日本オペレーションズ・リサーチ学会
2005年春季研究発表会

Partial Covering Bicriteria Location

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1. Introduction

In general maximin and minimax criteria have often been used to formulate such pushing and pulling forces. In such models, only extreme distances determine the objective functions' values. On the other hand, many governments today face severe financial problems. It often seems to be difficult to provide the same service to all inhabitants equally. Hence, in siting semi-obnoxious facilities, many types of negotiation and compensation through the transfer of benefits from host populations to a minority have been done in the form of monetary or non-monetary means.

Rather than full covering formulations which have extensively been introduced in past works, their partial-covering version may be more appropriate for semi-obnoxious facility location. The aim of this paper is to present a polynomial-time algorithm for analytically tracing out the efficient solutions and the tradeoffs within push-pull partial covering context.

2.Paritial Covering Location

Given a convex polygon Ω on a Euclidean plane where a facility can be built. Let I and $\{\mathrm{p}_1, \ldots, \mathrm{p}_{|I|}\}$ be the index and location sets of the affected inhabitants on the plane, respectively.

In the *partial anti-center location problem*, an exogenously specified number n^- of inhabitants will be resettled farther from the facility, and it may therefore be considered that their current location will be neglected. Hence, this is defined by

$$\max_{\mathbf{x}\in\Omega}\left(F_{n^-}(\mathbf{x}) \equiv \max_{J^-\subseteq I, |J^-|=|I|-n^-}\left(\min_{u\in J^-}\|\mathbf{x}-\mathbf{p}_u\|\right)\right). \tag{1}$$

We denote its optimal location by $\mathbf{a}^*_{n^-}$: see Figure 1.

In the *partial center location problem*, an exogenously specified number n^+ of inhabitants will remain unserved. Its mathematical description is

$$\min_{\mathbf{x}\in\Omega}\left(G_{n^+}(\mathbf{x}) \equiv \min_{J^+\subseteq I, |J^+|=|I|-n^+}\left(\max_{v\in J^+}\|\mathbf{x}-\mathbf{p}_v\|\right)\right). \tag{2}$$

Its solution is denoted by $\mathbf{c}^*_{n^+}$. Figure 2 shows $\mathbf{c}^*_{n^+}$ for the same inhabitant set with Figure 1.

3.Bicriteria Location

Consider the biobjective problem generated by combining (1) with (2). Let $E^*_{n^-,n^+}$ be the efficient set and $t^*_{n^-,n^+}$ be the biobjective values corresponding to $E^*_{n^-,n^+}$ in objective space.

Let $I^k_1, \ldots, I^k_{t(k)}$ be all possible subsets out of I whose cardinality is k, where $t(k) \equiv \frac{|I|!}{k!(|I|-k)!}$.

$$V_{I^k_i} \equiv \{\mathbf{x}\in\mathbb{R}^2 \mid \max_{u\in I^k_i}\|\mathbf{x}-\mathbf{p}_u\| \le \min_{v\notin I^k_i}\|\mathbf{x}-\mathbf{p}_v\|\}.$$

The union of all $V_{I^k_i}$'s is called the *order-k Voronoi diagram.* Since $n^- + n^+ < |I|$, we have $F_{n^-}(\mathbf{x}) < G_{n^+}(\mathbf{x})$ at any point $\mathbf{x}\in\Omega$. Thus, as shown in Ohsawa and Tamura(2003),

Proposition 1 $E^*_{n^-,n^+} \subseteq \partial V^{n^-+1} \cup \partial V^{|I|-n^+-1} \cup \partial\Omega.$

When $n^- = n^+ = 0$ this proposition reduces to the result by Ohsawa(2000).

As a consequence of Proposition 1, an algorithm for construction of the efficient solutions and the tradeoffs can be given by modifying the technique by Ohsawa and Tamura(2003) as follows:

Step 1. Set up the planar graph $N \equiv \partial V^{n^-+1} \cup \partial V^{|I|-n^+-1} \cup \partial\Omega$.
Step 2. Split the links of N into sublinks along which the (n^-+1)-th and $(|I|-n^+-1)$-th nearest inhabitants are both constant.
Step 3. Draw $(F_{n^-}(N), G_{n^+}(N))$ in objective space.
Step 4. Detect its south-eastward envelope.
Step 5. Specify the sublinks of N corresponding to the envelope in geographical space.

Proposition 2 *$E^*_{n^-,n^+}$ and $t^*_{n^-,n^+}$ can be found in $O((|I|^4+|\partial\Omega|)\log(|I|^4+|\partial\Omega|))$ time.*

An example of $E^*_{n^-,n^+}$ and $t^*_{n^-,n^+}$ are shown in Figures 3 and 4.

References

[1] Y. Ohsawa, Bicriteria Euclidean location associated with maximin and minimax criteria, *Naval Research Logistics*, 47(2000), 581–592.
[2] Y. Ohsawa and K. Tamura, Efficient location for a semi-obnoxious facility, *Annals of Operations Research*, 123(2003), 173–188.

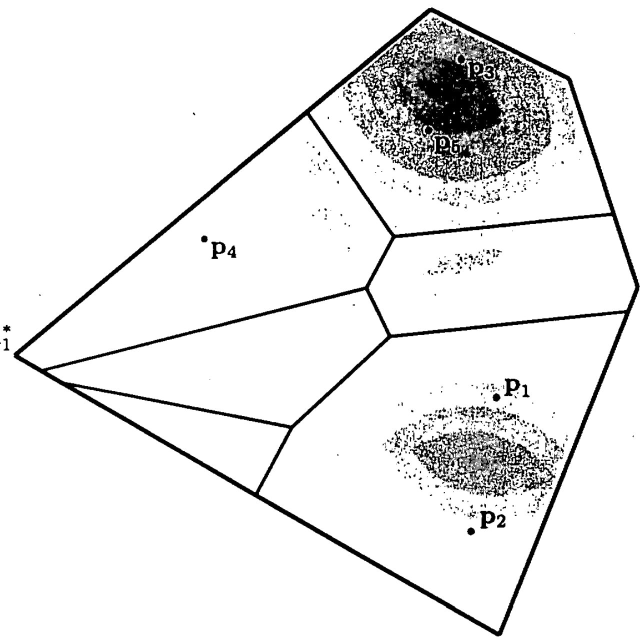

Figure 1: Partial anti-center location with
$n^{-} = 1$ and order-two Voronoi diagram

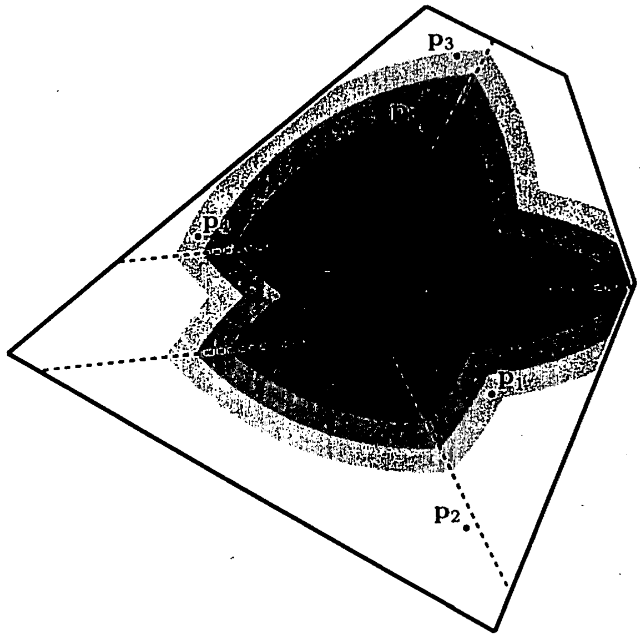

Figure 2: Partial center location with
$n^{+} = 1$ and order-three Voronoi diagram

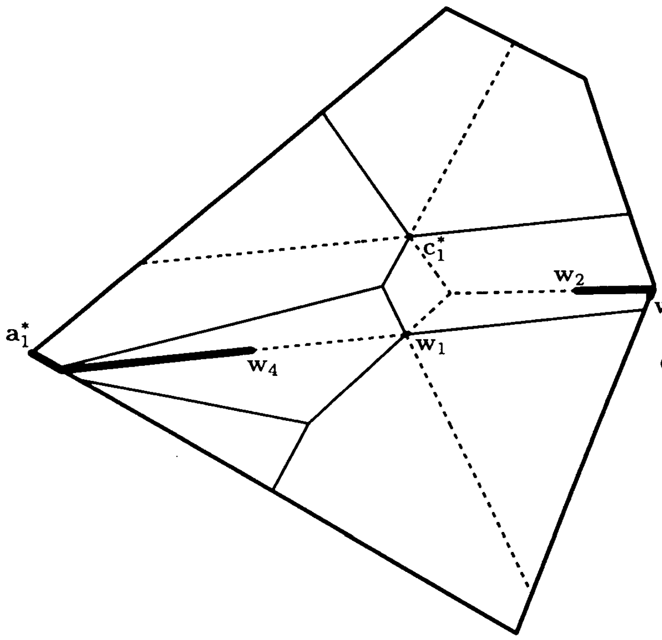

Figure 3: Efficient set for push-pull partial
covering with $n^{-} = 1$ and $n^{+} = 1$

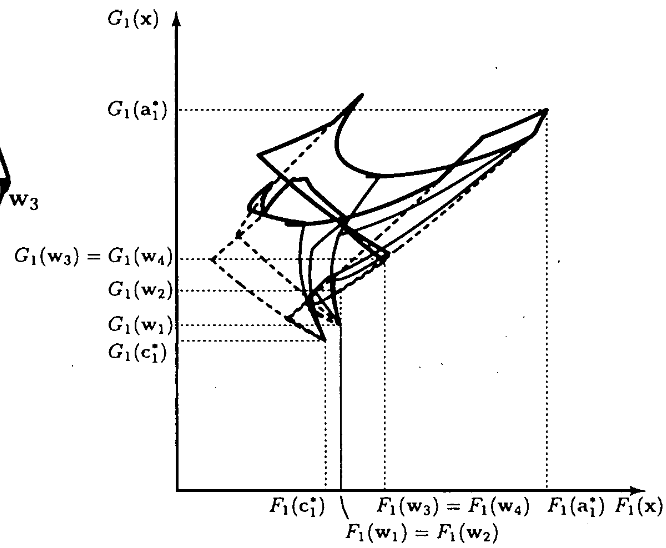

Figure 4: Tradeoff for push-pull partial
covering with $n^{-} = 1$ and $n^{+} = 1$