Indivisibilities and Economies of Scope in DEA

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1. Introduction

This paper concentrates on a multi-stage production process where idle capacity arises due to unequal length of production runs of intermediate stages, which leads to *scale* effects[1] when production is expanded. If the demand for output is downward slopping, then instead of scaling up existing output merely on basis of capacity utilization, the firm could also use the existing idle capacities to diversify into other products so as to enjoy economies of scope (ES).

The empirical estimation of cost structure has shed little light on the kinds of production processes that lead to *scope* economies. In this paper we have made an attempt to suggest a new data envelopment analysis (DEA) model to estimate a cost frontier revealing scope economies arising from task-specific idle capacities in the multistage production model.

2. Nature of production model

We look at production as a *task-specific process* in which production is broken down into its various principle stages. The idea is to bring out inherent hidden *indivisibilities* of the activities by observing the task-length associated with each stage. The main observation is that production process usually consists of more than one stage, and the task-lengths associated with various stages need not be equal. This is because different pieces of capital equipment used at different stages of production serve different purpose and are designed with respect to that purpose at hand with the existing technical know-how. Now the question for ES to hold good is whether the set of tasks executed at any given point of time allows full and continuous utilization of all factors of production or not. The answer largely depends on how the tasks are arranged in the production process. In our full paper we have considered an example of a production process for the manufacture of door-and-window frames to empirically show in a DEA framework how *scope* effects occur due to indivisibilities arising from unequal task-length associated with various stages of production.

[1] On the evolution of the concept of scale and its estimation procedures, see, among others, Sahoo *et al.* (1999) and Tone and Sahoo (2003a,b).

3. DEA model for measuring ES

Baumol *et al.* (1982) define ES to exist between two products if the cost of producing two products by one firm is less than the cost of producing them separately in specialized firms, i.e., $C(y_1,y_2)<C(y_1,0)+C(0,y_2)$, where $C(y_1,y_2)$ is the cost of joint production by the diversified firm, $C(y_1,0)$ and $C(0,y_2)$ are the respective costs of production of y_1 and y_2 by two specialized firms. So the local degree of economies of scope (DES) for firm j is defined as

$$\text{DES}_j=\frac{C(y_1,0)+C(0,y_2)-C(y_1,y_2)}{C(y_1,y_2)}.$$

DESj > 0 implies that the firm j exhibits economies of scope, DESj < 0 implies diseconomies of scope, and DESj = 0 implies that cost function $C(y_1,y_2)$ is additive in nature.

We assume here to deal with n diversified firms, each using m inputs to produce s outputs. For each firm 'o' (o = 1,2,…,n) we denote respectively the input/output vectors by $x_o \in R^m$ and $y_o \in R^s$. Given the unit input price vector $c_o \in R^m$ (> 0) for the input x_o of firm 'o', the cost efficiency (CE) is defined as

$$\gamma^*=c_o x_o^*/c_o x_o=\sum_{i=1}^{m}c_{io}x_i^*/\sum_{i=1}^{m}c_{io}x_i,$$

where x_o^* is an optimal solution of the following linear programming problem (LP):

$$\text{[Cost]}\quad C(y_o;c_o)=\min\sum_{i=1}^{m}c_{io}x_i$$

$$\text{subject to}\ \sum_{j=1}^{n}x_{ij}\lambda_j\le x_i\ (\forall i)$$

$$\sum_{j=1}^{n}y_{rj}\lambda_j\ge y_{ro}\,(\forall r)$$

$$\sum_{j=1}^{n}\lambda_j=1$$

$$\lambda_j\ge 0\,(\forall j).$$

Now we need to compare the minimal cost of these n *diversified* firms along with their observed outputs with a frontier consisting of *additive* firms satisfying the condition: DES = 0. These additive firms are hypothetical ones, which are all created from *specialized* firms. Assuming there are n_1 firms producing output y_1 alone and n_2 firms producing output y_2 alone. All possible permutations of the outputs and costs of these two sets of specialized firms are added pair wise to form the set of hypothetical additive firms. Let the number of additive firms be k *whose* output and cost of these firms are associated with superscript '+'. So in order to calculate economies of scope for the diversified firm 'o', we need to solve the following LP:

$$[\text{Cost\_m}] \quad C^+(y_o;c_o)=\min\sum_{i=1}^{m} c_{io}x_i$$

$$\text{subject to} \quad \sum_{j=1}^{k} x_{ij}^{+}\lambda_j \le x_i \ (\forall i)$$

$$\sum_{j=1}^{k} y_{rj}^{+}\lambda_j \ge y_{ro} (\forall r)$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\lambda_j \ge 0 (\forall j).$$

Here $C^+(y_o;c_o)$ represents the minimum cost of production of output vector y_o in the additive technology set when input price vector faced by firm 'o' is c_o. DES_o is defined as:

$$DES_o=\frac{C^+(y_o;c_o)}{C(y_o;c_o)}-1.$$

Tone (2002) observed several shortcomings in the above cost efficiency evaluation model, and suggested a new model to define CE as

$$\bar{\gamma}^* = \bar{ec}_o^* / \bar{ec}_o,$$

where $\bar{c}_o^*$ is an optimal solution of the LP below:

$$[\text{NCost}] \quad \bar{ec}^* = \min \sum_{i=1}^{m} e_i\bar{c}_i$$

$$\text{subject to} \quad \sum_{j=1}^{n} \bar{c}_{ij}\lambda_j \le \bar{c}_i \ (\forall i)$$

$$\sum_{j=1}^{n} y_{rj}\lambda_j \ge y_{ro} \ (\forall r)$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\lambda_j \ge 0 \ (\forall j).$$

Let us denote $\bar{ec}_j$ by $\bar{\bar{c}}_j$, i.e.,

$$\bar{\bar{c}}_j=\sum_{i=1}^{m}\bar{c}_{ij}=\sum_{i=1}^{m}x_{ij}c_{ij}. \ (j=1,\ldots,n)$$

$\bar{\bar{c}}_j$ is the total input cost of firm j for producing the output vector y_j. Using this notation, we have a new scheme as expressed by the following LP:

$$[\text{NCost-1}] \quad \bar{\bar{c}}^* = \min 1.\bar{\bar{c}}$$

$$\text{subject to} \quad \sum_{j=1}^{n}\bar{\bar{c}}_j\lambda_j - 1.\bar{\bar{c}} \le 0$$

$$\sum_{j=1}^{n} y_{rj}\lambda_j \ge y_{ro} \ (\forall r)$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\lambda_j \ge 0 \ (\forall j).$$

We can also express [Ncost-1] in a simple input oriented BCC model as follows:

$$[\text{NCost-1E}] \quad \theta^* = \min \theta$$

$$\text{subject to} \quad \sum_{j=1}^{n}\bar{\bar{c}}_j\lambda_j \le \theta\bar{\bar{c}}_o$$

$$\sum_{j=1}^{n} y_{rj}\lambda_j \ge y_{ro} \ (\forall r)$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\lambda_j \ge 0 \ (\forall j).$$

Analogous to the procedure discussed above, to compute DES for diversified firm 'o', we need to solve the following LP:

$$[\text{NCost-1E}] \quad \theta^{+*} = \min \theta^+$$

$$\text{subject to} \quad \sum_{j=1}^{k}\bar{\bar{c}}_j^{+}\lambda_j \le \theta^+\bar{\bar{c}}_o^{*}$$

$$\sum_{j=1}^{k} y_{rj}^{+}\lambda_j \ge y_{ro} \ (\forall r)$$

$$\sum_{j=1}^{k} \lambda_j = 1$$

$$\lambda_j \ge 0 \ (\forall j).$$

The degree of economies of scope (DES_o) is defined as θ^{+*} minus one, i.e.,

$$DES_o = \theta^{+*} - 1.$$

4. Concluding remark

We have discussed use of a new DEA model to estimate a new cost frontier exhibiting scope economies arising from *process indivisibilities*.

References

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