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拡張累積損傷モデル及びデータベースシステムへの応用

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1. Introduction

This paper considers a cumulative damage model with two kinds of shocks described: A system suffers two kinds of shocks which occur at a nonhomogeneous Poisson process with an intensity function $\gamma(t)$ and a mean-value function $\Gamma(t)$. We call that one is *failure shock* by which a system fails and the other is *damage shock* at which it suffers only damage. These damages accumulate additively and a system also fails when the total damage has exceeded a threshold level K. A system is replaced after failure. However, to lessen a replacement cost after failure, a system is also replaced before failure at scheduled time T as preventive maintenance.

Suppose that the probability that the damage shock occurs is p $(0 < p \leq 1)$ and the probability that failure shock occurs is $1-p$. It is noted that failure shocks occur at nonhomogeneous Poisson process with an intensity function $(1-p)\gamma(t)$, and damage shocks occur at nonhomogeneous Poisson process with an intensity function $\lambda(t) \equiv p\gamma(t)$ and a mean-value function $R(t) \equiv p\Gamma(t)$ [1]. Let $F(t) \equiv 1 - e^{-(1-p)\Gamma(t)}$ and $H_j(t) \equiv \frac{[R(t)]^j}{j!}e^{-R(t)}$. Further, an amount Y_j of damage due to the j-th damage shock has a probability distribution $G_j(x)$. Then, the total damage Z_j to the j-th damage shock has a distribution $G^{(j)}(x)$ where the asterisk mark represents the Stieltjes convolution.

2. Backup Model

In this paper, we apply the cumulative damage model to the backup of files in a database system [2], by putting *damage shock* by *update*, *failure shock* by *database failure* and *damage* by *dumped files*. To ensure the safety of data and to save hours, we make the following backup policy: If the total dumped files do not exceed a threshold level K, we perform the incremental backup where only new files since the previous full backup are dumped. Conversely, we perform the full backup at periodic time T, when the total files exceed K, or when the database system fails, whichever occurs first. The database system returns to an initial state by the full backup.

Let introduce the following costs: A cost c_1 is suffered for the incremental backup, a cost $c_2 + c_0(x)$ is suffered for the full backup at time T when the total files are x

$(0 \leq x < K)$, a cost $c_3 + c_0(K)$ is suffered for the full backup when the total files has exceeded a level K, and a cost $c_4 + c_0(x)$ is suffered for the recovery when a database system fails, where $c_1 \leq c_2 < c_3 \leq c_4$, $c_0(0) \equiv 0$. Then, the expected cost is

$$\begin{aligned} C(T) = \quad & \{c_0 \sum_{j=0}^{\infty} \int_0^K [1 - G^{(j)}(x)]dx \int_0^T \overline{F}(t)dH_j(t) \\ & + c_1 \sum_{j=0}^{\infty} G^{(j)}(K) \int_0^T H_j(t)\lambda(t)\overline{F}(t)dt + c_2\overline{F}(T) \sum_{j=0}^{\infty} H_j(T)G^{(j)}(K) \\ & + c_3 \sum_{j=0}^{\infty} [G^{(j)}(K) - G^{(j+1)}(K)] \int_0^T H_j(t)\lambda(t)\overline{F}(t)dt \\ & + c_4 \sum_{j=0}^{\infty} G^{(j)}(K) \int_0^T H_j(t)dF(t)\} / \sum_{j=0}^{\infty} G^{(j)}(K) \int_0^T H_j(t)\overline{F}(t)dt. \qquad (1) \end{aligned}$$

3. Optimal Policy

Suppose that a database is updated at a Poisson process with rate $p\lambda$, *i.e.*, $\lambda(t) = p\lambda$ and $H_j(t) = \frac{[p\lambda t]^j}{j!} e^{-p\lambda t}$. Further, $c_0(x) = c_0 x$, $G_j(x) = 1 - e^{-\mu_j x}$, and $1/\mu_j \equiv \alpha^{j-1}/\mu$.

A necessary condition that a finite T^* minimizes $C(T)$ is given by differentiating $C(T)$ with respect to T and setting it equal to zero as follows:

$$\sum_{j=0}^{\infty} [(c_3 - c_2 - c_0\alpha^j/\mu)G^{(j+1)}(K) - U(T)G^{(j)}(K)] \int_0^T H_j(t)p\lambda e^{-(1-p)\lambda t}dt = c_2, \qquad (2)$$

where

$$U(T) \equiv \frac{\sum_{j=0}^{\infty}(c_3 - c_2 - c_0\alpha^j/\mu)G^{(j+1)}(K)H_j(T)}{\sum_{j=0}^{\infty} G^{(j)}(K)H_j(T)}. \qquad (3)$$

Let $Q(T)$ be the left-hand side of (2). Note that if $G^{(j+1)}(K)/G^{(j)}(K)$ is strictly decreasing in j when $G_j(x) = 1 - e^{-\mu_j x}$. Thus, if $c_3 > c_2 + c_0/\mu$ then $U(T)$ is strictly decreasing in T, and hence the $Q(T)$ is strictly increasing in T. Therefore, if $Q(\infty) > c_2$ then there exists a finite and unique T^* $(0 < T^* < \infty)$ which satisfies (2), and the resulting cost is

$$C(T^*)/\lambda = pc_1 - c_2 + pc_3 + (1-p)c_4 - pU(T^*). \qquad (4)$$

References

[1] S. Osaki:*Applied Stochastic System Modeling.* (Springer Verlag, Berlin, 1992).

[2] K. Suzuki and K. Nakajima: Storage Management Software. *Fujitsu,* **46**(1995) 389-397.