1996年度日本オペレーションズ・リサーチ学会
春季研究発表会

# AUCTION BIDDING AND EVOLUTIONARY STABLE STRATEGIES

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## 1. INTRODUCTION

One of the most unusual applications of game theory in recent years has been to model how animal behavior evolves from generation to generation. Maynard-Smith [1974] modelled conflicts among animals in their mating behavior, for example, in terms of game theory by introducing the concepts of "strategy" and "fitness" ( i.e., payoffs in other terminology) that are considered to be "thought and used" by animals. He defined evolutionary stable strategies (ESS) and showed that animals use ESS in order to secure revolution of their species (this explains why snakes wrestle rather than bite). Now we understand that some examples are observed in the animal world that can be thought of as n-player game rather than two-player game. (Thomas [1984; pp. 186-187]). The aim of the present paper is to extend the two-player ESS originally defined by Maynard-Smith to the n-player ESS, and to apply this concept to some types of n-bidder auction. In Section 2 n-player ESS are defined by extending the concept of two-player ESS to the n-player case. Three types of n-bidder auctions, i.e., second-bid, first-bid, and sad-loser auctions (in Section 3) and a guessing game, as a variant of auction, (in Section 4), are discussed and a unique mixed-strategy ESS (if it exists) is derived for each of these four examples in auction.

## 2. EVOLUTIONARY STABLE STRATEGIES

Consider an n-player non-zero-sum, competitive, and "symmetric" game where the payoff to player i, when he uses a strategy $x_i$ and his opponents use a (n-1)-tuple of strategies $X^{-i}=(x_1,\dots x_{i-1},x_{i+1},\dots x_n)$, is represented by

$$M_i(x_1,\dots,x_n)=e(x_i,\ X^{-i}) \qquad i=1,\dots,n.$$

Here the function $e(x_1,\dots,x_n)$ is called an n-player fitness function and has the property that $e(x_i,X^{-i})$, for each i, is unchanged under any permutation over the components of $X^{-i}$.

We state the following definition. Let $x^{*n-1}$ be an (n-1) repetition of the same $x^*$. A strategy $x^*$ is an n-player evolutionary stable strategy (ESS) if it satisfies

$$e(x^*,x^{*n-1})\ge e(y,x^{*n-1}),\ \text{for any strategy } y\ne x^*, \tag{2.1}$$

and if $e(x^*,x^{*n-1})=e(y,x^{*n-1})$, then

$$e(x^*,y^{n-1})>e(y,y^{n-1}). \tag{2.2}$$

This is a natural extension of the two-player ESS, originally given by Maynard-Smith [1974], to the n-player case. For each player, an ESS $x^*$ is the best response to $x^{*n-1}$ employed by his competitors, and if there exists an equivalent strategy $y\ne x^*$, then, to $y^{n-1}$, $x^*$ is a better response than $y^*$. Note that, if $x^*$ is an n-player ESS, $x^{*n}$ will be an equilibrium n-tuple for the corresponding non-zero-sum game, but not all such equilibrium n-tuples are n-player ESS because of (2.2).

## 3. ESS IN AUCTION BIDDING

In sealed auctions n players bid for an item which has an identical worth V>o for all players. The bidder who bids highest obtains the item, and when more than one bidder bid highest the item is given to a single bidder chosen by an equal-chance lottery. According to how the price is paid to the seller by the winner we have several types of auction. (See Thomas [1984; Chapter 9] and Vickrey [1962]). We discuss 1. Second-Bid auction, 2. First-Bid auction and 3. Sad-Loser auction. In the last type of auction the loser forfeits the amount of his bid, and the winner obtains the item without paying the price. The fitness function is

$$e(x_1,x_2,\ldots,x_n)=\begin{cases} -x_1, & \text{if } x_1<y_{-1} \\ (V-x_1)/m_1(x), & \text{if } x_1=y_{-1} \\ V, & \text{if } x_1>y_{-1} \end{cases}$$

where $y_{-i}=\max_{j\neq i} x_j$ and $m_i(x)=|\{j\,|x_j=x_i\}|$, $i=1,\ldots,n$. We prove

Proposition 3. In the sad-loser auction with the above fitness function the mixed strategy

$$F^*(x)=\left(\frac{x}{V+x}\right)^{1/(n-1)}, \quad 0\leq x<\infty$$

is an n-player ESS. The equilibrium value is zero.

## 4. A GUESSING GAME

As a final example we will give an auction-like n-player symmetric game. This is a Guessing Game as follows: Player O picks up a random number t uniformly distributed on $0\leq t\leq 1$. Each of n players guesses t independently of other players' guess. They know only that the distribution of t is uniform. A player, whose guess is nearest to t among the guesses not greater than t, wins, and is rewarded by a unit amount. The loser gets nothing. Each player seeks to find the strategy that maximizes his expected reward. The fitness function in this game is given by

$$e(x_1,x_2,\ldots,x_n)=(z_1(x)-x_1)/m_1(x), \quad x=(x_1,\ldots,x_n)\in[0,1]^n$$

where $z_i(x)=\begin{cases} \min\{x_j|x_j>x_i\}, & \text{if } x_j>x_i \text{ for some } j \\ 1, & \text{if otherwise} \end{cases}$

A unique mixed-strategy ESS when it exists is derived.

References

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Full paper is in Math. Japonica, 37 (1992) 221-229