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RANKING ALTERNATIVES FROM THE DECISION MAKER'S PREFERENCES: AN APPROACH BASED ON UTILITY AND THE NOTION OF MARGINAL ACTION

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Abstract This paper proposes an operational approach (based on marginal actions) to specify multi-attribute utility functions, which allows us to easily rank alternatives from criteria relevant to the manager, the customer or any other decision maker without heuristic remedies, imprecision or loss of generality. Marginal actions are fictitious alternatives which play a key role in preference analyses as a tool recently introduced for the ranking treatment of decision tables. As usual in multi-attribute value theory, preferential independence is assumed. A practical large-scale case in the textile industry to rank commercial fabrics –characterized by seven quality factors– is developed.

Keywords: Mathematical modeling, multi-attribute ranking, microeconomic ordinal utility, marginal actions

1. Introduction

An ongoing problem in management is how to rank alternatives from criteria relevant to the manager, the customer or any other decision maker. Economists rather than rule-of-thumb analysts feel the ranking should be performed according to utility theory. However, there is a drawback that applicability from this theory appears doubtful, as most managers are unable to specify multi-attribute utility functions. A decision table can be constructed either facing uncertainty or only using nonrandom information. In both cases, the rows are alternatives to be ranked while the meaning of columns widely differs from one case to another. Under uncertainty the columns are states of nature –one and only one of them can truly occur. Then, in the Bayesian context of probabilities/likelihoods assigned to the states, the ranking solution commonly derives from Von Neumann's and Morgenstern's [15] expected utility theory, whereas in the strict uncertainty context various solutions are obtained either from security-based decision rules or other criteria. As Keeney and Raiffa ([8] p. 220, footnote) say, "there is no standardized terminology" about value functions and utility functions so that in the Bayesian context "utility" is termed "preference functions, cardinal utility functions, Von Neumann utility functions, probabilistic utility functions, and utility functions", while in other contexts "value" is referred to as "worth functions, ordinal utility functions, preference functions, Marshallian utility functions, and even utility functions." The cited work by Keeney and Raiffa especially focuses on uncertainty cases, although tradeoffs under certainty are less extensively treated there with assumptions such as preferential independence. With nonrandom information (i.e., the certainty case) the columns are criteria often named attributes, but "there is a profusion of terminology in this area of the literature, and there is not complete agreement on usage" ([7] p. 105).

In this paper, the decision maker faces a decision table whose columns refer to attributes (maybe, characteristics or aspects inherent in the alternatives) and whose rows refer to alternatives, each row being a vector of levels of achievement against the attributes. In this certainty environment, the aim of the paper is to propose an ordinal utility-based approach to ranking. This utility will be named microeconomic ordinal utility function (MOUF), according to some of the different terms in the literature. A decisive piece of the approach is marginal actions –a tool proposed in ranking problems elsewhere [2], which will be here used to elicit preferences and specify the decision maker's utility function. In sum, our proposal is a departure from heuristic weighting, outranking relations and even Keeney's and Raiffa's model. However, our approach is not intended to replace other techniques in the literature, as all of them can be useful depending on environments and circumstances.

Despite the fact that MOUF is a firmly established model of strong rationality in social sciences with more followers than the rest of approaches put together, it is hardly employed to rank decision tables of nonrandom attributes. This is not a paradox but the consequence of operational restrictions. Although the utility form is often assumed a priori –without empirical investigation–, even researches have difficulties in specifying the parameters of a MOUF, and managers yet more. Therefore, the specification problem undertaken herein is relevant.

The paper is organized as follows. After an introductory background (Section 2) where the essentials of MOUF and its competing approaches are reviewed, the ranking theory proposed in the paper is formulated in Section 3 while the step-by-step method in Section 4. An illustrative case from the textile industry is developed in Section 5. The paper closes with concluding remarks.

2. Background

Even now, there are two reasons to keep untouched ordinal utility in decision theory, namely, (a) utility is not measurable, and (b) ordering preferences is sufficient for choice. These reasons prove persuasive to most economists as decision theory has nothing to do with welfare economics and happiness theory [12]. In a MOUF-based analysis, the decision maker prefers one choice to another (or both are indifferent to him), so that measurement of his utility is not significant to fix the order. Under some conditions of reasonableness, the analyst can establish a MOUF explaining the decision maker's preferences, but he cannot compare utility between two different decision makers and much less obtain an aggregate utility for society as a whole. Indeed, the validity of the microeconomic ordinal utility model requires axioms such as rationality and certainty of relevant information, consistency and transitivity of choice, assignation of a utility index to each alternative from the preference order, and diminishing marginal rate of substitution. This axiomatic set is a portion of the axiomatic basis in general equilibrium theory [6]. Linearity is ruled out. The linear simplification was already dropped even in the early years of utility theory because linearity involves both independent utilities of the various attributes and non-decreasing marginal utility, which is quite unrealistic.

If comparing the approach in this paper with weighting techniques of ranking such as Analytic Hierarchy Process (AHP, [14]) and even with Keeney's and Raiffa's ([8] ch. 3) tradeoffs under certainty, advantages of MOUF/marginal actions are: (i) to preserve the economic basis of utility, which involves rejecting measurability and especially linearity; and (ii) to avoid ranking from weighting, which seems desirable as different determinations of weights from one technique to another lead to antagonistic results. Concerning AHP, it requires numerous pairwise comparisons using a built-in scale valued from 1 to 9, which forces the decision maker to seek quick decisions involving measurements, the set of comparisons often showing inconsistencies which are treated with somewhat artificial remedies. Another method is ELECTRE [13], which involves the notions of outranking and indifference and preference thresholds. PROMETHEE [4] and GAIA [11] have advantages over AHP such as: (a) the decision maker defines his own scales of measure to reveal his preferences for each criterion, so that a large number of comparisons are not needed; and (b) sensitivity analyses can be performed in a visual manner. As to Keeney's and Raiffa's tradeoffs, the axiomatic basis and results are consistent and also somewhat appealing; however, they are a departure from classic economics.

3. Theory

Let us focus on the certainty case starting from a decision table or matrix with a finite number of alternatives a_i (i= 1,2,...M) such as products, designs, activities or any other objects available for choice. The manager or another decision maker wants to make his choice in a rational way by defining a finite number of significant criteria or attributes X_j (j= 1,2,...n). In his view, these criteria are sufficient to describe each alternative for practical managerial purposes, while a lot of minor criteria are deemed worthless for description. As usual in research that concentrates attention on ranking, we do not cope here with prior issues –how to ascertain the finite set of attributes and how to measure each variable. From value theory and utility theory, weak preference ordering is assumed. Thus, if the decision maker weakly prefers a to a', then the expression to be written is $a \succ\sim a'$, which reads a is preferred to a' or both are indifferent to the decision maker. The following matrix is now constructed:

$$\begin{array}{c|cccccc}
\hline
\text{Existing alternatives} & \multicolumn{6}{c}{\text{Attributes}} \\
 & X_1 & X_2 & \dots & X_j & \dots & X_n \\
\hline
a_1 & v_{11} & v_{12} & \dots & v_{1j} & \dots & v_{1n} \\
a_2 & v_{21} & v_{22} & \dots & v_{2j} & \dots & v_{2n} \\
\vdots & & & & & & \\
a_i & v_{i1} & v_{i2} & \dots & v_{ij} & \dots & v_{in} \\
\vdots & & & & & & \\
a_M & v_{M1} & v_{M2} & \dots & v_{Mj} & \dots & v_{Mn} \\
\hline
\text{Marginal actions} & & & & & & \\
\text{(fictitious alternatives)} & & & & & & \\
a_{010} & v_1^* & v_{2^*} & \dots & v_{j^*} & \dots & v_{n^*} \\
a_{020} & v_{1^*} & v_2^* & \dots & v_{j^*} & \dots & v_{n^*} \\
\vdots & & & & & & \\
a_{0j0} & v_{1^*} & v_{2^*} & \dots & v_j^* & \dots & v_{n^*} \\
\vdots & & & & & & \\
a_{0n0} & v_{1^*} & v_{2^*} & \dots & v_{j^*} & \dots & v_n^* \\
\hline
\end{array} \; . \quad (1)$$

Matrix (1) is split into two parts, a horizontal line separating them. The upper part includes the M alternatives available for choice, which are there called existing alternatives. Notice that every existing alternative is structured as a vector of specific levels of achievement against the attributes ($v_{ij} \geq 0$). As to the lower part of the matrix, it includes the marginal actions whose definition and usefulness will be shown later. Some attributes behave as "more is better" while others as "more is worse". Behaviors neither "more is better" nor "more is worse", although possible in less frequent problems, are not here considered (see Appendix 1).

Assumption 1. *Preferential independence.* In matrix (1), every attribute is preferentially independent of the others.

This is a commonly accepted assumption in the literature (see, e.g., [7] p. 107 and 117), and thus much simpler theoretical results are obtained. Also, Keeney and Raiffa [8] use the assumption of preferential independence in the sense that "any pair of attributes is preferentially independent of the others" (p. 118). In Appendix 1, a standard statement of Assumption 1 is recalled. Throughout this paper, it is postulated that the property of preferential independence holds for every attribute with respect to the remaining attributes.

Definition 1. *Marginal actions.* They are fictitious alternatives denoted by $a_{0j0}(j=$ 1,2,... $n)$ with the following row vector structure:

$$a_{0j0} = (v_{1^*}, v_{2^*}, \dots v_{j-1^*}, v_j^*, v_{j+1^*}, \dots v_{n^*}), \tag{2}$$

where
v_{1*} = The worst level of achievement in column 1 among the existing alternatives, namely, the worst v_{i1} value (i= 1,2,...M).
v_{2*} = The worst level of achievement in column 2 among the existing alternatives.
...
v_j^* = The best level of achievement in column j among the existing alternatives.
...
v_{n*} = The worst level of achievement in column n among the existing alternatives.

Consistency of the above definitions of v_j^* and v_{j*} as the best and the worst values, respectively in the j^{th} column, is assured if preferential independence holds. Let us describe a case in which preferential independence is not satisfied. A consumer wants to rank several equally-sized boxes of a certain food containing two different fats, say, soy oil and fish oil in different proportions. According to our consumer's preferences, the total quantity of fat (soy oil plus fish oil) behaves as "more is better" over a range between 0 and 20 grams per box, a total fat in excess of 20 grams being deemed inadequate. Moreover, at least 2 grams of soy oil and 3 grams of fish oil per box are required by the consumer for healthcare purposes. Suppose now that our consumer has fixed soy oil at a level of 17 grams. Then, the best quantity of fish oil will be 3 grams, not 20 grams at all. In contrast, suppose that the consumer has fixed soy oil at a level of 14 grams. Then, the best quantity of fish oil will be 6 grams, not 20 grams either. In this example, we cannot consistently define the best fish oil quantity since it is not independent of the soy oil level previously fixed. Indeed, we have:

$$\text{(17 grams, 3 grams)} \succ\sim \text{(17 grams, 6 grams)}$$
$$\text{(14 grams, 3 grams)} \prec\sim \text{(14 grams, 6 grams)}$$

Henceforward, the ranking of the existing alternatives will be derived from weak preference ordering of marginal actions, this weak ordering being elicited by an interactive dialogue between the analyst and the decision maker as explained below (Section 4).

(i) *Marginal actions weak preference ordering.* To establish it, the analyst poses the following question to the decision maker: "Focus on the fictitious action $(v_{1^*}, v_{2^*}, \dots v_{j^*}, \dots v_{n^*})$

whose components are all the worst values of the attributes. Suppose you can improve one and only one component by replacing its worst value by its best value, while the other components are kept at their worst values. Which component will you single out to attain this improvement?" Maybe the decision maker would answer saying: "I would pick component number 4". The analyst asks again: "Suppose your favorite component 4 cannot be improved. Then, which component will you choose?" Perhaps the decision maker would now answer: "I am indifferent to components number 2 and 5". This dialogue goes on this way until all components are over. As a result, the marginal actions are ranked in the following weak preference ordering:

$$a_{040} \succ a_{020} \sim a_{050} \cdots,$$

or otherwise written,

$$(v_{1^*}, v_{2^*}, v_{3^*}, v_4^*, v_{5^*}, ...) \succ (v_{1^*}, v_2^*, v_{3^*}, v_{4^*}, v_{5^*}, ...) \sim (v_{1^*}, v_{2^*}, v_{3^*}, v_{4^*}, v_5^*, ...) \ldots$$

Remark 1. *Labeling.* For ease of notation and without loss of generality, the original labeling of attributes can be rearranged to label the marginal actions in a natural order from a_{010} (corresponding to the least preferred one) to a_{0n0}(corresponding to the most preferred one). Moreover, the fictitious action $(v_{1^*}, v_{2^*}, \ldots v_{j^*}, \ldots v_{n^*})$ will be labeled a_{000}. Hence, the string of preference relationships is written:

$$a_{0n0} \succ\sim \ldots \succ\sim a_{0j0} \succ\sim \ldots \succ\sim a_{020} \succ\sim a_{010} \succ\sim a_{000}. \tag{3}$$

(ii) *Marginal actions utility indexes deduced from their weak preference ordering.* First, the lowest utility index $\mathrm{U}_{000}= 1$ is assigned to the fictitious action a_{000}, as this action is the least preferred. Let U_{0j0} be the utility index for the marginal action a_{0j0}. From (3), we have:

$$a_{0(j-1)0} \prec a_{0j0} \Rightarrow U_{0(j-1)0} < U_{0j0} \Rightarrow U_{0j0}/U_{0(j-1)0} = q_j, \tag{4}$$

where q_j is a positive parameter greater than one, and

$$a_{0(j-1)0} \sim a_{0j0} \Rightarrow U_{0(j-1)0} = U_{0j0}. \tag{5}$$

Relationships (4) and (5) are obvious implications. In fact, if $a_{0j0}$is preferred to $a_{0(j-1)0}$, then the utility index of the first action must be greater than the utility index of the second action, and consequently, the first utility index is necessarily equal to the second index times a factor q_j greater than 1. Analogously, relationship (5) is obvious.

Assumption 2. *Laplace's principle of insufficient reason* [9]. From this principle, all q_j parameters are fixed at equal level $q_j = q > 1$, where the q value will be later proven to be irrelevant for the purpose of ranking.

Given a decision matrix, Laplace's principle of insufficient reason can be used in different environments of uncertainty or incomplete information. Let us explain this principle and justify its use in our context. Notice first that ratios $U_{0j0}/U_{0(j-1)0}$ are given by equation (4) under incomplete information as parameters $q_j$are unknown. Consider any two q_j ($j = h$ and $j = h'$) and compare them by the quotient $\lambda = q_h/q_{h'}$. Extremely large and small values of this quotient such as λ= 5000 and λ= 1/5000= 0.0002 are described by the decision maker as highly "improbable", namely, unlikely. Less extreme values such as λ= 5 and λ= 1/5= 0.2 are described as less "improbable". From these descriptions, λ is viewed by the decision maker like a log-normally distributed variable with mean and mode corresponding to $\lambda = 1$ (because $\log 1 = 0$). Therefore, the decision maker chooses the “most probable” value $\lambda = 1$, which implies $q_h = q_{h'}$. This is the exact meaning of Laplace’s principle. Another brief but superficial explanation is to say that the decision maker reveals that he prefers a_{0j0} to $a_{0(j-1)0}$ but he does not reveal how much the former is preferred to the latter.

(iii) *Specifying the utility function to rank the existing alternatives.* Let us start with the following:

Assumption 3. *Cobb-Douglas utility.* The decision maker’s preferences are sufficiently accurately described by the Cobb-Douglas utility function:

$$U_i = v_{i1}^{\alpha_1} v_{i2}^{\alpha_2} \ldots v_{ij}^{\alpha_j} \ldots v_{in}^{\alpha_n}, \tag{6}$$

where $\alpha_j \geq 0$ for all j. Assumption 3 is justified from the wide use that researchers make of Cobb-Douglas functions to describe utility environments, especially for applications. Concerning this point, Coleman [5] contends that Cobb-Douglas functions are capable of reflecting any kind of utility maps. Although this opinion might be controversial, the use of Cobb-Douglas utility as a sufficiently approximated approach can be sensibly defended.

Remark 2. *“More is worse” attributes.* If some attribute j behaves as “more is worse”, then its values in decision matrix (1) should be converted into “more is better” by the equation:

$$v_{ij} = v'_{j^*} + {v'_j}^* - v'_{ij}, \tag{7}$$

where variables v'_{j^*}, ${v'_j}^*$and v'_{ij} refer to the original values in the decision matrix. This transformation is necessary as every attribute in the utility function should be positively oriented.

As equation (6) can be extended to both the existing and the marginal actions, the utility ranking index for a_{0j0} is given by:

$$U_{0j0} = v_{1^*}^{\alpha_1} v_{2^*}^{\alpha_2} \ldots v_j^{*\alpha_j} \ldots v_{n^*}^{\alpha_n}. \tag{8}$$

If $v_{j^*} \neq 0$, then we get from equation (8):

$$U_{0j0} = (v_j^*/v_{j^*})^{\alpha_j} (v_{1^*}^{\alpha_1} v_{2^*}^{\alpha_2} \ldots v_{j^*}^{\alpha_j} \ldots v_{n^*}^{\alpha_n}) = (v_j^*/v_{j^*})^{\alpha_j} K, \tag{9}$$

where K is a positive constant. If $v_{j^*} v_{j-1^*} \neq 0$ and $v_{j-1}^* \neq 0$, then equation (9) yields:

$$\frac{U_{0j0}}{U_{0(j-1)0}} = \frac{(v_j^*/v_{j^*})^{\alpha_j}}{(v_{j-1}^*/v_{j-1^*})^{\alpha_{j-1}}}\ . \tag{10}$$

From equations (4), (5) and (10) as well as from Assumption 2, we obtain:

$$a_{0(j-1)0} \prec a_{0j0} \ \ \Rightarrow \ \ \frac{(v_j^*/v_{j^*})^{\alpha_j}}{(v_{j-1}^*/v_{j-1^*})^{\alpha_{j-1}}} = q, \tag{11}$$

$$a_{0(j-1)0} \sim a_{0j0} \ \ \Rightarrow \ \ \frac{(v_j^*/v_{j^*})^{\alpha_j}}{(v_{j-1}^*/v_{j-1^*})^{\alpha_{j-1}}} = 1. \tag{12}$$

From (11)-(12), relationships (3) and Assumption 2, we obtain:

Property 1. *Cobb-Douglas utility specification rule.* In the common case $v_j^* \neq v_{j*}$ and $v_{j*} \neq 0$ for all j, the α_j exponent is given by

$$\alpha_j = \frac{p_j/(\log v_j^* - \log v_{j^*})}{\sum\limits_{s=1}^{n} p_s/(\log v_s^* - \log v_{s^*})}, \tag{13}$$

where parameter p_j is the number of symbols ($\succ$) existing before a_{0j0} in the string of preference relationships (3). In the special case $v_j^* = v_{j^*}$ for some j, this j^{th} attribute should be removed from decision matrix (1) as it has no impact on the utility ranking index at all. Concerning case $v_{j^*} = 0$, see Appendix 3.

In Appendix 2, Property 1 is proven. Hereafter, we assume that (13) is valid in case of $v_{j^*} = 0$ for some j.

(iv) *Interpreting results.* To highlight the meaning of the α_j exponents given by equation (13) and the role they play in the ranking index, it is convenient to convert Cobb-Douglas utility (6) into its logarithmic form. If overlooking the factor of proportionality $1/\sum$ (irrelevant for ranking), we have:

$$\log U_i = \sum_{j=1}^{n} p_j (\log v_{ij})/(\log v_j^* - \log v_{j^*}), \tag{14}$$

which is another expression of the utility ranking index. Thus, according to the microeconomic notion of marginal utility, the ranking index does not increase in proportion to values v_{ij}, but rather increases with the logarithm of values. By adding and subtracting the same amount $K = \sum (\log v_{j^*})/(\log v_j^* - \log v_{j^*})$ to the right-hand side of equation (14), which alters nothing of this expression, we have:

$$\log U_i = K + \sum_{j=1}^{n} p_j (\log v_{ij} - \log v_{j^*})/(\log v_j^* - \log v_{j^*}), \tag{15}$$

where K is irrelevant for ranking. Hence, the α_j exponents (13) play a clear role of normalizers. Indeed, notice that:

$$0 \leq \frac{\log v_{ij} - \log v_{j^*}}{\log v_j^* - \log v_{j^*}} \leq 1 \text{ for all } j. \tag{16}$$

In Appendix 4, some properties to underpin this theory are stated.

4. The Method

From the above theory, a step-by-step method of ranking easy to apply will be developed henceforward. For the sake of clarity, this method will be explained through the following example. Imagine that the problem is to rank 5 brands of a certain ready-to-eat food containing fish and vegetables. The decision maker focuses on 4 criteria, say, three quality factors A, B and C, which behave as "more is better", and a measure of potential deterioration, which obviously behaves as "more is worse". These criteria or attributes are defined as follows.

- *Factor A*. Quality of cooking, appealing color of the food and taste
- *Factor B*. Cleanness of the cooked food, namely, the less the presence of fish bones and skin the more the cleanness.

- *Factor C*. Marketing external presentation (box design, easy opening of the box, external image, etc.).
- *Potential deterioration*. From samples of the brands kept in the same atmosphere for a long time, an index of potential deterioration is estimated for each brand.

In Table 1, upper part, the five brands (namely, the existing actions) are row headings, while the four attributes (namely, the three quality factors and potential deterioration) are column headings.

While the original values of attributes are displayed in the columns to the left of the table, the normalized logarithmic values (16) are displayed to the right. Just below the brands, the table includes the fictitious action a_{000} and the four marginal actions. In the lower part of the table, some ancillary data to compute the α exponents by equation (13) are recorded.

Table 1: Decision table: An example with 5 alternatives (brands) and 4 attributes

		Original values						Normalized logarithmic values				
	Brands	Quality Factors			Potential Deterior-ation	Potential preserva-tion	Utility ranking index	Quality factors			Potential preserva-tion	Utility ranking index
		A	B	C				A	B	C		
Existing actions	a_1	25	13	21	242	647	58.787	0.200	0	0	0.906	2.917
	a_2	85	49	29	363	526	97.088	0.945	0.751	0.239	0.798	5.320
	a_3	93	38	56	114	775	128.666	1	0.607	0.727	1	6.668
	a_4	18	43	81	775	114	64.410	0	0.677	1	0	3.355
	a_5	58	76	32	613	276	85.668	0.712	1	0.312	0.461	4.721
	Fictitious action a_{000}	18	13	21		114	31.964	0	0	0	0	0
Marginal actions	a_{010}	93	13	21		114	39.388	1	0	0	0	1
	a_{020}	18	76	21		114	48.536	0	1	0	0	2
	a_{030}	18	13	81		114	48.536	0	0	1	0	2
	a_{040}	18	13	21		775	59.809	0	0	0	1	3
Ancillary data	v_j^*	93	76	81		775						
	v_{j^*}	18	13	21		114						
	$\log v_j^*$	4.533	4.331	4.394		6.653						
	$\log v_{j^*}$	2.890	2.565	3.045		4.736						
	$\frac{1}{(\log v_j^* - \log v_{j^*})}$	0.609	0.566	0.741		0.522						
	p_j	1	2	2		3						
	$\frac{p_j}{(\log v_j^* - \log v_{j^*})}$	0.609	1.132	1.482		1.566						

- *First step*. Check that the best and worst values for each attribute, namely, v_j^* and v_{j^*} (j= 1,2,3,4) are consistently defined (preferential independence is a sufficient condition for it). For this purpose, take, e.g., factor C. From Table 1, we have:

$$(25,\ 13,\ v_3^*,\ 242) \succ\!\sim (25,\ 13,\ v_{k3},\ 242)$$
$$(85,\ 49,\ v_3^*,\ 363) \succ\!\sim (85,\ 49,\ v_{k3},\ 363)$$
$$(93,\ 38,\ v_3^*,\ 114) \succ\!\sim (93,\ 38,\ v_{k3},\ 114)$$
$$(18,\ 43,\ v_3^*,\ 775) \succ\!\sim (18,\ 43,\ v_{k3},\ 775)$$
$$(58,\ 76,\ v_3^*,\ 613) \succ\!\sim (58,\ 76,\ v_{k3},\ 613)$$

where $v_{k3} \leq v_3^* = 81$. Analogous relationships hold if v_3^* is replaced by v_{3^*}, symbol $\succ\!\sim$ is replaced by symbol $\prec\!\sim$, and v_{k3} is greater than (or equal to) v_{3^*} for any k. Indeed, the whole quality of the product improves when the marketing external presentation improves while the other attributes remain constant, no matter the levels at which quality of cooking, cleanness of food, and potential deterioration are fixed. Compare the above numerical preference relationships to the numerical relationships concerning the soy oil - fish oil example (Section 3) where preferential independence does not hold. Therefore, unlike the soy oil - fish oil example, preferential independence is here satisfied for factor C and analogously for each of the other attributes.

- *Second step.* From Remark 2, the potential deterioration values are converted into potential preservation by equation (7).
- *Third step.* From Section 3, paragraph (i), the marginal actions weak preference ordering is elicited through the following dialogue.

  Question 1. "Imagine you have the fictitious brand (18, 13, 21, 114) whose components quality factor A, B, C, and potential preservation are all at their lowest (worst) levels. Suppose also that you can improve one and only one component by replacing its worst value by its best value, while the other components of the fictitious brand are kept at their lowest levels. Which of the four components will you choose to attain this improvement?"

  Answer 1. "I would choose potential preservation".

  Question 2. "Imagine now that unfortunately potential preservation cannot be improved. Which of the other three components will you choose to improve the fictitious brand?"

  Answer 2. "Both quality factors B and C would be indifferent to me. Indeed, I prefer to improve any of these factors rather than improve quality factor A."

  This dialogue leads to the following marginal actions weak preference ordering:

  $$(18,\ 13,\ 21,\ 775) \succ (18,\ 76,\ 21,\ 114) \sim (18,\ 13,\ 81,\ 114) \succ (93,\ 13,\ 21,\ 114) \succ (18,\ 13,\ 21,\ 114)\ ,$$

  or tantamount in symbols:

  $$a_{040} \succ a_{020} \sim a_{030} \succ a_{010} \succ a_{000}.$$

- *Fourth step.* From equations (6) and (13), the utility ranking index for the i^{th} brand (i= 1,2,3,4,5) is given by:

  $$U_i = \left[v_{i1}^{1/(\log 93 - \log 18)} v_{i2}^{2/(\log 76 - \log 13)} v_{i3}^{2/(\log 81 - \log 21)} v_{i4}^{3/(\log 775 - \log 114)} \right]^{1/H},$$

  where

  $$H = \frac{1}{\log 93 - \log 18} + \frac{2}{\log 76 - \log 13} + \frac{2}{\log 81 - \log 21} + \frac{3}{\log 775 - \log 114} = 4.788,$$

  that is, $1/H$= 0.2088, or tantamount (see Table 1, last row),

  $$U_i = (v_{i1}^{0.609} v_{i2}^{1.132} v_{i3}^{1.482} v_{i4}^{1.566})^{0.2088}.$$

These utility indexes are displayed in Table 1, sixth column, giving rise to the following ranking:

$$a_3 \succ a_2 \succ a_5 \succ a_4 \succ a_1.$$

By using the right-hand side of the table (normalized logarithmic values), the same result is found.

Sensitivity analysis. To compare results from different hypotheses on the marginal actions weak preference ordering, it is interesting to ascertain how the ranking of existing actions is affected by changes in the preferences. The right-hand side of Table 1 allows us to achieve sensitivity analyses quickly in a visual manner. What if preferences for marginal actions were either ($a_{040} \succ a_{030} \succ a_{010} \sim a_{020}$) or ($a_{040} \succ a_{010} \sim a_{020} \succ a_{030}$) or ($a_{040} \succ a_{010} \sim a_{030} \succ a_{020}$) or ($a_{040} \succ a_{010} \succ a_{020} \sim a_{030}$) or ($a_{040} \succ a_{010} \sim a_{020} \sim a_{030}$)? Then, the ranking of the five existing actions is the same for any of these five hypotheses, namely, ($a_3 \succ a_2 \succ a_5 \succ a_1 \succ a_4$). However, this ranking involves a change with respect to the marginal actions weak preference ordering ($a_{040} \succ a_{020} \sim a_{030} \succ a_{010}$) assumed in Table 1. In fact, while the top three existing actions keep the same order, the last two switch their order. What if preferences for marginal actions were either ($a_{010} \sim a_{020} \sim a_{030} \succ a_{040}$) or even if all four marginal actions were equally preferred? Then, only the existing actions a_1 and a_4 switch their order with respect to ($a_{040} \succ a_{030} \succ a_{010} \sim a_{020}$) while there is no change relating to the preference hypothesis in Table 1.

5. A Large-scale Case of Ranking in the Textile Industry

In what follows, a problem of ranking 132 upholstery/curtain fabrics from a real world textile firm catalog is undertaken. In [3], a very different methodology is applied to solve this problem, so that although the empirical information herein comes from the cited paper, both approaches are a departure from one another. Seven quality factors are considered as attributes in the utility approach, say, tensile strength (the utmost longitudinal tension a fabric can endure without tearing apart), shrinkage (after the action of thermal, chemical and mechanical agents through the manufacturing process), pilling (small balls on the fabric surface), density, fastness to light (damage due to the action of daylight), fabric weight (as heavy fabrics help make a majestic impression of richness and sumptuousness) and flame retardance (flammability depending on fiber blends, construction and hairiness). Some of these attributes (shrinkage and pilling) behave as "more is worse", and consequently, they have to be converted into "more is better" attributes as explained in Remark 2. Each attribute is preferentially independent of the others. Their laboratory values for each fabric in the catalog are recorded on the left-hand side of Table 2, columns A-G. Each attribute measurement is performed by the Spanish standard UNE 40085, 25077, 1049-2, 40187, 40339 and 1021-2 with the exception of pilling which is measured by ISO/FDIS 12945-2.

Let us show that the quality factors meet preferential independence. Take, e.g., tensile strength, whose values will be denoted by v_1. From Table 2, we have:

$$(v_1^*, 1, 4, 4, 8, 247, 2) \succ\sim (v_{k1}, 1, 4, 4, 8, 247, 2)$$

$$\dots$$

$$(v_1^*, 2, 4, 4, 3, 226, 4) \succ\sim (v_{k1}, 2, 4, 4, 3, 226, 4)$$

where the best value v_1^* of tensile strength is greater than (or equal to) every v_{k1}. Analogous relationships hold if v_1^* is replaced by v_{1^*}, symbol $\succ\sim$ is replaced by symbol $\prec\sim$, and v_{k1} is greater than (or equal to) v_{1^*} for any k. Indeed, the whole quality of the fabric

Table 2: Laboratory data on seven quality factors identifying 132 fabrics from a sales catalog (fragment)

	Quality factors													
Code	Original values							Normalized logarithmic values						
	A	B	C	D	E	F	G	A	B	C	D	E	F	G
a_1	159	1	4	4	8	247	2	0.743	0.356	1	1	1	0.330	0.500
a_2	147	1	4	3	7	286	1	0.654	0.356	1.000	0.792	0.904	0.560	0
a_3	153	3	2	2	2	258	2	0.699	0.712	0.500	0.500	0	0.398	0.500
$\vdots$														
a_{51}	176	6	3	2	7	318	3	0.860	1	0.792	0.500	0.904	0.725	0.792
$\vdots$														
a_{62}	106	5	4	1	2	272	1	0.280	0.921	1	0	0	0.481	0
$\vdots$														
a_{132}	148	2	4	4	3	226	4	0.661	0.565	1	1	0.292	0.191	1
a_{000}	83	1	1	1	2	200	1	0	0	0	0	0	0	0
a_{010}	199	1	1	1	2	200	1	1	0	0	0	0	0	0
a_{020}	83	7	1	1	2	200	1	0	1	0	0	0	0	0
a_{030}	83	1	4	1	2	200	1	0	0	1	0	0	0	0
a_{040}	83	1	1	4	2	200	1	0	0	0	1	0	0	0
a_{050}	83	1	1	1	8	200	1	0	0	0	0	1	0	0
a_{060}	83	1	1	1	2	379	1	0	0	0	0	0	1	0
a_{070}	83	1	1	1	2	200	4	0	0	0	0	0	0	1

Definitions of the quality factors: $A=$ Tensile strength; $B=$ Shrinkage; $C=$ Pilling; $D=$ Density; $E=$ Fastness to light; $F=$ Fabric weight; $G=$ Flame retardance.

improves when tensile strength improves while the other quality factors remain constant, no matter the levels at which shrinkage, pilling, etc. are fixed. This result is straightforwardly extended to each of the other quality factors.

On the right-hand side of Table 2, columns *A*-*G*, the normalized logarithmic values are displayed for the 132 existing fabrics as well as for the marginal actions. According to the patterns established in Section 4 for preference elicitation dialogues, the author posed questions to a decision maker in the textile industry. Depending on the household use the fabric is intended for (curtains, upholstery and others for sale in different markets), the decision maker's answers led to six weak preference orderings of marginal actions. For each ordering, the 132 utility ranking indexes were computed from the data on the right-hand side of Table 1. If focusing on the top five indexes obtained for each of the six cases, the results are as follows.

- If the marginal actions weak preference ordering (MAWPO) is $a_{070} \succ a_{020} \succ a_{030} \sim a_{050} \succ a_{010} \sim a_{040} \sim a_{060} \succ a_{000}$, then the top existing five fabrics are $a_{21} \succ a_{103} \succ a_{80} \succ a_{24} \succ a_{39}$.
- If the MAWPO is $a_{070} \succ a_{020} \sim a_{030} \succ a_{050} \succ a_{010} \sim a_{040} \sim a_{060} \succ a_{000}$, then the top five are $a_{21} \succ a_{103} \succ a_{80} \succ a_{39} \succ a_{24}$.

- If the MAWPO is $a_{070} \succ a_{030} \succ a_{020} \sim a_{050} \succ a_{010} \sim a_{040} \sim a_{060} \succ a_{000}$, then the top five are $a_{21} \succ a_{103} \succ a_{80} \succ a_{24} \succ a_{39}$.
- If the MAWPO is $a_{070} \succ a_{020} \sim a_{030} \sim a_{050} \succ a_{040} \sim a_{060} \succ a_{010} \succ a_{000}$, then the top five are $a_{103} \succ a_{21} \succ a_{80} \succ a_{113} \succ a_{24}$.
- If the MAWPO is $a_{070} \succ a_{020} \sim a_{030} \sim a_{050} \succ a_{010} \sim a_{040} \sim a_{060} \succ a_{000}$, then the top five are $a_{21} \succ a_{103} \succ a_{80} \succ a_{24} \succ a_{39}$.
- If the MAWPO is $a_{070} \succ a_{030} \succ a_{020} \sim a_{040} \sim a_{060} \succ a_{010} \sim a_{050} \succ a_{000}$, then the top five are $a_{103} \succ a_{39} \succ a_{21} \succ a_{60} \succ a_{61}$.

In three of these cases, the top five fabrics appear ranked in the same order. Concerning the first two cases, fabrics 24 and 39 switch their order while the first three ranked fabrics in both cases keep their order exactly. However, as to the remaining cases, sharper changes in the ranking appear with respect to the others.

6. Conclusions

They are summarized as follows.

(i) No unusual assumption has been needed to underpin the approach but only standard assumptions, namely: (a) preferential independence, which is a well-known postulate in multi-attribute value theory to establish robust axiomatic bases for ranking models such as Keeney's and Raiffa's trade-offs; (b) Laplace's principle of insufficient reason, which is a classic paradigm from the 19^{th} century; and (c) Cobb-Douglas utility, which has a long tradition of describing multi-attribute preference maps in microeconomic applications. From the standard utility theory perspective, Cobb-Douglas utility (equivalent to additive decomposability through the logarithmic transformation) is a function more adequate than linear functions used in other approaches, as Cobb-Douglas meets desirable properties (for example, decreasing marginal utility).

(ii) Using marginal actions as a tool to specify Cobb-Douglas utility functions is new and conclusive in this paper.

(iii) As typical in models consistently built, the limitations from preferential independence should not be forgotten. Therefore, the practitioner should apply the proposed approach after checking that every attribute is preferentially independent of the others.

(iv) Eliciting preferences by dialogues referred to marginal actions does not require defining scales of measures or performing large cumbersome sets of comparisons. As such dialogues do not involve questions of the type "how much you prefer X to Y", they are very simple and easily understood by the decision maker.

(v) Computing the utility ranking indexes from the normalized logarithmic values on the right-hand side of Table 1 is remarkably easy by using Excel, so that the analyst only takes a few minutes to do it.

(vi) Results obtained either from Table 1 or from the large-scale case in Section 5 suggest that the utility ranking indexes are moderately sensitive to changes in the marginal actions weak preference ordering.

Future research could be conducted to compare results with other ranking models and techniques in the literature. Indeed, the bunch of results provided by heterogeneous approaches requires comparisons not in terms of superiority of one technique over others but in terms of appropriateness to a given problem in each scenario.

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Appendix 1.

Standard formulation of preferential independence: A reminder

Preferential independence (Assumption 1) means the following:

$$\begin{aligned}&\text{For all } v_{hj}, v_{rj} \in X_j (j = 1, 2, \ldots n),\\ &(v_{hj}, \alpha) \succsim (v_{rj}, \alpha) \text{ for some } \alpha \Rightarrow (v_{hj}, \beta) \succsim (v_{rj}, \beta) \text{ for all } \beta,\end{aligned} \tag{A1}$$

where α, β are row vectors in matrix (1), whose j^{th} component has been removed. Given the j^{th} attribute, consider the weak preference order

$$(v_{i_1 j}, \alpha) \succsim (v_{i_2 j}, \alpha) \succsim ...(v_{i_M j}, \alpha), \tag{A2}$$

for some fixed α. From (A1), this order (A2) implies

$$(v_{i_1 j}, \beta) \succsim (v_{i_2 j}, \beta) \succsim ...(v_{i_M j}, \beta), \tag{A3}$$

for all β. Hence, we can consistently define the best v_j^* and the worst v_{j^*} values in the j^{th} column by the identities

$$v_j^* = v_{i_1 j} \text{ and } v_{j^*} = v_{i_M j} \ . \tag{A4}$$

Cases "more is better" and "more is worse". Since values v_{ij} are levels of achievement, the following cases can occur.

- Case 1. In relationship (A3), the v values monotonically decrease from the first to the last, namely,

$$v_{i_1 j} \geq ... \geq v_{i_M j} \ . \tag{A5}$$

This is called "more is better". In other words, the j^{th} attribute would be positively oriented in that the decision maker would prefer higher scores.

- Case 2. In relationship (A3), the v values monotonically decrease from the last to the first, namely,

$$v_{i_M j} \geq ... \geq v_{i_1 j} \ . \tag{A6}$$

This is called "more is worse". In other words, the j^{th} attribute would be negatively oriented in that the decision maker would prefer lower scores.

- Case 3. In less frequent environments, the attributes can be nonmonotonic so neither "more is better" nor "more is worse" need to hold. For example, distance functions are not monotonic in location.

As utility theory assumes monotonicity not only in ordinal approaches but even also in linear cardinal approaches, case 3 is discarded in this paper.

Appendix 2.

Proof of Property 1. In equation (13), notice first that the factor of proportionality $1/\sum$ is irrelevant for ranking; however, it identifies Cobb-Douglas expression (6) as a standard utility function characterized by decreasing marginal utility with respect to each j^{th} attribute.
(a) Case $v_j^* \neq v_{j^*}$ for all j. According to Section 3, paragraph (ii), a utility index U_{000}= 1 is assigned to the fictitious action a_{000}. Let $a_{010}, \ldots \ a_{0h0}$ be the set of less preferred marginal actions which are indifferent to the decision maker. The α exponents derived from them are as follows (if overlooking the factor of proportionality $1/\sum$, which is irrelevant for ranking):

$$\alpha_1 = \log q/(\log v_1^* - \log v_{1^*}), \tag{A7}$$

$$\ldots,$$

$$\alpha_h = \log q/(\log v_h^* - \log v_{h^*}). \tag{A8}$$

Let $a_{0(h+1)0}, \ldots \ a_{0h'0}$ be the subsequent set of marginal actions which are indifferent to the decision maker. The α values derived from them are:

$$\alpha_{h+1} = \log q^2/(\log v_{h+1}^* - \log v_{h+1^*}) = 2\log q/(\log v_{h+1}^* - \log v_{h+1^*}), \tag{A9}$$

$$\ldots,$$

$$\alpha_{h'} = \log q^2/(\log v_{h'}^* - \log v_{h'^*}) = 2\log q/(\log v_{h'}^* - \log v_{h'^*}), \tag{A10}$$

and so on. From (A7)-(A10) and so on, $\log q$ is a common factor of all exponents. Therefore, the utility index $U_i^{1/\log q}$can be used instead of U_i so that parameter q has no impact on the ranking. Thus, the first part of Property 1 is demonstrated.
(b) Case $v_j^* = v_{j^*}$ for some j. Then, all the v_{ij} values are equal for this j^{th} attribute. Notice that equation (13) is now affected by an indeterminate form ∞/∞, which is easily solved by the limit:

$$\lim_{v_{j^*} \to v_j^*} \alpha_j = 1. \tag{A11}$$

Thus, the second part of Property 1 is demonstrated.

Appendix 3.

Zero values. If some $v_{j^*} = 0$ appears in decision matrix (1), then the corresponding term in equation (15) is affected by the indeterminate form ∞/∞, which is solved by taking the limit:

$$\lim_{v_{j^*} \to 0} p_j(\log v_{ij} - \log v_{j^*})/(\log v_j^* - \log v_{j^*}) = p_j \quad \text{if } v_{ij} \neq v_{j^*} \text{ , and} \tag{A12}$$

$$\lim_{v_{j^*} \to 0} p_j(\log v_{ij} - \log v_{j^*})/(\log v_j^* - \log v_{j^*}) = 0 \quad \text{if } v_{ij} = v_{j^*} \text{ .} \tag{A13}$$

Therefore, the normalization range (16) is not altered after taking the above limit.

Appendix 4.

Consistency and other properties. The following statements are interesting to enhance the approach.

Property 2. *Consistency of utility ranking indexes of marginal actions versus their weak preference ordering.* Both the marginal actions ranking from the Cobb-Douglas utility approach and the marginal actions weak preference ordering (3) necessarily coincide.

Proof. Consider the marginal action a_{0h0}= $(v_{1^*}, v_{2^*}, \ldots v_h^*, \ldots v_{n^*})$. These values specified for the utility ranking index (15) lead to:

$$\begin{aligned}\log U_{0h0} =& K + p_h(\log v_h^* - \log v_{h^*})/(\log v_h^* - \log v_{h^*}) + \sum_{j \neq h} p_j(\log v_{j^*} - \log v_{j^*})/(\log v_j^* - \log v_{j^*}) \\ =& K + p_h,\end{aligned} \tag{A14}$$

for every h, where K is irrelevant for ranking. Thus, Property 2 is demonstrated.

In Table 1, this property can be checked numerically. Either using original values or normalized logarithmic values, the utility ranking indexes of the marginal actions in the table exactly replicate the marginal actions weak preference ordering.

Property 3. *Domination.* If two existing alternatives a_h and a_k in matrix (1) have values $v_{hj} \geq v_{kj}$ for every j^{th}attribute (at least some of these relationships being a strict inequality), then the utility ranking index of a_h must be greater than the utility ranking index of a_k.

Proof. By specifying the a_h and a_k values for the Cobb-Douglas utility function (6), we have:

$$U_h = v_{h1}^{\alpha_1} v_{h2}^{\alpha_2} \ldots v_{hj}^{\alpha_j} \ldots v_{hn}^{\alpha_n} > v_{k1}^{\alpha_1} v_{k2}^{\alpha_2} \ldots v_{kj}^{\alpha_j} \ldots v_{kn}^{\alpha_n} = U_k \text{ .} \tag{A15}$$

Thus, Property 3 is demonstrated.

Property 4. *Independence of irrelevant alternatives.* The utility ranking indexes obtained by equations (6) and (13) do not change by adding a new alternative to matrix (1) providing that this adjunction does not alter the extreme values v_j^* and v_{j*} of any attribute.

Proof. The adjunction considered does not change the α_j exponents (13), and therefore, the utility ranking indexes (6) are kept. Thus, Property 4 is demonstrated.

Remark. *Risk aversion, marginal rate of substitution and the meaning of the above property.* As noted at the beginning of the paper, certainty is assumed in our approach. However, considering independence/dependence of irrelevant alternatives involves introducing some uncertainty, namely, the more or less probable existence of ignored alternatives for which either the j^{th} highest value or lowest value or both lie outside the range (v_{j*},v_j^*). As the decision maker is afraid of the fact that such ignored alternatives can exist, a problem of risk aversion arises. Recall that Arrow's ([1] p. 94) relative risk aversion coefficient for the j^{th} attribute is:

$$R_{Rj} = 1 - \alpha_j, \tag{A16}$$

where α_j is given by (13). As v_{j*} tends to v_j^* for some j, namely, as the range between the greatest and the lowest j^{th} values tends to zero, the utility curve:

$$u_j = v_{ij}^{\alpha_j}, \tag{A17}$$

becomes a small segment of the tangent line, which characterizes risk neutrality with $\alpha_j = 1$. More generally put, the smaller the range (v_{j*},v_j^*) the lower the risk aversion, or otherwise, the larger the range the higher the risk aversion. In Cobb-Douglas utility (6), this implies that the marginal rate of substitution:

$$MRS(j,h) = \frac{\alpha_j/v_{ij}}{\alpha_h/v_{ih}} \tag{A18}$$

changes as a function of risk aversion (A16). Therefore, the utility ranking indexes of two existing alternatives can change if new previously ignored alternatives were added to matrix (1) providing that these adjunctions alter the range $(v_{j*}v_j^*)$. In short, this range should be sufficiently large and representative of the j^{th} values to assure that independence of irrelevant alternatives always holds.

In a very different context such as strict uncertainty, Luce and Raiffa ([10] ch. 13) and French ([7] ch. 2) state several axioms which may be mutatis mutandis desirable for the above approach. Others vary from author to author or are meaningless in our context. Two of them, domination and independence of irrelevant alternatives have just been considered.

Property 5. *Complete ranking.* Utility (6) provides a complete ranking of the existing alternatives a_i (i= 1,2,...M).

Property 6. *Independence of labeling.* Suppose a second decision matrix is constructed from (1) by permuting the rows and columns so that subscript i turns into $\pi(i)$ while subscript j turns into $\tau(j)$; moreover, the same value v_{ij} is inserted into the $[\pi(i),\tau(j)]$ cell. This change in labeling does not alter the α exponents (13) nor the utility indexes (6). Therefore, the property holds.

Property 7. *Independence of value scale.* Suppose a second decision matrix is constructed from (1) by using now another scale to measure the j^{th} values so that the levels on the new scale are proportional to the old ones. As every v_{ij} in the j^{th} column is thus multiplied by a constant C, we have that equation (13) is kept unchanged, while utility index (6) is multiplied by an irrelevant factor. Therefore, the property holds. For example, imagine the j^{th} attribute is quantity of cakes, the unit of measurement on the original scale being one cake while the unit on the new scale is one tray with ten cakes. Then, using either one cake or one tray as a measurement unit does not alter the marginal actions weak preference ordering nor the difference $(\log v_j^* - \log v_{j^*}) = (\log Cv_j^* - \log Cv_{j^*})$.

Property 8. *Continuity.* The ranking indexes should be given by a continuous function of the v_{ij} values. This happens with Cobb-Douglas utility (6), and therefore, the property holds.

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