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Offline Traffic Engineering Design and Optimization in Communication Networks with Risk Analysis

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1 Introduction

In [1] and [2], the stochastic traffic engineering problems have been studied. In [3], we presented an analysis for the loss rate constraint in such problems and showed the impact of the loss rate constraint on the network performance by numerical results. In this paper, we present an analysis for the model presented in [3] to derive the optimal bandwidth capacity with a linear penalty cost. We also analyze the risk of network profit shortfall by using mean-variance approach.

2 System Model

A Communication Network (CN) should derive its revenue by serving traffic demand including voice, packet data, image and full-motion video. For unit bandwidth capacity allocated to the CN, an unit cost will be charged. For unsatisfied unit traffic demand with the limitation of network bandwidth, a linear penalty cost will be added in the objective function. The objective of this system is to maximize the CN mean profit.

Uncertain total traffic demand in the CN denoted by $D > 0$ is characterized by a random distribution with the probability density function $f(x)$ and the cumulative distribution function $F(x)$.

Let $b > 0$ denote the amount of bandwidth capacity provisioned in the CN, r denote the unit revenue of the CN by serving the traffic demand, c denote the unit cost for unit bandwidth capacity allocated in the CN, and q denote the linear penalty cost for the unsatisfied unit demand. Let $P(b \geq \delta D) \geq 1-\epsilon$ denote the loss rate constraint, and let $C_{max} > 0$ denote the maximal capacity that can be allocated in the CN. To avoid unrealistic cases, we make the following assumptions:

(1) System parameters are: $r > q > 0$, $r > c > 0$.
(2) Loss rate constraint parameter is: $0 \leq \delta \leq 1$.
(3) Confidence level is: $0 \leq 1-\epsilon \leq 1$.

3 Optimal Bandwidth Allocation with Penalty Cost

Let $\pi(b, D)$ denote the random profit function by serving traffic demand in the network with the linear penalty cost, namely,

$$\pi(b, D) = r(b \wedge D) - q(D-b)^{+} - cb \qquad (1)$$

where $\wedge$ represents to choose the smaller one between two components, and $^{+}$ represents to choose the positive part.

Let $\Pi(b, D)$ denote the mean profit function with the linear penalty cost as follows:

$$\begin{aligned}\Pi(b, D) &= r\int_0^b xf(x)dx + rb\int_b^{+\infty} f(x)dx \\ &\quad -q\int_b^{+\infty}(x-b)f(x)dx - cb. \qquad (2)\end{aligned}$$

The objective function of the system is

$$\Pi^* = \max_{b>0}\{\Pi(b, D)\}, \qquad (3)$$

subject to $P(b \geq \delta D) \geq 1-\epsilon$ and $b \leq C_{max}$. Π^* is the optimal profit function.

Next, we analyze the property of the mean profit function $\Pi(b, D)$. The first order derivative of $\Pi(b, D)$ with respect to b is given as follows:

$$\frac{d\Pi(b, D)}{db} = (r+q-c) - (r+q)F(b). \qquad (4)$$

The second order derivative of $\Pi(b, D)$ with respect to b is given as follows:

$$\frac{d^2\Pi(b, D)}{db^2} = -(r+q)f(b) \leq 0. \qquad (5)$$

Therefore, we can say that $\Pi(b, D)$ is a concave function of b. So, the optimal bandwidth capacity without constraints is $F^{-1}\left(\frac{r+q-c}{r+q}\right)$, where $F^{-1}(\cdot)$ is the inverse function of $F(\cdot)$.

We define the loss rate as the probability that the traffic demand can not be served by the CN. Therefore, the loss rate constraint is equivalent to $b \in [\delta F^{-1}(1-\epsilon), +\infty)$. If we consider the maximal capacity constraint, the optimal bandwidth capacity for the CN can be given by

$$\left[F^{-1}\left(\frac{r+q-c}{r+q}\right) \vee \delta F^{-1}(1-\epsilon)\right] \wedge C_{max} \qquad (6)$$

where $\vee$ represents to choose the larger one between two components.

We consider a fully distributed network, where the traffic demand is assumed to follow a uniform distribution as a special example. We give some numerical results to show the impact of penalty cost on the bandwidth capacity (see Fig. 1). Horizontal axis q/r of Fig. 1 corresponds to the increase of penalty cost. Ordinate axis of Fig. 1 corresponds to the percentage difference of the optimal bandwidth capacity from the optimal bandwidth capacity without penalty. For comparing with the model of [1], we choose system parameters for two cases as follows: (1) $r = 7.5, c = 1.5$, and (2) $r = 7.5, c = 0.5$, in the interval [0, 1].

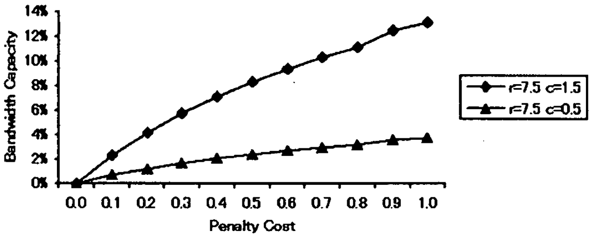

Figure 1: Impact on bandwidth capacity.

The numerical results shown in Fig. 1, including the results presented in [1], reveal a distinct impact of penalty cost on the CN bandwidth capacity. With the same penalty cost, the larger the unit cost c is, the greater the impact on the bandwidth capacity is.

4 Risk Analysis with Penalty Cost

Due to the uncertain traffic demand, the profit is also uncertain, we define the risk as the deviation from the optimal profit in this paper.

We analyze the risk of profit shortfall by using the mean-variance approach. The objective function, which is denoted by Φ^*, is given as follows:

$$\Phi^* = \max_{b>0} \{\Pi(b, D) - \alpha Var[\pi(b, D)]\} \qquad (7)$$

where α ($0 \le \alpha \le 1$) is a risk averseness parameter, $\pi(b, D)$ is the random profit function given by Eq. (1), and $Var[\pi(b, D)]$ is the variance of $\pi(b, D)$.

When the traffic demand distributed in the whole network is assumed to follow the uniform distribution, we can obtain the mean function by

$$\Pi(b, D) = -\frac{r+q}{2}b^2 + (r+q-c)b - \frac{q}{2}. \qquad (8)$$

The variance function is obtained by

$$Var[\pi(b, D)] = -\frac{1}{4}(q+r)^2 b^4 + (2r^2 + 2q^2 + c^2 + 4qr - 3rc - 3qc)b^3 - (\frac{1}{2}q^2 + \frac{1}{2}qr)b^2 + \frac{1}{12}q^2. \qquad (9)$$

Moreover, the objective function Φ^* presented in Eq. (7) is given by

$$\Phi^* = \max_{b>0}\{[-\frac{r+q}{2}b^2 + (r+q-c)b - \frac{q}{2}] -\alpha[-\frac{1}{4}(q+r)^2 b^4 + (2r^2 + 2q^2 + 4qr - 3rc - 3qc + c^2)b^3 - (\frac{1}{2}q^2 + \frac{1}{2}qr)b^2 + \frac{1}{12}q^2]\}. \qquad (10)$$

With the same system parameters for Fig. 1, we give some numerical results to show the impact of risk averseness on objective function (see Fig. 2). Horizontal axis of Fig. 2 corresponds to the increase of risk averseness. Ordinate axis of Fig. 2 corresponds to the percentage difference of the objective function from the objective function without risk.

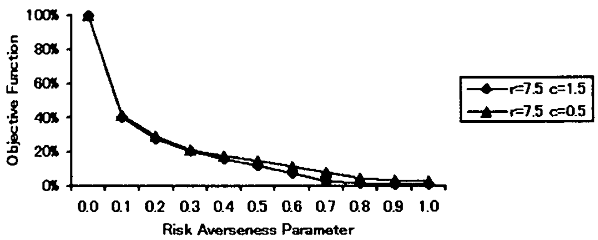

Figure 2: Impact on objective function.

The numerical results shown in Fig. 2, including the results presented in [1], reveal a distinct impact of risk on the CN objective function. With the same risk averseness, the larger the unit cost c is, the less the impact on the objective function is.

5 Conclusion

In this paper, we presented a stochastic model for optimizing bandwidth allocation in Communication Networks (CNs) and derived the optimal bandwidth capacity with the penalty cost. We also analyzed the risk averseness in CNs under the mean-variance framework. Numerical results revealed the impacts of the penalty cost and the risk averseness on the network performance.

Acknowledgments

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