Warranty Cost Analysis: Modelling the Repair Time in Free Replacement Renewing Warranty

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The growth of using the product warranty as a strategic tool has increased quite significantly over the past decade. A product warranty is an agreement offered by a producer to a consumer to repair or replace a faulty item, or to partially or fully reimburse the consumer in the event of a failure (see [1, 2]). There are two types of warranty coverage used in industry and discussed in the literature :

- Non-renewing warranty: A newly sold item is covered by a warranty for some calendar time of duration T. The warranter assumes all, or a portion of the expenses for the failure of the product until the expiration of the warranty coverage. Non-renewing warranty with non-zero repair time has been studied in [3].

- Renewing warranty: The warranter agrees to repair or replace any failed item up to time T from the time of purchase. At the end of the repair time within an existing warranty, the item is warranted anew for a period of length T.

We aim to evaluate the warranty expenses under renewing free replacement warranty over the warranty coverage and over the life cycle of the item.

Acronyms

cdf	cumulative distribution function
iid	independent and identically distributed
pdf	probability density function

Notation

X	lifetime of the product with cdf $F_X(x)$ and pdf $f_X(x)$
Y	warranty repair time with cdf $F_Y(y)$ and pdf $f_Y(y)$
C_i	random cost of the i^{th} repair
T	predetermined warranty period
W_T	random warranty coverage with warranty period of T

Model

We consider the following model: Initially the item is in operating condition for length of time X_1. Then the repair condition starts and the item remains in it for a time Y_1. After the repair, the item is operative for a time X_2 which is followed by a repair for a time Y_2 and so on. It is assumed that $\{X_i\}_1^\infty$ and $\{Y_i\}_1^\infty$ are independent sequences of iid random variables. The cost of the i^{th} repair has the form $C_i = A + \delta Y_i$ and it is assumed that $E(C) = E(C_i) = A + \delta E(Y_i) = A + \delta E(Y)$ where A and δ are prespecified constants. At the end of the repair time, the item is warranted anew for a period of length T. Life cycle of a product is defined as a time while the product is still usable and contemporary. It is assumed, that during the life cycle, after the expiration of the warranty coverage for the initially purchased item, at the time of the first off warranty failure, the consumer purchases an identical to the initial item with the same warranty.

Expected cost under renewing warranty coverage

The warranty coverage W_T can be represented as

$$W_T = \begin{cases} T, & \text{if } X_1 > T \\ T + \sum_{i=1}^{n}(X_i + Y_i), & \text{if } X_1 \le T, \cdots, X_n \le T, X_{n+1} > T \text{ for some } n. \end{cases}$$

Then, the warranty cost $C(W_T)$ over the warranty coverage is a random variable with geometric distribution with parameter $(1 - F_X(T))$. Thus

$$E(C(W_T)) = \frac{F_X(T)}{1 - F_X(T)} \quad (A + \delta E(Y))$$

Expected costs under renewing warranty coverage over life cycle

Let L^* be a prespecified time during which a product is considered to be contemporary and competitive with similar products in the market. Let L be the time of the first off-warranty failure of the product after L^*. Then, we call $(0, L)$ a life cycle of an item. Let ξ represent the time between two consecutive purchases, i.e.,

$$\xi = \begin{cases} X_1 & \text{if } X_1 > T \\ \sum_{i=1}^{n}(X_i + Y_i) + X_{n+1} & \text{if } X_1 \leq T, \cdots, X_n \leq T, X_{n+1} > T \text{ for some } n. \end{cases}$$

Then, the expected cost over $(0, L)$ is given by

$$E(C(L)) = (m_\xi^*(L) + 1)E(C(W_T)), \tag{1}$$

where $m_\xi^*(t)$ is the renewal function of the renewal process generated by ξ. By introducing the age parameter τ for ξ, we derive an integral equation for the pdf, $g_\xi(\tau, t)$, namely

$$g_\xi(\tau, t) = f_X(t) + \int_0^{t-T} g_\xi(\tau + s, t - s) f_T(s) ds, \quad t > T, \tau \geq 0 \tag{2}$$

where $f_T(s) = \int_0^{T \wedge s} f_X(u) f_Y(s - u) du$. Exploiting the specific form of (2), we use `Mathematica` to write a code for the numerical procedure for obtaining the density $g_\xi(t)$. We thank Prof. Estate Khmaladze for the useful discussions during this stage of our study.

We use the results of the numerical procedure to evaluate the renewal function generated by ξ needed in (1). Based on [4], a renewal equation solver has been written by Dr Richard Arnold in programming language `R`. We thank him for the permission to use his code. As an illustration of the ideas we consider an example assuming that X and Y are exponentially distributed. The same procedure is valid for general distributions of X and Y.

References

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