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OPTIMAL STOPPING GAMES FOR BIVARIATE UNIFORM DISTRIBUTION

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Abstract We consider a class of two-person time-sequential games called optimal stopping games. Let (X_i, Y_i), $i=1, \cdots, n$, be an *iid* sequence of r.v.'s sampled from bivariate uniform distribution on [0, 1]. At each time $i=1,2, \cdots$, each of two players I and II is dealt with a hand X_i and Y_i, respectively. After looking at his hand *privately*, each player can then choose either to accept (A) his hand or to reject (R) it. If the players' choice pair is A–A, then the game ends with the predetermined payoffs to the players. If the choices are R–R, then the current sample is rejected and the game continues to facing a next sample (X_{i+1}, Y_{i+1}). If the choices are A–R(R–A)then a lottery is used to the effect that either A–A or R–R is enforced to the players with probability p_1 (p_2) and $\overline{p}_1$ $(\overline{p}_2)$, respectively, where $\overline{p}_i = 1-p_i$. Each player wants to maximize his expected payoff at the termination time of the game. We explicitly derive the solutions of (1) zero-sum game, where the terminal payoff to I is $X_\tau - Y_\tau$, (2) non-zero-sum game, where the terminal payoffs are $E(X_\tau)$-$E(Y_\tau)$, where τ is the time at which the game is stopped.

1. Introduction and Summary
2. Zero-Sum Sequential Game
3. Non-Zero-Sum Sequential Game

§3 のみを概説する.

$\{(X_i, Y_i)\}_{i=1}^{n}$ is *iid* with bivariate uniform with *pdf*

$$f(x, y) = 1 + \gamma(1-2x)(1-2y), \quad (x,y) \in [0,1]^2, \quad |\gamma| \leqq 1.$$

I(II) observes $X=x(Y=y)$ privately. If the choice-pair A–R[R–A] is chosen, then lottery (A–A, R–R; p_1, $\overline{p}_1$)[(A–A, R–R; p_2, $\overline{p}_2$)] is performed. OE is

$$(u_n, v_n) = \text{eq. pval} \begin{array}{|c|c|} \hline x, \quad y & p_1 x + \overline{p}_1 u_{n-1}, \ p_1 y + \overline{p}_1 v_{n-1} \\ \hline p_2 x + \overline{p}_2 u_{n-1}, \ p_2 y + \overline{p}_2 v_{n-1} & u_{n-1}, \ v_{n-1} \\ \hline \end{array}$$

$$(n \geqq 1; \quad u_0 = v_0 \equiv 0)$$

where

$$\text{eq. pval}\left[A^1(x,y), A^2(x,y)\right] = \underset{(\alpha(\cdot), \beta(\cdot))}{\text{eq. val}} \left[M^1(\alpha, \beta), M^2(\alpha, \beta)\right].$$

and

$$M^i(\alpha, \beta) \equiv E\left\{(\alpha(X), \overline{\alpha}(X)) A^i(X, Y) \begin{bmatrix} \beta(Y) \\ \overline{\beta}(Y) \end{bmatrix}\right\}, \qquad i = 1, 2.$$

Eq. strategy-pair at the first stage is to (accept X iff $X > u_{n-1}$)-(accept Y iff $Y > v_{n-1}$), where (u_{n-1}, v_{n-1}) is determined by a simultaneous recursion.

It is shown that in the special case of $p_1 = p_2 = p$, we obtain $u_n = v_n$ for all n, and if $\gamma = 0$ additionally, then u_n converges, as $n \to \infty$, to a unique root u_∞ in $[0, 1]$ of the equation $(2p-1)u^2 + (2-p)u - 1 = 0$. Moreover we obtain in case of $p = \gamma = 0$, a two-person non-zero-sum-game version of the well-known Moser's sequence of numbers $V_n = \frac{1}{2}(1 + V_{n-1}^2)$. That is,

$$u_n = u_{n-1} + \frac{1}{2}(1 - u_{n-1})^3, \quad (n \geqq 1 \ ; u_0 = 0)$$

Theorem 2. *For the non-zero-sum sequential game* $G(n, p_1, p_2)$ *over bivariate uniform distribution* (3.1) *with* $0 \leqq \gamma \leqq 1$*, the equilibrium values* (u_n, v_n) *satisfy the recurrence relation*

$$u_n = a + \frac{1}{2}\left\{(1 - \bar{p}_1 b)\bar{a}^2 - p_2 \bar{b} a^2\right\} + \gamma b \bar{b}\left\{\frac{1}{6}\bar{p}_1 + \left(\frac{1}{2}a^2 - \frac{1}{3}a^3\right)\Delta\right\} \tag{3.3a}$$

$$v_n = b + \frac{1}{2}\left\{(1 - \bar{p}_2 a)\bar{b}^2 - p_1 \bar{a} b^2\right\} + \gamma a \bar{a}\left\{\frac{1}{6}\bar{p}_2 + \left(\frac{1}{2}b^2 - \frac{1}{3}b^3\right)\Delta\right\} \tag{3.3b}$$

$$(n = 1, 2, \cdots \ ; u_0 = v_0 \equiv 0, \ a_0 = b_0 = 0)$$

with a *and* b *replaced by* u_{n-1} *and* v_{n-1}*, respectively.*

The equilibrium strategy-pair at the first stage is

$$\alpha^*(x) = 0, \ \text{if } x < u_{n-1} \ ; = 1, \ \text{if } x > u_{n-1} \tag{3.4a}$$

$$\beta^*(y) = 0, \ \text{if } y < v_{n-1} \ ; = 1, \ \text{if } y > v_{n-1}. \tag{3.4b}$$

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