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On the Optimal Control of Discrete Event Systems Based on Supervisory Control

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1 Introduction

Discrete event systems (DESs) are those systems that are driven for often occurring finite events. Supervisory control of DESs was presented by Ramadge and Wonham [1] with two bran-ches, the event feedback control and state feedback control.

Passino and Antsaklis [2] studied an optimal control problem to minimize the total cost among strings from the initial state to some given target state subset. Tsitsiklis [3] presented a dynamic programming model to solve the supervisory control synthesizing problem. Kumar and Garg [4] studied an optimal static control problem with two cost functions $c(q,\sigma)$ and $p(q)$ that occur only once.

But all the above researches have not concerned on general frameworks for optimal control of DESs. This paper will present and study a new framework and an analysis for the optimal control problem of DESs.

2 Notations and Preliminaries

A discrete event system based on automatons is $G=\{Q,\ \Sigma,\delta,q_0\}$, where Q is a countable state space, Σ is a finite event set, δ is a partial function from $\Sigma\times Q$ to Q, while $q_0\in Q$ is the initial state. We denote by Σ^* (or Σ^ω) the set of all finite (or infinite) strings on Σ including the empty string ϵ. δ is generalized to $\delta(s,q)$ for $s\in\Sigma^*$, and $\delta(s,q)!$ denotes that $\delta(s,q)$ is well defined.

Let $\Sigma(q)=\{\sigma|\delta(\sigma,q)!\}$ and $\Sigma(s)=\{\sigma|\ \delta(s\sigma, q_0)!\}$ be sets of events that may occur at state q or after string s, respectively. The language and the infinite language generated by G is defined by $L(G)=\{s\in\Sigma^*|\delta(s,q_0)!\}$ and $L^\omega(G)=\{s\in\Sigma^\omega\mid t\in L(G),\forall t\leq s\}$, respectively.

The event set Σ is divided into a uncontrollable event set Σ_u and a controllable event set Σ_c. The set of control inputs is $\Gamma:=\{\gamma:\Sigma_u\subset\gamma\subset\Sigma\}$. A policy (or supervisor) is a map $\pi:L(G)\to\Gamma$. A stationary policy (or state feedback) is $f:Q\to\Gamma$. The two branches in the supervisory control, the event feedback control and the state feedback control, are based on $\pi\in\Gamma^{L(G)}$ and $f\in\Gamma^Q$, respectively.

We denote by $L(\pi/G)$ and $L^\omega(\pi/G)$, respectively, the language and the infinite language generated by the system supervised under π, and by $L(\pi/G,s)$ the language similarly as $L(\pi/G)$ but with an initial state $\delta(s,q_0)$. And Let $L^\omega(\pi/G,s)$ be the similar set of infinite strings.

3 Optimal Control Model

Supervisory control of DESs belongs to the logic level for control of DESs [1], since its essential task is to constrain the system's behavior, described by a set of strings of occurring events or a state subset in a given region.

The control task for the supervisory control is hard where some behaviors are allowed and other behaviors are strictly prohibited.

The new framework for the optimal control problem of DESs presented in this paper belongs to the performance level among all possible strings that the system generates. Its performance measure is to minimize the maximal discounted total cost. Then this framework can describe soft control problems where it is better if some behaviors are realized. We will show the optimality equation and obtain its optimal solution.

Suppose that there is a cost function $c(s,\sigma)$ for occurring an event σ after a string s. Let $\beta>0$ be a discount factor. Let

$$v_s(t)=\sum_{k=0}^{n}\beta^k c(st_k,\sigma_k)$$

be the discounted total cost occurring $t=\sigma_0\cdots\sigma_n$ after s has been occurred, where $t_k=\sigma_0\cdots\sigma_{k-1}$ for $k\geq 1$ ($t_0=\epsilon$). When the string t is infinite, $n=\infty$. We call $v_s(t)$ *the cost of* t *after* s.

We introduce a fictitious event $\sigma_J\notin\Sigma$ and a fictitious state $q_J\notin Q$ such that σ_J leads the system from any state to q_J without any cost.

For convenience, we also write this new DES by G. Then any finite string $s\in L(G)$ is corresponding to an infinite string $s\sigma_J^\infty$ in the new DES, and

$v_s(t) = v_s(t\sigma_J^\infty)$ for strings s and t. Therefore, we define the objective function by

$$I(\boldsymbol{\pi}, s) = \max_{t \in L^\omega(\boldsymbol{\pi}/G,s)} v_s(t), \quad s \in L(G)$$

to be the maximal discounted total cost under policy $\boldsymbol{\pi}$, which should be minimized as follows:

$$I^*(s) = \min_{\boldsymbol{\pi} \in \Gamma^{L(G)}} I(\boldsymbol{\pi}, s)$$

for $s \in L(G)$. We say a supervisor $\boldsymbol{\pi}^*$ to be optimal if $I(\boldsymbol{\pi}^*, s) = I^*(s)$ for all $s \in L(G)$.

Condition (N): $c(s, \gamma)$ is nonpositive.

Condition (P): $c(s, \gamma)$ is nonnegative.

Condition (D): $c(s, \gamma)$ is uniformly bounded by a positive constant M, and $\beta \in (0, 1)$.

Under (N), (P) or (D), $v_s(t)$ and therefore $I(\boldsymbol{\pi}, s)$ are well defined and are nonpositive, nonnegative, or are uniformly bounded by $(1 - \beta)^{-1}M$, respectively.

For $\gamma \in \Gamma$ and $\boldsymbol{\pi} \in \Gamma^{L(G)}$, let $\Sigma_\gamma(s) = \Sigma(s) \cap \gamma$. Under condition (D) or (P), $I^*(s)$ is a solution of the following optimality equation (OE):

$$I(s) = \min_{\gamma \in \Gamma} \max_{\sigma \in \Sigma_\gamma(s)} \{c(s, \sigma) + \beta I(s\sigma)\}, \quad s \in L(G).$$

If a supervisor $\boldsymbol{\pi}^*$ attains the minimum of OE, then $\boldsymbol{\pi}^*$ is optimal. Moreover, I^* is the unique (or the smallest nonnegative) solution of OE under (D) (or (P)). While under (N), $I^*(s)$ is the largest nonpositive solution of OE and a supervisor $\boldsymbol{\pi}^*$ is optimal if $I(\boldsymbol{\pi}^*)$ is a solution of OE.

We then consider the stationary cost function $c(q, \sigma)$ defined in $Q \times \Sigma$ such that $c(s, \sigma) = c(\delta(s, q_0), \sigma)$. Let $v_q(t) = \sum_{k=0}^\infty \beta^k c(q'_k, \sigma_k)$ be the discounted total cost during string $t = \sigma_0\sigma_1 \cdots$ at the state q, where $q'_{k+1} = \delta(q'_k, \sigma_k)$ with $q'_0 = q$ and $k \geq 0$.

We define $I(f, q) = \max\{v_q(t) | t \in L^\omega(f/G, q)\}$ to be the maximal discounted total cost of the system supervised by f in the state q, and define $I^*(q) = \min_{f \in F} I(f, q), q \in Q$ to be the optimal value function. We say f^* to be optimal if $I(f^*, q) = I^*(q)$ for all $q \in Q$.

Therefore, $I^*(\delta(s, q_0)) \geq I^*(s)$. Under condition (D) or (N), the reverse case is also true, and so $I^*(s) = I^*(\delta(s, q_0))$ for $s \in L(G)$. Moreover, the OE has the following simpler form:

$$I(q) = \min_{\gamma \in \Gamma} \max_{\sigma \in \Sigma_\gamma(q)} \{c(q, \sigma) + \beta I(\delta(\sigma, q))\}, \ q \in Q.$$

4 Link to Logic Level

By applying the above model and results to the supervisory control, we could result that the state feedback control is a stationary case of the event feedback control.

For any given language $L \subset L(G)$, suppose that the cost function $c(s, \sigma)$ is nonnegative and satisfies the condition that $c(s, \sigma)$ is bounded in $s \in L$ and $\sigma \in \Sigma$ while $c(s, \sigma) = \infty$ for $s \notin L, \sigma \in \Sigma$. Then, $L^* = \{s \mid I^*(s) < \infty\}$ is the maximal control invariant sub-language of L and $MC(L^*) := \{s \in L^* \mid t \in L^* \text{ for all } t \leq s\}$ is the maximal closed controllable sub-language of L.

For the state feedback control, for any given predicate $P \subset Q$, suppose that $c(q, \sigma)$ is bounded in $q \in P$ and $\sigma \in \Sigma$ while $c(q, \sigma) = \infty$ for $q \notin P$, $\sigma \in \Sigma$. Then, $P^* = \{q \mid I^*(q) < \infty\}$ is the maximal control-invariant sub-predicate of P.

L^* and P^* are not only the solutions in the supervisory control, but also the solutions with finite optimal values for an optimal control problem. So the meaning of control invariant languages is more strong than just "control invariance" in the supervisory control [1].

Hence, we show a link existing between the logic level and the performance level for the control of DESs.

5 Conclusion

In this paper, we presented a new framework and an analysis for optimal control of discrete event systems based on the supervisory control. We derived the optimality equation and obtained its optimal solution.

References

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