Figure 1.

3 Software Availability Analysis and Measurement

Letting $p_n(t)=\Pr\{X(t)=W_n\}$ and $q_n(t)=\Pr\{X(t)=R_n\}$ $(n=0,\ 1,\ 2,\ \cdots)$, we get the following difference-differential equations with respect to $p_n(t)$ and $q_n(t)$:

$$\left.\begin{array}{l} p_0'(t) = -Dp_0(t) \\ p_n'(t) = -Dk^n p_n(t) + \mu q_{n-1}(t) \ (n=1,2,\cdots) \\ q_n'(t) = -\mu q_n(t) + Dk^n p_n(t) \ (n=0,1,2,\cdots) \end{array}\right\}, \tag{2}$$

where the initial conditions are given as follows:

$$\left.\begin{array}{l} p_0(0)=1, \quad q_0(0)=0 \\ p_n(0)=q_n(0)=0 \qquad (n=1,\ 2,\ \cdots) \end{array}\right\}. \tag{3}$$

Solving (2) with respect to $p_n(t)$ and $q_n(t)$ under the initial conditions (3) yields

$$p_n(t) = \sum_{l=0}^{n} A_{n,l}^{p} e^{-Dk^l t} + \sum_{l=0}^{n-1} B_{n,l}^{p} t^l e^{-\mu t}, \tag{4}$$

$$q_n(t) = \sum_{l=0}^{n} (A_{n,l}^{q} e^{-Dk^l t} + B_{n,l}^{q} t^l e^{-\mu t}), \tag{5}$$

where we postulate $\sum_{l=0}^{-1} \cdot = 0$ in (4) for $n=0$. Constant coefficients $A_{n,l}^{p}$ and $A_{n,l}^{q}$ are given by

$$A_{n,l}^{p} = \frac{\mu^n k^{\frac{1}{2}n(n-1)}}{(\mu - Dk^l)^n \prod_{\substack{j=0 \\ j\neq l}}^{n} (k^j - k^l)}$$
$$(n=0,\ 1,\ 2,\ \cdots;\ l=0,\ 1,\ 2,\ \cdots,\ n), \tag{6}$$

$$A_{n,l}^{q} = \frac{\mu^n D k^{\frac{1}{2}n(n+1)}}{(\mu - Dk^l)^{n+1} \prod_{\substack{j=0 \\ j\neq l}}^{n} (k^j - k^l)}$$
$$(n=0,\ 1,\ 2,\ \cdots;\ l=0,\ 1,\ 2,\ \cdots,\ n), \tag{7}$$

respectively, where we postulate $\prod_{\substack{j=0 \\ j\neq 0}}^{0} \cdot = 1$ in (6) and (7) for $n=0$. Constant coefficients $B_{n,l}^{p}$ and $B_{n,l}^{q}$ can be calculated by using the following recursion formulas:

$$B_{n,n-1}^{p} = \frac{\mu^n D^n k^{\frac{1}{2}n(n-1)}}{(n-1)! \prod_{j=0}^{n} (Dk^j - \mu)} \quad (n=1,\ 2,\ \cdots), \tag{8}$$

$$B^p_{n,n-1-i} = \frac{-\sum_{r=0}^{i-1}\left[(n-1-r)!B^p_{n,n-1-r}\left\{\sum_{u=1}^{\binom{n+1}{i-r}}\prod_{j\in S^n_{i-r,u}}(Dk^j-\mu)\right\}\right]}{(n-1-i)!\prod_{j=0}^{n}(Dk^j-\mu)}$$

$$(n=2,\ 3,\ \cdots;\ i=1,\ 2,\ \cdots,\ n-1), \qquad (9)$$

$$B^q_{n,n} = \frac{\mu^n D^{n+1} k^{\frac{1}{2}n(n+1)}}{n!\prod_{j=0}^{n}(Dk^j-\mu)} \quad (n=0,\ 1,\ 2,\ \cdots), \qquad (10)$$

$$B^q_{n,n-i} = \frac{-\sum_{r=0}^{i-1}\left[(n-r)!B^q_{n,n-r}\left\{\sum_{u=1}^{\binom{n+1}{i-r}}\prod_{j\in S^n_{i-r,u}}(Dk^j-\mu)\right\}\right]}{(n-i)!\prod_{j=0}^{n}(Dk^j-\mu)}$$

$$(n=1,\ 2,\ \cdots;\ i=1,\ 2,\ \cdots,\ n), \qquad (11)$$

respectively. Let S^n and S^n_m (n=1, 2, $\cdots$; m=0, 1, 2, $\cdots$, n) denote a set of integers from 0 to n, i.e. $S^n=\{0, 1, 2, \cdots, n\}$, and a family of those S^n's subsets which contain (n+1−m) elements, respectively. We can represent a element of S^n_m as $S^n_{m,u}$ (u=1, 2, $\cdots$, $\binom{n+1}{m}$), where $\binom{n+1}{m}=(n+1)!/[(n+1-m)!m!]$ is the number of combination containing m elements which are extracted from S^n.

Provided that the initial fault content in the system prior to the testing, N, is known, the instantaneous software availability (see Shooman [2]) is defined as

$$A(t) \equiv \sum_{n=0}^{N} p_n(t). \qquad (12)$$

That is, $A(t)$ in (12) represents the probability that the software system is operating at specified time point t.

4 Numerical Examples

The instantaneous software availabilities, $A(t)$'s in (12) for various values of k representing the decreasing ratio in the hazard rate are shown in Figure 2, where N = 10, D = 0.1, and μ = 0.5. We can see that the probability that the software is operational becomes larger as k decreases. The instantaneous software availabilities $A(t)$'s for various fault correction rates, μ's, are shown in Figure 3 where N = 10, D = 0.1, and k = 0.8. The fault correction rate, μ, can be regarded as a maintainability index of a software. That is, the software system has high maintainability with increasing μ. Figure 3 shows that the increase in maintainability improves the software availability.

References

[1] Moranda, P. B., "Event-altered rate models for general reliability analysis", *IEEE Trans. Reliability*, vol. R-28, no. 5, pp. 376–381, Dec. 1979.

[2] Shooman, M. L., *"Software Engineering: Design, Reliability, and Management"*, McGraw-Hill, New York, 1983.

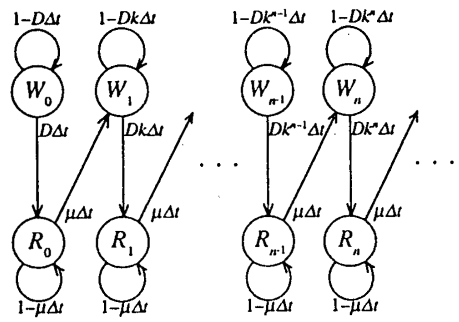

Figure 1: A diagrammatic representation of state transitions between $X(t)$'s.

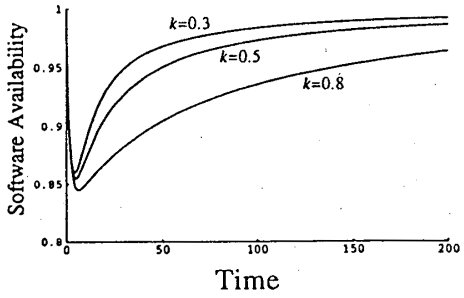

Figure 2: Dependence of decreasing ratio of the hazard rate, k, in $A(t)$ (N =10, D =0.1, μ =0.5).

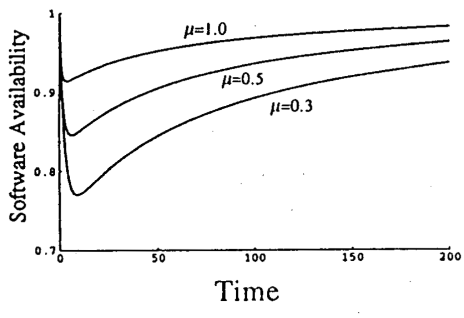

Figure 3: Dependence of fault correction rate μ in $A(t)$ (N =10, D =0.1, k =0.8).