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CUSTOMERS SELECTION PROBLEM WITH IDLING PROFIT WHERE MULTIPLE CUSTOMERS CAN BE HELD IN THE SYSTEM

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□ *Introduction* This paper deals with the problem of selecting profitable customers to accept out of sequentially arriving ones in a custom production company. In the problem, if orders of all arriving customers are accepted irrespective of their profitabilities, the production process would soon become full; as a result, orders of customers arriving after that could not be accepted however high profitabilities they have. On the contrary, excessively refraining from accepting orders under the apprehension of system's being full causes a few number of backorders. Then production process may soon become idle. This two cases cause the diminishment of long run profit to the company. Hence the objective here is to find the optimal customers selection rule so as to maximize the expected long run profit through keeping an appropriate level of backorder by controlling the number of orders to accept in advance. This problem is usually called the customers selection problem. In the problem, it has been implicitly assumed that a customer *first* offers the price of his order, and judging from this, the company decides whether or not to accept it; let us refer to this problem as that of *customer-first-case* [1, Ikuta] [2, Lippman and Ross] [3, Miller].

In the paper, by n let us denote the maximum permissible number of orders which can be held in the system at any instant. In [4, Son] we discussed the problem in detail on the assumption of $n = 1$, so that we only examine the case of $n \geq 2$ where a new problem arises of clarifying the relationship of the optimal customers selection rule with the number of backorders.

□ *Model* The model examined in the paper is defined on the following assumptions, which are not taken into considerations in any three papers cited above.

1) It is only when search is conducted by paying a *search cost* $c \geq 0$ at a point in time that an customer arrives at the next point in time with a probability λ $(0 < \lambda \leq 1)$. The introduction of search cost inevitably yields the option whether to skip the search or not.
2) We also consider the *system-first-case* that the system first offers the price z for an order, judging from this, the customer decides whether or not to place the order with the company. Let us assume that each appearing customer has a maximum permissible ordering price w, implying that if and only if $z \leq w$, the customer is willing to place the order with the system (then the ordering price is z). Then the probability of the customer placing the order with the system is given by $p(z) = \Pr\{z \leq w\}$.
3) When there exists no backorder in the system, an *idling profit* $s \geq 0$ is yielded by engaging in other economic activities using the idle production line.

Further, let the prices offered by subsequently appearing customers, $w, w', \cdots$, in the customer-first-case and the maximum permissible ordering prices of subsequently appearing customers, $w, w', \cdots$, in the system-first-case be both independent identically distributed random variables having a known continuous distribution function $F(w)$ with a finite expectation μ. With a probability q $(0 < q < 1)$ an order in the system at a certain point in time is completed and goes out of the system up to the next point in time. Let the discount factor be denoted by $\beta < 1$.

The objective is to find the optimal decision rule so as to maximize the total expected present discounted net profit gained over an infinite planning horizon, the total expected present discounted value of prices of orders accepted or placed *plus* the idling profits *minus* the total expected present discounted value of search costs.

□ *Optimal Equations* To begin with, let us define $m = \mu$ for the customer-first-case and $m = \max_z p(z)z$ for the system-first-case. If no customer appears with probability $1 - \lambda$ at the present point in time, we shall refer to such a situation as "the system has a *fictitious order* ϕ". Then by $u(\phi, i)$ we shall denote the maximum of the total expected present discounted net profit starting from a state of having the fictitious order ϕ and $i\,(0 \leq i \leq n)$ orders in the system; let us refer to such a situation as the state (ϕ, i). By the notations C, K, A, and R let us denote the decisions of, respectively, continuing the search, skipping the search, accepting an order, and rejecting an order. Now, let us define

$$h_i = u(\phi, i) - u(\phi, i+1), \quad 0 \le i < n. \tag{0.1}$$

1. Customer-first-case: By $u(w, i)$ let us denote the maximum of the total expected present discounted net profit starting with $i\,(0 \le i < n)$ orders in the system and an arriving customer who offers a price w.

$$u(\phi, 0) = \max \begin{cases} \mathtt{C}: \beta\big(\lambda \mathbf{E}[u(\xi, 0)] + (1-\lambda)u(\phi, 0)\big) - c + s, \\ \mathtt{K}: \beta u(\phi, 0) + s, \end{cases} \tag{0.2}$$

$$u(\phi, i) = \max \begin{cases} \mathtt{C}: (1-q)\beta\big(\lambda \mathbf{E}[u(\xi, i)] + (1-\lambda)u(\phi, i)\big) \\ \qquad +q\beta\big(\lambda \mathbf{E}[u(\xi, i-1)] + (1-\lambda)u(\phi, i-1)\big) - c, \quad 1 \le i < n, \\ \mathtt{K}: (1-q)\beta u(\phi, i) + q\beta u(\phi, i-1), \end{cases} \tag{0.3}$$

$$u(\phi, n) = \max \begin{cases} \mathtt{C}: (1-q)\beta u(\phi, n) + q\beta\big(\lambda \mathbf{E}[u(\xi, n-1)] + (1-\lambda)u(\phi, n-1)\big) - c, \\ \mathtt{K}: (1-q)\beta u(\phi, n) + q\beta u(\phi, n-1), \end{cases} \tag{0.4}$$

$$u(w, i) = \max \left\{ \begin{array}{l} \mathtt{A}: w + u(\phi, i+1) \\ \mathtt{R}: u(\phi, i) \end{array} \right\} = \max\{w - h_i, 0\} + u(\phi, i), \quad 0 \le i < n. \quad \square \tag{0.5}$$

2. System-first-case: By $u(1, i)$ let us denote the maximum of the total expected present discounted net profit starting with $i\,(0 \le i < n)$ orders in the system and an arriving customer to whom the system offers an ordering price z. Then the optimal equations in state $u(\phi, i)$ for $0 \le i \le n$ can be written in almost the same way as those in Eqs. (0.2) to (0.4) except that $\mathbf{E}[u(\xi, i)]$ is replaced with $u(1, i)$. Further, we have

$$u(1, i) = \max_z \{p(z)\big(z + u(\phi, i+1)\big) + (1 - p(z))u(\phi, i)\} = \max_z p(z)(z - h_i) + u(\phi, i), \quad 0 \le i < n.$$

By $z(h_i)$ let us designate the z attaining the maximum of $p(z)(z - h_i)$ on $z \in (-\infty, \infty)$. □

□ *Conclusions* We have proved the existence of $s_i (0 \le i \le n)$ and $s^* (0 < s^* \le s_0)$ which are closely related to the optimal decision rules and the monotonicity of h_i, respectively. Then the optimal decision rules are described as follows.

(a) Let $\lambda\beta m \le c$ or "$\lambda\beta m > c$ and $s_0 \le s$". Then $\langle \mathtt{K} \rangle_0$[1], hence not conducting the search, i.e., skipping the search is optimal.

(b) Let $\lambda\beta m > c$ and $s < s_0$. Then since $\langle \mathtt{C} \rangle_{0 \le i < n}$, it is optimal to conduct the search by paying a search cost c. If $i = n$, any of continuing the search and skipping the search may be optimal: more precisely, if $s < s_n$, then $\langle \mathtt{C} \rangle_n$, or else $\langle \mathtt{K} \rangle_n$.

  1 Let $s^* \le s$. Then h_i is not always nondecreasing in i; in other words, there exists a $i^*(s) \ge 1$ such that h_i is decreasing in $i \le i^*(s)$ and increasing in $i > i^*(s)$.

  2 Let $s < s^*$. Then h_i is nondecreasing in i with $h_0 \le h_1$ where if $h_0 < h_1$, then h_i is strictly increasing in i.

In the system-first-case it should be noted that the monotonicity of h_i in i stated above is inherited to the optimal price $z(h_i)$.

References

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[1] The notation $\langle \mathtt{K} \rangle_0$ implies that skipping the search is optimal in state $(\phi, 0)$.