日本オペレーションズ・リサーチ学会
2004年春季研究発表会

Structural Analysis of Optimal Investment Strategy for Project Management via Real Option Approach

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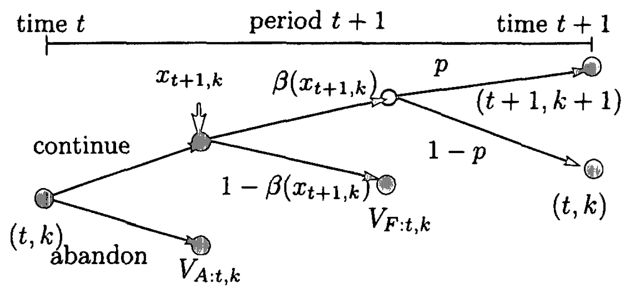
1 Introduction

In this research, based on a broader definition of ROA(Real Option Approach), a Dynamic Programming approach is proposed. The optimal investment strategy is incorporated explicitely within the decision structure of the model, thereby extending the previous work by Huchzermeier and Loch [1] substantially. Structural properties of the optimal investment strategy are investigated in detail, establishing certain monotonicity properties of the optimal project value and the optimal investment amount.

2 Model Description

We consider a project management problem over T periods. At time $t = 0$, the present project value S_0 is obtained as the expected discounted cash flow to be generated by this project when the project is completed as planned. In managing this project toward the end of period T, one has an option to terminate the project at the beginning of each period t, $1 \le t \le T$, with salvage value $V_{A:t+1,k}$. The decision criterion for this option will soon become clear. If it is decided to continue the project, the investment amount for this period $x_{t+1,k}$ should be determined. The project may be carried out successfully to the next period $t+1$ with probanility $\beta(x_{t+1,k})$ or may fail with probanility $1-\beta(x_{t+1,k})$ where $0 \le \beta(x_{t+1,k}) \le 1$. The success probability may depend on time and the investment amount. When successful, the value of the underlying asset increases by a factor of u with probability p or decreases by a factor of d with probability $1-p$ where $0 < d \le u$ and $0 \le p \le 1$. When the project fails in period t, the project is forced to stop with salvage value $V_{F:t+1,k}$.



3 Formulation of Optimal Investment Policy: Dynamic Programming Approach

For notational convenience, let $G(\lambda, A, B)$ be defined by $G(\lambda, A, B) = \lambda A + (1-\lambda)B$ where $0 \le \lambda \le 1$.

Problem $(DP - \hat{V})$ (without Option)

$$\max_{[\hat{x}_{t,k}]} \hat{V}_{0,0}$$

subject to

$$\hat{V}_{t,k} = G\Big(\beta(\hat{x}_{t+1,k}), G(p, \hat{V}_{t+1,k+1}, \hat{V}_{t+1,k}), V_{F:t+1,k}\Big)e^{-r} - \hat{x}_{t+1,k},$$
$$x_{t+1,k} \ge 0,\ 0 \le k \le t,\ 0 \le t \le T-1;$$

$$S_{T,k} = S_0 u^k d^{T-k}, \quad 0 \le k \le T; \qquad (3.1)$$
$$\hat{V}_{T,k} = S_{T,k} - X_T, \quad 0 \le k \le T;$$
$$V_{F:t+1,k} = W_F(S_{t,k}), \qquad (3.2)$$
$$0 \le k \le t, \quad 0 \le t \le T-1; \ and$$
$$V_{A:t+1,k} = W_A(S_{t,k}), \qquad (3.3)$$
$$0 \le k \le t, \quad 0 \le t \le T-1.$$

Problem $(DP - V)$ (with Option)

$$\max_{[x_{t,k}]} V_{0,0}$$

subject to (3.1), (3.2), (3.3) and

$$V_{t,k} = \max \Big[G\Big(\beta(x_{t+1,k}), G(p, V_{t+1,k+1}, V_{t+1,k}), V_{F:t+1,k}\Big)e^{-r} - x_{t+1,k}, V_{A:t+1,k}\Big],$$
$$x_{t+1,k} \ge 0,\ 0 \le k \le t,\ 0 \le t \le T-1;$$
$$V_{T,k} = S_{T,k} - X_T, \quad 0 \le k \le T.$$

It is natural to assume that $\beta(x)$,$W_F(x)$ and $W_A(x)$ are zero without investment and are strictly increasing and concave.

For the option value at state (t, k), we define

$$ROV^*_{t,k} = V^*_{t,k} - \hat{V}^*_{t,k}.$$

Of particular interest is to find the option value $ROV^*_{0,0}$ and the associated optimal investment strategy $(x^*_{t,k})$ at the start of the project for the optimal investment policy problem.

4 Structural Properties of DP-$\hat{V}$ and DP-V

Theorem 4.1 *Whenever it is decided to continue the project at* (t,k), *both* $DP-\hat{V}$ *and* $DP-V$ *have the unique optimal investment amounts* $\hat{x}^*_{t+1,k}$ *and* $x^*_{t+1,k}$ *respectively for all* k, $0 \le k \le t$, *and all* t, $0 \le t \le T-1$.

Theorem 4.2 *Let* $0 \le k \le t-1$ *for* $1 \le t \le T$. *Then:*

a) $\hat{V}^*_{t,k+1} > \hat{V}^*_{t,k}$

b) $V^*_{t,k+1} > V^*_{t,k}$

Theorem 4.3 *For* $0 \le k \le t$ *and* $0 \le t \le T-1$, *the following statements hold.*

a) $\hat{V}^*_{t,k}$ *is strictly increasing in* p, u, *and* d.

b) $V^*_{t,k}$ *is nondecreasing in* p.

c) $V^*_{t,k}$ *is strictly increasing in* u *and* d.

Theorem 4.4 *Let* $\beta = \beta(x,b) = \frac{bx}{1+bx}$. *Then* $\hat{V}^*_{t,k}$ *is strictly increasing and* $V^*_{t,k}$ *is nondecreasing in* b.

Theorem 4.5 *For* $0 \le k \le t$ *and* $0 \le t \le T$, $ROV^*_{t,k} = V^*_{t,k} - \hat{V}^*_{t,k} \ge 0$.

Theorem 4.6 *If it is decided to continue the project at state (t,k), then* $x^*_{t+1,k} \ge \hat{x}^*_{t+1,k}$, $0 \le k \le t$, $0 \le t \le T-1$.

Assumption 4.7

a) $W_F(x) = \alpha x$, $\alpha > 0$

b) $pu + (1-p)d > 1$

For notational convenience,

$$\begin{aligned}
\Delta_k a_k &= a_k - a_{k-1}, \quad k \ge 1; \\
\Delta_z \xi(z) &= \xi(z_1) - \xi(z_2) \quad where \quad z_1 > z_2; \\
\hat{A}_{t,k} &= G(p, \hat{V}^*_{t+1,k+1}, \hat{V}^*_{t+1,k})e^{-r}; \\
A_{t,k} &= G(p, V^*_{t+1,k+1}, V^*_{t+1,k})e^{-r}; \\
B_{t,k} &= V_{F:t+1,k}e^{-r}.
\end{aligned}$$

Theorem 4.8 *Under assumption 4.7, one has for* $0 \le k \le t$, $0 \le t \le T-1$*:*

a) $\Delta_k \hat{A}_{t,k} - \Delta_k B_{t,k} \ge 0$; $\Delta_k A_{t,k} - \Delta_k B_{t,k} \ge 0$

b) $x^*_{t+1,k}$ *and* $\hat{x}^*_{t+1,k}$ *are strictly increasing in* k.

Theorem 4.9

a) *Let* $u_1 > u_2$ *and suppose Assumption 4.7 is satisfied where* $u = u_2$. *Then*

$$\min\{\Delta_u A_{t,k}(u), \Delta_u \hat{A}_{t,k}(u)\} > \Delta_u B_{t,k}(u)$$

and both $x^*_{t,k}$ *and* $\hat{x}^*_{t,k}$ *are strictly increasing in* u.

b) *Let* $d_1 > d_2$ *and suppose Assumption 4.7 is satisfied where* $d = d_2$. *Then*

$$\min\{\Delta_d A_{t,k}(d), \Delta_d \hat{A}_{t,k}(d)\} > \Delta_d B_{t,k}(d)$$

and both $x^*_{t,k}$ *and* $\hat{x}^*_{t,k}$ *are strictly increasing in* d.

Theorem 4.10 *Let* $p_1 > p_2$. *Then*

$$\min\{\Delta_p A_{t,k}(p), \Delta_p \hat{A}_{t,k}(p)\} > \Delta_p B_{t,k}(p)$$

and both $x^*_{t,k}$ *and* $\hat{x}^*_{t,k}$ *are strictly increasing in* p.

5 Concluding Remarks

In this research, two DP models have been developed to investigate the optimal investment policy $(x^*_{t,k})$, the present value of the project $(V^*_{0,0})$, and the option value $ROV^*_{0,0}$ based on the broader definition of ROA. Structural properties of $\hat{V}^*_{t,k}$ and $V^*_{t,k}$ are examined analytically, proving the unique existence of optimal investment policy and yielding useful monotonicity results. Analytical results combined with extensive numerical experiments revealed the following observations.

(5.1) Both the project value at time t with k upward successes, $V^*_{t,k}$, and the associated investment amount $x^*_{t,k}$ increase as a function of k.

(5.2) As the risk-potential of the project increases: 1) $V^*_{0,0}$ decreases to the lower bound determined by the value of doing nothing from the very beginning, and then stays at the level; 2) $\hat{V}^*_{0,0}$ decreases; and the option value $ROV^*_{0,0}$ increases.

References

[1] Huchzermeier, A. and C. H. Loch, "Project Management Under Risk: Using the Real Option Approach to Evaluate Frexibility in R&D," Management Science, Vol. 47, No.1, Jnauary 2001 pp. 85-101