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Trunk Reservation Effects on Multi-Server System with Batch Arrivals of Loss and Delay Customers

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1 Introduction

We treat a multi-server system with trunk reservation scheme. The system is offered by two types of customers (class-1 and class-2). They arrive in independent batch Poisson streams with parameter $\lambda_i (i=1,2)$and have an exponentially distributed service time with mean μ^{-1}. Class-1 customers will be lost or rejected if they find all S servers busy on their arrivals. Class-2 customers will use at most $S'=S-R$ servers and enter a queue with N capacity if they find the number of idle servers less than or equal to R on their arrivals. Here, R is the number of reserved servers for class-1 customers. An example of the system is realized in NTT's facsimile communications network F-NET [1].

2 Steady State Probabilities

Let us introduce some notations given by

$$a_i = \lambda_i/\mu, \quad (i=1,2),$$
$$g=\sum_{i=1}^{\infty} i g_i, \quad h=\sum_{i=1}^{\infty} i h_i,$$
$$\bar{g}_i=\sum_{k=i}^{\infty} g_k, \quad \bar{h}_i=\sum_{k=i}^{\infty} h_k,$$
$$\tilde{g}_{i,j}=\sum_{k=i}^{j} g_k, \quad \tilde{h}_{i,j}=\sum_{k=i}^{j} h_k,$$

where $\{g_i\}$ and $\{h_i\}$ denote the batch size distributions of class-1 and class-2 customers, respectively. Denoting by p/w Partial/Whole Batch Acceptance Strategy, we also introduce the following notations

$$g_{i,j}^{(m)}=\begin{cases} 1 & m=p \\ \tilde{g}_{i,j} & m=w \end{cases}, \quad h_{i,j}^{(n)}=\begin{cases} 1 & n=p \\ \tilde{h}_{i,j} & n=w \end{cases}$$

The steady state is specified by (i,j), where i is the number of busy servers and j is the number of occupied waiting rooms. Under m and n strategies for class-1 and class-2 customers, the steady state probabilities $\{\pi_{i,j}\}$ of state (i,j) satisfy the following equations:
for $1 \le i \le S', j=0$

$$\begin{aligned} i\pi_{i,0} &= \sum_{k=0}^{i-1} a_1(\bar{g}_{i-k}\delta_{m,p}+\tilde{g}_{i-k,S-k}\delta_{m,w})\pi_{k,0} \\ &+ \sum_{k=0}^{i-1} a_2(\bar{h}_{i-k}\delta_{n,p}+\tilde{h}_{i-k,S'+N-k}\delta_{n,w})\pi_{k,0}, \end{aligned}$$

for $S'+1 \le i \le S, 0 \le j \le N$

$$\pi_{i,j}=\begin{cases} \sum_{k=0}^{i-1} a_{S-i+1,k}^{(0)}\pi_{k,0} & j=0 \\ \sum_{k=S'}^{i-1} a_{S-i+1,k}^{(j)}\pi_{k,j}+\sum_{k=0}^{j-1} b_{S-i+1,k}^{(j)} & j\neq 0, \end{cases}$$

for $i=S', 1 \le j \le N$

$$\begin{aligned} S'\pi_{S',j} &= \sum_{k=0}^{S'} a_1(\bar{g}_{S'+1-k}\delta_{m,p} \\ &\quad +\tilde{g}_{S'+1-k;S-k}\delta_{m,w})\pi_{k,0} \\ &\quad -(S'+1)\pi_{S'+1,0} \\ &+ \sum_{k=1}^{j-1}(a_1(\bar{g}_1\delta_{m,p}+\tilde{g}_{1,S-S'}\delta_{m,w})\pi_{S',k} \\ &\quad -(S'+1)\pi_{S'+1,k}) \\ &+ \sum_{k=0}^{S'} a_2((\bar{h}_{S'+j-k}\delta_{n,p} \\ &\quad +\tilde{h}_{S'+j-k,S'+N-k}\delta_{n,w})\pi_{k,0} \\ &\quad +(\bar{h}_{j-k}\delta_{n,p} \\ &\quad +\tilde{h}_{j-k,S'+N-k}\delta_{n,w})\pi_{S',k}), \end{aligned}$$

where $a_{S-i+1,k}^{(j)}, b_{S-i+1,k}^{(j)}$ are coefficients which can be evaluated by $\lambda_1, \lambda_2, \mu, \{g_i\}, \{h_i\}$, and $\{\pi_{i,j}\}$ and $\delta_{i,j}$ is Kronecker delta. Note that steady state probabilities $\{\pi_{i,j}\}$ are recursively solved, provided that one parameter, say $\pi_{0,0}$, is given.

3 Performance Measures

Loss Probability

Let us define $\pi_k^{(1)}$ as the probability that the k servers busy. By definition

$$\pi_k^{(1)} = \begin{cases} \pi_{k,0} & 0 \le k < S' \\ \sum_{j=0}^{N} \pi_{k,j} & S' \le k \le S. \end{cases}$$

Since the probability that an arbitrary class-1 customer arrives in batch of size i is equal to ig_i/g, we have

$$B^{(1)} = \sum_{i=1}^{\infty} ig_i B_i^{(1)}/g,$$

where $B_i^{(1)}$ is the customer loss probability under the condition that the customer arrives in batch of size i. Expressing $B_i^{(1)}$ in terms of $\pi_k^{(1)}$, we finally obtain $B^{(1)}$ as

Under PBAS

$$B^{(1)} = \sum_{k=0}^{S} \sum_{i=S-k+1}^{\infty} (i+k-S) g_i \pi_k^{(1)}/g.$$

Under WBAS

$$B^{(1)} = \sum_{k=0}^{S} \sum_{i=S-k+1}^{\infty} ig_i \pi_k^{(1)}/g.$$

Overflow Probability

As in the case of the loss probability, we introduce $\pi_k^{(2)}$ as the probability that the k servers and/or waiting rooms are busy or occupied, which is given by

$$\pi_k^{(2)} = \begin{cases} \pi_{k,0} & 0 \le k < S' \\ \sum_{i=S'}^{S} \pi_{i,k-S'} & S' \le k \le S'+N. \end{cases}$$

The customer overflow probability $B^{(2)}$ of an arbitrary class-2 customer can be expressed by $\{\pi_k^{(2)}\}$ as in the case of the customer loss probability of an arbitrary class-1 customer. We only give final expressions.

Under PBAS

$$B^{(2)} = \sum_{k=0}^{S'+N} \sum_{i=S'+N-k+1}^{\infty} (i+k-S'-N) h_i \pi_k^{(2)}/h.$$

Under WBAS

$$B^{(2)} = \sum_{k=0}^{S'+N} \sum_{i=S'+N-k+1}^{\infty} ih_i \pi_k^{(2)}/h.$$

Note that the $B^{(1)}$ and $B^{(2)}$ are related to the carried load a_c

$$a_c = a_1 g(1 - B^{(1)}) + a_2 h(1 - B^{(2)}),$$

which is also equal to the expected number of simultaneously busy servers given by $\sum_{i=0}^{S} i\pi_i^{(1)}$.

Mean Queue Length and Mean Waiting Time

Using the steady state probabilities $\{\pi_k^{(2)}\}$, the mean queue length L_q of an arbitrary class-2 customer is given by

$$L_q = \sum_{k=S'}^{S'+N} (k - S')\pi_k^{(2)}.$$

Applying Little's formula, the mean waiting time of an arbitrary class-2 customer W is written as

$$W = L_q/(\lambda_2 h(1 - B^{(2)})).$$

4 Numerical Examples

One of our numerical examples is shown in Fig. 1 which indicates the customer loss probability $B^{(1)}$ of an arbitrary class-1 customer with various reservation parameter R. As R increases, $B^{(1)}$ decreases, which is intuitively consistent.

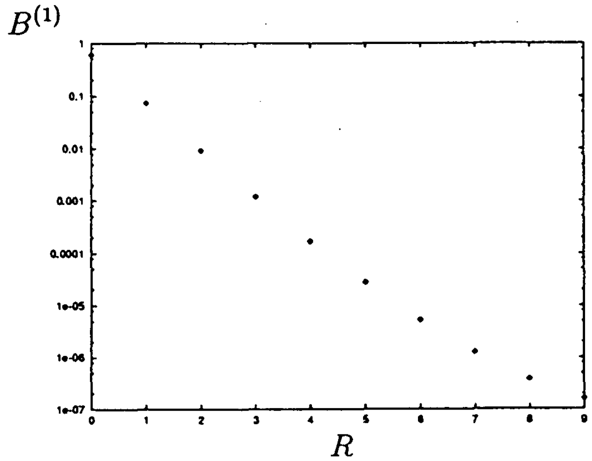

Figure 1: The customer loss probability $B^{(1)}$ of an arbitrary class-1 customer vs. reservation parameter R.

References

[1] H. Fuju, T. Konishi, Y. Takeyama, and H. Miyaji: A multi-image communication system design, *NTT R&D*, **43**, 841–848 (1994). (in Japanese)