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The Discrete-Time Opportunistic Replacement Models with Application to Scheduled Maintenance for Electric Switching Device

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1. INTRODUCTION

In this paper, we consider the discrete-time opportunistic replacement models with application to scheduled maintenance for electric switching devices. The electric switching devices equiped with telegrapf poles have to be replaced preventively before they fail and the electric current is off over an extensive area. On the other hand, the electric switching device can be replaced if the telegrapf pole is removed for any construction before its age has elapsed a threshold level. This problem is reduced to a simple oppoutunity based age replacement model [1-3] exept that it is considered in a discrete-time setting.

Ordinarily, the discrete-time models [4, 5] are considered as trivial analogies of the continuous-time ones. However, we show in this paper that a replacement model with more than two maintenance options can be classified into some kinds of models by the priority of maintenance options. This implies that the discrete-time model has more delicate aspects for analysis than the continuous one.

2. MODEL DESCRIPTION

First, we consider a discrete-time model corresponding to Iskandar and Sandoh [2]. Let us consider the single-unit system with a non-repairable item in a discrete-time setting. Suppose that the interval between opportunities for replacements X obeys the geometric distribution $\Pr\{X = x\} = g_X(x) = p(1-p)^{x-1}$ ($x = 1, 2, \cdots; 0 < p < 1$) with survivor function $\Pr\{X \geq x\} = (1-p)^{x-1} = \overline{G}_X(x-1)$, mean $\mathrm{E}[X] = 1/p$ and variance $\mathrm{Var}[X] = (1-p)/p^2$. Then, the unit may be replaced at a first opportunity after time S (S is a non-negative integer) even if it does not fail. The failure Y follows the common distribution $\Pr\{Y = y\} = f_Y(y)$ with survivor function $\Pr\{Y \geq y\} = \overline{F}_Y(y-1)$ and failure rate $r_Y(y) = f_Y(y)/\overline{F}_Y(y-1)$. Without any loss of generality, we assume that $f_Y(0) = g_X(0) = 0$. If the failure occurs before a prespecified preventive replacement time T ($T = 1, 2, \cdots$), the corrective replacement is executed. On the other hand, if the unit does not fail up to the time T, the preventive replacement is made at time T. The cost components under consideration are the following;

c_1 (> 0): corrective replacement cost per failure

c_2 ($< c_1$): cost for each preventive replacement

c_3 ($< c_2$): cost for each opportunistic replacement.

It should be noted that the discrete-time model above has to be treated carefully. At an arbitrary discrete point of time, the decision maker can select three options, failure (corrective) replacement $\mathrm{F_a}$, preventive replacement $\mathrm{S_c}$ and opportunistic replacement $\mathrm{O_p}$. We introduce the following symbol for the priority relationship;

Definition 2.1: The option P has priority to the option Q if $\mathrm{P} \succ \mathrm{Q}$.

From Definition 2.1, if both options occur at the same time point, the option with higher priority P is executed. In our model setting, consequently, it is possible to consider 6 different models. At the moment, we consider the following two models;

Model 1 : $\mathrm{S_c} \succ \mathrm{F_a} \succ \mathrm{O_p}$

Model 2 : $\mathrm{F_a} \succ \mathrm{S_c} \succ \mathrm{O_p}$

The other variations, $\mathrm{S_c} \succ \mathrm{O_p} \succ \mathrm{F_a}$, $\mathrm{O_p} \succ \mathrm{S_c} \succ \mathrm{F_a}$, $\mathrm{O_p} \succ \mathrm{F_a} \succ \mathrm{S_c}$ and $\mathrm{F_a} \succ \mathrm{O_p} \succ \mathrm{S_c}$, may be analyzed in the similar manner.

For Model 1, the probability that the system is replaced at time n ($n = 0, 1, 2, \cdots$) is

$$h_1(n) = \begin{cases} f_Y(n) & (0 \leq n \leq S) \\ f_Y(n)\overline{G}_X(n-1-S) + \overline{F}_Y(n)g_X(n-S) & \\ & (S+1 \leq n \leq T-1) \\ \overline{F}_Y(n-1)\overline{G}_X(n-1-S) & (T = n) \\ 0 & (T+1 \leq n) \end{cases} \quad (1)$$

where $\sum_{n=0}^{\infty} h(n) = 1$. From Eq.(1), the mean time length of one cycle is

$$\begin{aligned} A_1(T) &= \sum_{n=0}^{S} n f_Y(n) \\ &+ \sum_{n=S+1}^{T-1} n\{f_Y(n)\overline{G}_X(n-1-S) \\ &+\overline{F}_Y(n) g_X(n-S)\} \\ &+T\overline{F}_Y(T-1)\overline{G}_X(T-1-S) \\ &= \sum_{j=1}^{S} \overline{F}_Y(j-1) \\ &+ \sum_{j=S+1}^{T-1} \overline{F}_Y(j-1)\overline{G}_X(j-S-1) \\ &+\overline{F}_Y(T-1)\overline{G}_X(T-1-S). \end{aligned} \tag{2}$$

On the other hand, the total expected cost during one cycle is

$$\begin{aligned} B_1(T) &= c_1 \sum_{n=0}^{S} f_Y(n) \\ &+c_1 \sum_{n=S+1}^{T-1} f_Y(n)\overline{G}_X(n-1-S) \\ &+c_2\overline{F}_Y(T-1)\overline{G}_X(T-1-S) \\ &+c_3 \sum_{n=S+1}^{T-1} \overline{F}_Y(n) g_X(n-S). \end{aligned} \tag{3}$$

Then the expected cost per unit time in the steady-state is, from the falimiar renewal reward argument,

$$\begin{aligned} C_1(T) &= \lim_{n\to\infty} \frac{\text{E[total cost on } (0,n]]}{n} \\ &= B_1(T)/A_1(T). \end{aligned} \tag{4}$$

On the other hand, for Model 2, the mean time length of one cycle is $A_2(T) = A_1(T)$ and the total expected cost for one cycle is

$$B_2(T) = B_1(T) + (c_1 - c_2) f_Y(T)\overline{G}_X(T-1-S). \tag{5}$$

Hence the resulting expected cost per unit time in the steady-state becomes $C_2(T) = B_2(T)/A_2(T)$.

3. OPTIMAL POLICIES

In this section, we consider two models, Model 1 and Model 2, and derive the respective optimal preventive replacement policies which minimize the expected costs per unit time in the steady-state. Define the functions;

$$\begin{aligned} q_1(T) &\equiv \frac{1}{1-p}\Big\{(c_1 - c_2)R_Y(T) + p(c_3 - c_2)\Big\}A_1(T) \\ &\quad -B_1(T), \end{aligned} \tag{6}$$

$$\begin{aligned} q_2(T) &\equiv \Big\{(c_1 - c_2)r_Y(T+1) + \frac{p(c_3 - c_2)}{1-p}\Big\}A_2(T) \\ &\quad -B_2(T), \end{aligned} \tag{7}$$

where

$$R_Y(T) \equiv r_Y(T)/(1 - r_Y(T)). \tag{8}$$

Lemma 3.1: The function $R_Y(T)$ is strictly increasing [decreasing] if the failure time distribution is strictly IFR (Inceasing Failure Rate) [DFR (Deceasing Failure Rate)].

Theorem 3.2: (1) Suppose that the failure time distribution is strictly IFR for Model j ($j = 1, 2$).

(i) If $q_j(S+1) < 0$ and $q_j(\infty) > 0$, then there exists a finite and unique optimal preventive replacement time T^* ($S+1 < T^* < \infty$) which satisfies $q_j(T^*-1) < 0$ and $q_j(T^*) \geq 0$.

(ii) If $q_j(\infty) \leq 0$, then the optimal preventive replacement time is $T^* \to \infty$ and it is optimal to carry out the failure replacement or the opportunistic one.

(iii) If $q_j(S+1) \geq 0$, then the optimal preventive replacement time is $T^* = S+1$ and it is optimal to carry out the failure replacement or the preventive one.

(2) Suppose that the failure time distribution is DFR for Model j ($j = 1, 2$). Then the optimal preventive replacement time is $T^* \to \infty$ or $T^* = S+1$.

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