Optimization and simulation model analyses for the ambulance location problem

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1. Introduction

Ambulance center location problems have been investigated for long time by many researches. We try to solve this problem by applying optimization methods and computer simulation techniques. Two objectives are taken into account in our analyses. One is the efficiency of the ambulance service system, which is measured by the total traveling distance of the ambulance cars. Another is the uniformity of the services, which is measured by the differences among the individual ambulance teams with respect to the total number of calls and the total traveling distance. We first give the mathematical formulation of the problem, and report the application results on a part of Tokyo metropolitan area. Queuing simulations are also given finally.

2. Optimization model

Given a set of blocks, denoted by N, in the ambulance service area, each block $j \in N$ receives p_j calls for the ambulance service in one year. Suppose K ambulance cars are available in this area. Which blocks should the ambulance car stations be placed? We first try to minimize the total traveling distance of the ambulance cars, leading to the following formulation of the ambulance location problem:

$$\begin{array}{lll} \text{minimaze} & \sum_{i,j=1}^{n} d_{ij}x_{ij} & \\ \text{subject to} & \sum_{i=1}^{n} x_{ij} \geq p_j & j \in N \\ & \sum_{j=1}^{n} x_{ij} \leq M z_i & i \in N \\ & \sum_{i=1}^{n} z_i = K & \\ & x_{ij} \geq 0,\ z_i \in \{0,1\} & \end{array} \qquad (1)$$

where

- $z_i = 1$, if an ambulance station is placed on the block i; $z_i = 0$, otherwise.
- x_{ij} is the number of calls in the block j which are covered by the ambulance placed on the block i.
- p_j is the number of calls in the block j.
- d_{ij} is the distance between the centroid of the block i and j.
- M is the maximum capacity of an ambulance team for one year.
- K is the number of ambulance cars to be located.

This is a (version of) capacitated facility location problem. The capacity M plays a role to maintain uniformity in the number of calls covered by each ambulance team. In this model we use the capacity M as a parameter to find a solution with a balance between the efficiency and uniformity.

3. Model application

We apply the model (1) to Shinjuku ward in Tokyo metropolitan area. Shinjuku ward has 151 blocks ($N = \{1, 2, \ldots, 151\}$) and received approximately 25000 ambulance calls on weekdays in 2002. There are 10 ambulance cars in Shinjuku ward ($K = 10$). First we analyze the actual location of ambulance stations. Fig. 1 shows actual location of ambulance stations by ○ or □. Since there are two stations on each two ambulance cars stand by under actual deployment, eight stations are shown in the figure. Those stations which equips two ambulances are designated by □, others are shown by ○. We solved the model (1) by fixing the variables z_i to the actual positions and $M = 6000$. According to the optimal solution x_{ij} the partition of the area are shown in Fig. 1. The optimal value of objective function is shown in the second column of Table.1. To find the uniformity of the service we investigate the variance of the total traveling distance of the individual ambulance car. Let $L^* = \{i : z_i = 1\}$ be the set of positions which ambulance station is located. For $i \in L^*$, $d_i = \sum_{j=1}^{n} d_{ij}x_{ij}$ is the total traveling distance of the ambulance car located at i. We calculate the sample standard deviation of d_i, that is

$\hat{s}_d = \sqrt{\frac{1}{K-1}\sum_{i=1}^{K}(d_i - \bar{d})^2}$, where $\bar{d} = \frac{1}{K}\sum_{i=1}^{K} d_i$. It is shown in the third column of Table. 1. We also calculate the total number of calls covered by the individual ambulance, that is, $p_i = \sum_{j=1}^{n} x_{ij}$ for $i \in L^*$, and their sample standard deviation $\hat{s}_p$, which is in the forth column of Table. 1.

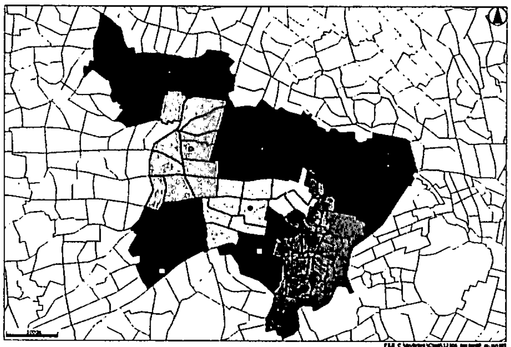

Figure 1: Actual location of ambulance stations and the partition obtained by model (1)

M	obj. func.	$\hat{s}_d$	$\hat{s}_p$
actual	17979340	1052640	1492
2600	10465261	603474	123
2700	10303365	583571	272
2800	10249020	580480	328
2900	10227482	582549	351
3000	10191301	548377	443
3500	10078745	458255	667
4000	10052582	435792	787

Table 1: Optimal value and the standard deviations for various M.

Varying the parameter M gives several solutions of (1). Table 1 also shows the corresponding result. Fig. 2 shows the optimal location of ambulance stations(designated by ○) and the partition obtained by solving the model (1) with the parameter $M = 2600$. In Table 1 we can see that the actual location is inferior to any other solutions of various M in both the sense of objective function value and standard deviations of the individual traveling distance or number of covered calls. Another finding is that the standard deviation of the number of individual covered calls $\hat{s}_p$ decreases as the parameter M decreases. The individual traveling distance, however, increases as M decreases. This results follows because the number of calls is unequally distributed in the area. The small M value makes the ambulance stations concentrated to heavy demand points.

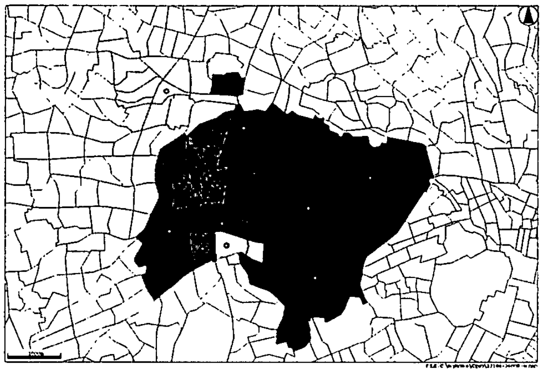

Figure 2: Optimal solution($M = 2600$)

4. Simulation result

We use a stochastic simulation to analyze the solutions of (1). In the simulation we note the operation rate of each ambulance and ratio of calls which are operated by the nearest ambulance station. We call this ratio the covered rate. Table 2 summarize the result. The optimal solution for $M = 2600$ reduces the gap in operation rate, although the covered rate has very small change.

	actual		$M = 2600$	
	operation rate	covered rate	operation rate	covered rate
stdev.	0.11	0.13	0.03	0.11
max.	0.62	0.70	0.51	0.69
min.	0.27	0.33	0.40	0.33

Table 2: Simulation result

References

[1] H. Kawai: Optimization and simulation model analysis of the ambulance deployment for the efficient service(in Japanese). Policy Proposal Paper, National Graduate Institute for Policy Studies, 2004.

[2] O. Berman and D. Krass: Facility location problems with stochastic demands and congestion. In *Facility Location: Applications and Theory*, Z. Drezner and H. W. Hamacher(eds.), pp. 329–371, Springer, 2002.