A versatile algorithm for designing robust and reliable communication networks and its complexity analysis

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1. Introduction

A do-loop enumeration type algorithm is proposed to design robust and reliable communication networks. The network design problem to be handled is stated in Sec.2, its complexity when solving it through enumeration is analyzed in Sec. 3, and an algorithm to solve the problem is proposed in Sec. 4. Future direction is suggested in Sec. 5.

2. Problem statement

The network design problem considered in this paper is the *M*-best economical network design problem (MENDP) as stated below: Given a node-to-node traffic matrix, a GoS specification, cost data, and a network topology, enumerate the best *M* solutions (link size and node size vectors) that minimize the total cost (node cost + link cost) while satisfying the GoS constraints. The algorithm to solve this problem should be robust and versatile in the sense that the best *M* (eg. best 10) most economical designs need to be enumerated and link- and node-cost functions for an arbitrary shape must be accommodated.

Moreover, robustness (ie, survivability) is required in designed networks. This can be achieved by using double-homing and/or triple-homing topologies.

3. Analysis of computational complexity

From reliability specifications such as node-to-node availability and robustness against transit-node (T-node) group breakdown, the network topology to be used is determined.
In this paper, we focus on two-level networks, where nodes consist of T-nodes (or level-2 nodes) and local nodes (L-nodes or level-l nodes).

First, consider a simple example. Let the number of nodes in a network be n. Then, in the case of a two-level one-area double-homing topology, the M best solutions should be selected from among $C(n, 2)$ solutions, where two T-nodes (or one T-node pair) are selected from among the n nodes. When a pair of T-nodes are selected, the other n-2 nodes all belong to this pair. Let $Q(n, k, h)$ be the computational amount required to calculate the cost of a two-level n-node k-area h-homing-multiplicity network, when its topology (T-node group location and L-node homing pattern), node-to-node traffic matrix, GoS specification, and cost data are given. Then, the total computational amount for this case is determined by Eq. (1).

$$P(n,\ 1,\ 2)=\frac{n(n-1)}{2}Q(n,\ 1,\ 2) \qquad (1)$$

Here, $P(n, k, h)$ is the total computational amount required to solve MENDP(n, k, h), the M-best economical network design problem with two-level, n-node, k-area, h-homing-multiplicity topology, through an exhaustive enumeration.

Next, consider a less simple topology with k=2 and h=2. In this case, four T-nodes are selected from among the n nodes, and three patterns can be used to make two T-node pairs out of the four T-nodes. For each pattern out of the $3\times C(n,4)$ patterns, each node (including the T-node) needs to decide which pair it will belong to. Finally, the best M solutions can be selected from among $3\times C(n,4)\times 2^n$ solutions, and $P(n, 2, 2)$ is determined by Eq. (2).

$$P(n,\ 2,\ 2)=3\times C(n,\ 4)\times 2^n\times Q(n,\ 2,\ 2) \qquad (2)$$

The index $R(n, k, h)$, defined by Eq. (3), can be introduced to measure the computational complexity of the M-best economical network design problem MENDP(n, k, h) when solving it through an exhaustive enumeration.

$$R(n,\ k,\ h)=\frac{P(n,\ k,\ h)}{Q(n,\ k,\ h)} \qquad (3)$$

This index $R(n, k, h)$ is generally evaluated by Eq. (4).

$$R(n,\ k,\ h) = S(n,\ k,\ h) \times T(k,\ h) \times U(n,\ k)$$
$$= \frac{n!}{(n-kh)!k!}\frac{k^n}{(h!)^k} \tag{4}$$

$$S(n,\ k,\ h) = C(n,\ kh) \tag{5}$$

$$T(k,\ h) = \frac{(kh)!}{(h!)^k k!} \tag{6}$$

$$U(n,\ k) = k^n \tag{7}$$

Here, $S(n, k, h)$ is the number of combinations when taking kh nodes (as T-nodes) out of n nodes, $T(k, h)$ is the number of T-node location patterns for a fixed set of T-nodes, and $U(n, k)$ is the number of homing patterns for a fixed T-node location pattern.

More generally, when the homing multiplicity of area i is h_i, $S(n, k, \boldsymbol{h})$ and $T(k, \boldsymbol{h})$ are evaluated as below ($\boldsymbol{h} = \{h_i\}$).

$$S(n,\ k,\ \boldsymbol{h}) = C(n, \sum_{i=1}^{k} h_i) \tag{8}$$

$$T(k,\ \boldsymbol{h}) = \frac{\left(\sum_{i=1}^{k} h_i\right)!}{\left\{\prod_{i=1}^{k}(h_i!)\right\} \times k!} \tag{9}$$

4. Algorithm

We propose an enumerative-type two-stage algorithm for solving MENDP(n, k, h). The outline of the M-best enumeration algorithm for h=2 is shown in Fig. 1, where all the T-node location patterns are enumerated at the outer loop (STPLOC) and the M-best solutions are updated at the inner loop (SPHOME) by examining all or part of the homing patterns for a given T-node location pattern.

Note that in solving the SPHOME problem (the inner loop) for a given T-node location pattern, the most economical solution can be obtained by following a simple rule: Home node j to the area with the least cost required to connect node j and the area.

The proposed M-best enumeration algorithm not only can handle various actual cost structures and accommodation conditions at nodes, but can also cope with other intangible, intractable, or difficult-to-formulate conditions such as preferences with respect to T-node selection, partly fixed T-node locations, and uncertainty in node-to-node traffic demand.

5. Remarks

It is feared that the computational time required to solve the MENDP(n, k, h) will grow explosively with n, k and h. With current workstations, 100-best economical solutions cannot be obtained within an hour for parameters n>40 and k>4 (h=2). Speeding up of the algorithm by using parallel computation, integrating it with some meta-heuristic algorithms and/or by pre-fixing node subsets at L-nodes and T-nodes, is a topic for future research.

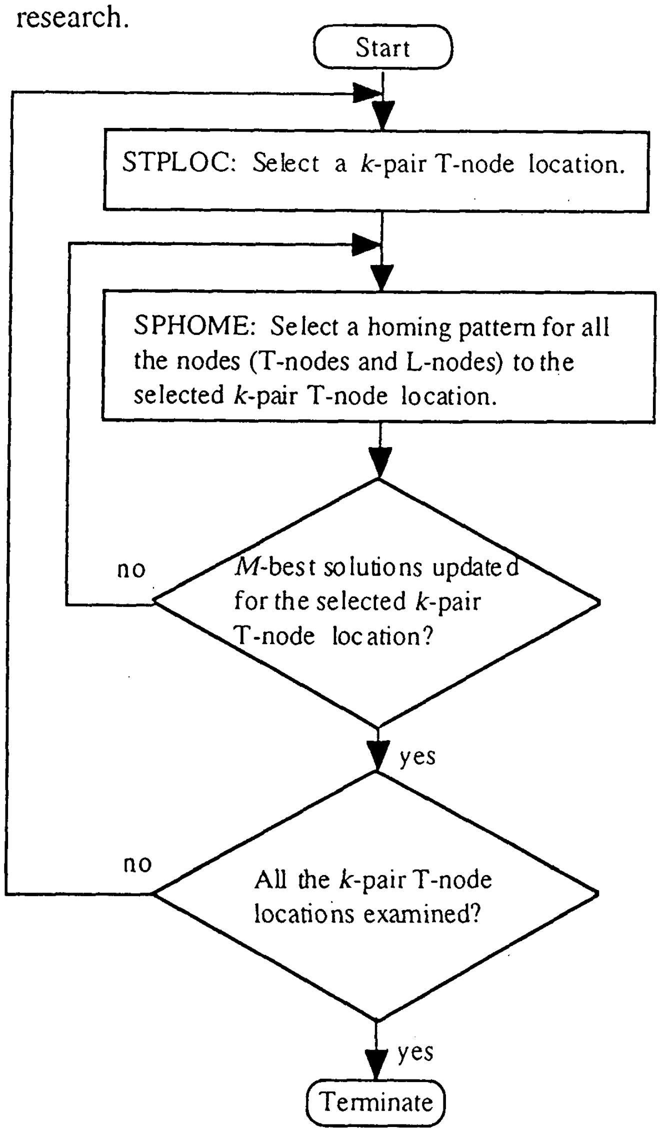

Fig. 1 Outline of the M-best enumeration algorithm for solving MENDP(n, k, 2)