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Optimal Checkpoint Strategy Subject to System Failures Caused by a Renewal Process

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1. INTRODUCTION

This paper considers a probabilistic model for a database recovery action with checkpoint generations when system failures occur according to a renewal process whose renewal density depends on the cumulative operation period since the last checkpoint. Necessary and sufficient conditions on the existence of the optimal checkpoint interval which maximizes the ergodic availability are analytically derived. The model condidered in this paper generalizes that by Young [1] and is different from those by Gelenbe [2] and Sumita, *et al.* [3].

2. DATABASE MANAGEMENT MODEL

Let $X(t)$ be the cumulative operation time for the database system at time t since the last checkpoint. Also, let the renewal process be governed by a sequence of i.i.d. non-negative random variables $D_i (i = 1, 2, \cdots)$ having $F(x) = \Pr\{D_i \leq x\}$ and

$$f(x) = \frac{d}{dx}F(x). \tag{1}$$

Let

$$f^{(k)}(x) = \int_0^x f^{(k-1)}(x-y)f(y)dy, \qquad (k \geq 2) \tag{2}$$

and $f^{(1)}(x) = f(x)$, where $f^{(k)}(\cdot)$ is the k-fold convolution of $f(\cdot)$ of itself $(k \geq 2)$. Then the probability that a system failure on the primary memory occurs in the time interval $(x, x+\delta)$ is given by $m(x)\delta + o(\delta)$, where $m(x) = \sum_{k=1}^{\infty} f^{(k)}(x)$ is the renewal density and the corresponding renewal function is defined as

$$M(x) \equiv \int_0^x m(y)dy. \tag{3}$$

Upon a failure, a rollback recovery takes place where the buffer information saved at the last checkpoint creation and the log of transactions are used for restoring the database to a usable state. The length of the rollback recovery is assumed to depend on the number of transactions in the log, *i.e.*, on the value of $X(t)$ at the time of failure. We employ a generic random variable V_x denoting the length of the rollback recovery given that a failure occured at time t with $X(t) = x$. The distribution of V_x is denoted by $B(y) = \Pr\{V_x \leq y\}$.

Intervals between two consecutive checkpoints are determined by the total operation time in the interval excluding rollback periods. The i-th checkpoint is generated as soon as the total operation time since the $(i-1)$st checkpoint reaches the length S_i $(i = 1, 2, \cdots)$. Assume that S_i $(i = 1, 2, \cdots)$ constitutes a sequence of random variables with common distribution $A(x) = \Pr\{S_i \leq x\}$. Times (overheads) required for creating checkpoints also form a sequence of i.i.d. random variables $C_i (i = 1, 2, \cdots)$ with $W(z) = \Pr\{C_i \leq z\}$.

Let T_i $(i = 1, 2, \cdots)$ be the actual time interval between the $(i-1)$st and the i-th checkpoints. Then, since T_i $(i = 1, 2, \cdots)$ is a sequence of i.i.d. random variables, checkpoints are clearly regenerative points. From the renewal argument, it is sufficient to consider the model in one cycle and we drop the discrete time index i $(i = 1, 2, \cdots)$ in the following discussion. Since the one cycle is defined as the time period commencing at the end of one checkpoint and ending the end of another checkpoint, the mean time of one cycle is $\mathrm{E}_A[S] + \mathrm{E}_W[C] + R$ and the mean operating time for one cycle is $\mathrm{E}_A[S]$, where R is the total mean time of rollback recovery and is expressed by

$$R = \int_0^{\infty} dA(x) \int_0^x m(s)\mathrm{E}_B[V_s]ds. \tag{4}$$

Then, the ergodic availability is formulated as

$$\Pi = \frac{\mathrm{E}_A[S]}{\mathrm{E}_A[S] + \mathrm{E}_W[C] + R}. \tag{5}$$

From Eq. (5), the problem is to seek the optimal checkpoint strategy which maximizes Π.

3. OPTIMAL CHECKPOINT STRATEGY

Let us derive the optimal checkpoint interval for an arbitrary failure time distribution. Define the following functions;

$$h(x) \equiv m(x)\mathrm{E}_B[V_x], \tag{6}$$

$$H(x) \equiv \int_0^x h(s)ds. \tag{7}$$

The function $h(x)$ is called *switching function* in this paper and satisfies the following formula;

$$\begin{aligned}\mathrm{E}_A[H(x)] &= \int_0^\infty dA(x)\int_0^x h(s)ds \\ &= \int_0^\infty H(x)dA(x).\end{aligned} \tag{8}$$

Further, we define a set of all $A(x)$s with fixed expectation $T \in [0,\infty)$ as J_T. The following theorem will be useful to characterize the condition on the existence of the optimal checkpoint interval.

Lemma 3.1: Let J_T be a set of all $A(x)$s with fixed expectation $T \in [0,\infty)$. If the switching function $h(x)$ is increasing, then the element $A(x)$ of the set J_T which maximizes the ergodic availability Π is

$$A(x) = U(x-T) = \begin{cases} 1 & \text{if } x \geq T \\ 0 & \text{if } x < T, \end{cases} \tag{9}$$

where $U(\cdot)$ is the unit function.

From Lemma 3.1, the randomized policy $A(x)$ is translated to the constant policy T and we can replace Π and $\mathrm{E}_A[S]$ to $\Pi(T)$ and T, respectively, that is, the problem is formulated as

$$\max_{0<T<\infty} : \Pi(T) = \frac{T}{T + \mathrm{E}_W[C] + H(T)}. \tag{10}$$

Now, we specify the random variable V_x denoting the length of the rollback recovery. Following the literature [1-3], put

$$\mathrm{E}_B[V_x] = \alpha x + \beta, \qquad (\alpha > 0, \beta \geq 0), \tag{11}$$

where the first term denotes the mean time needed to reprocess transactions processed in time interval $[0,x]$ and the second term is the mean time concerned with reloading the information stored at the checkpoint back into primary memory. More concretely, we consider the following Markovian model [2]: If transactions arrive at the system according to a homogeneous Poisson process with intensity $\lambda(>0)$, processing requirements of transactions for both initial processing and reprocessing are i.i.d. having a common exponential distribution with mean $1/\kappa(>0)$. Then the parameter α is interpreted as $\alpha = k\lambda/\kappa$, where $k(>0)$ is the rate of transactions to be restored after any failure. From Eq. (6), the switching function becomes

$$h(x) = m(x)(\alpha x + \beta). \tag{12}$$

Differentiating $\Pi(T)$ with respect to T and setting equal to zero implies the equation $q(T) = 0$, where

$$q(T) \equiv \mathrm{E}_W[C] + H(T) - Th(T). \tag{13}$$

Using Eq. (13), we have the main result in this paper.

Theorem 3.2:

(i) Suppose that the switching function $h(x)$ is strictly increasing. Then, there exists a finite and unique optimal checkpoint interval T^* $(0 < T^* < \infty)$ satisfying the nonlinear equation $q(T^*) = 0$ and the corresponding ergodic availability is

$$\Pi(T^*) = \frac{1}{1 + h(T^*)}. \tag{14}$$

(ii) Suppose that the switching function $h(x)$ is constant. Then, the optimal checkpoint interval is $T^* \to \infty$, *i.e.*, no checkpoint should be generated and the corresponding ergodic availability is

$$\Pi(\infty) = \frac{1}{1 + h(\infty)}. \tag{15}$$

4. REMARKS

It is not easy for an arbitrary distribution to get the analytical expression for the nonlinear equation, since $q(T)$ involves the renewal function $M(T)$ and its associated quantities. Then the approximation procedures will be useful to estimate the optimal checkpoint intervals. In the conference, we will introduce four approximation methods in the cases where the failure mechanism is unknown but the first three moments are known and where the failure time distribution is inferred, respectively, and will report results on the comparison of them.

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