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R&D COMPETITION IN ALTERNATIVE TECHNOLOGIES: A REAL OPTIONS APPROACH

Michi Nishihara
Osaka University

Atsuyuki Ohyama
Kyoto University

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Abstract We study a problem of R&D competition using a real options approach. We extend the analysis of Weeds [34] in which the project is of a fixed standard to the case where the firms can choose the target of the research from two alternative technologies of different standards. We show that the competition affects not only the firms' investment time, but also their choice of the standard of the technology. Two typical cases, namely the de facto standard case and the innovative case, are examined in full detail. In particular, in the de facto standard case, the firms could develop a lower-standard technology that would never appear in a noncompetitive situation. This provides a good account of a real problem resulting from too bitter R&D competition.

Keywords: Finance, decision making, real options, R&D, de facto standard, preemption, stopping time game

1. Introduction

Real options approaches have become a useful tool for evaluating irreversible investment under uncertainty such as R&D investment (see [6]). Although the early literature on real options (e.g., [4, 26]) treated the investment decision of a single firm, more recent studies provoked by [11] have investigated the problem of several firms competing in the same market from a game theoretic approach (see [1] for an overview). Grenadier [12] derived the equilibrium investment strategies of the firms in the Cournot–Nash framework and Weeds [34] provided the asymmetric outcome (called *preemption equilibrium*) in R&D competition between the two firms using the equilibrium in a timing game studied in [9]. In [18, 33], a possibility of mistaken simultaneous investment resulting from an absence of rent equalization that was assumed in [34] was investigated.

On the other hand, there are several studies on the decision of a single firm with an option to choose both the type and the timing of the investment projects. In this literature, [5] was the first study to pay attention to the problem and Décamps et al. [3] investigated the problem in more detail. In [8], a similar model is applied to the problem of constructing small wind power units.

Despite such active studies on real options, to our knowledge few studies have tried to elucidate how competition between two firms affects their investment decisions in the case where the firms have the option to choose both the type and the timing of the projects. This paper investigates the above problem by extending the R&D model in [34] to a model where the firms can choose the target of the research from two alternative technologies of different standards with the same uncertainty about the market demand[1], where the technology

[1]We assume that the two technologies are applied to homogeneous products.

standard is to be defined in some appropriate sense. As in [34], technological uncertainty is taken into account, in addition to the product market uncertainty. We assume that the time between project initiation and project discovery (henceforth the research term) follows the Poisson distribution[2] with its hazard rate determined by the standard of the technology. This assumption is realistically intuitive since a higher-standard technology is likely to require a longer research term and is expected to generate higher profits at its completion.

In the model, we show that the competition between the two firms affects not only the firms' investment time, but also their choice of the technology targeted in the project. In fact, we observe that the effect on the choice of the standard consists of two components. The presence of the other firm straightforwardly changes the value of the technologies. We call this the *direct* effect on the choice of the project type. In addition to the direct effect, the timing game caused by the competition affects the firms' choice of the targeted technology. This is due to the hastened timing through the strategic interaction with the competitor; accordingly we call this the *indirect* effect, distinctively from the direct effect.

We highlight two typical cases that are often observed in a market and, at the same time, reveal interesting implications. The first case is that a firm that completes a technology first can monopolize the profit flow regardless of the standard of the technology. *De facto standardization struggles* such as VHS vs Betamax for video recorders are true for this case (henceforth called the de facto standard case). In such cases, a firm can impose its technology as a de facto standard by introducing it before its competitors. Once one technology becomes the de facto standard for the market, the winner may well enjoy a monopolistic cash flow from the patent of the de facto standard technology for a long term. It is then quite difficult for other firms to replace it with other technologies even if those technologies are superior to the de facto standard one. Indeed, it has been often observed in de facto standardization races that the existing technology drives out a newer (superior) technology, which can be regarded as a sort of *Gresham's law*[3]. In conclusion, what is important in the de facto standard case is introducing the completed technology into the market before the opponents.

The other case is where a firm with higher-standard technology can deprive a firm with lower-standard technology of the cash flow by completing the higher-standard technology. This case applies to technologies of the innovative type (henceforth called the innovative case). As observed in evolution from cassette-based Walkmans to CD- and MD-based Walkmans, and further to flash memory- and hard drive-based digital audio players (e.g., iPod), the appearance of a newer technology drives out the existing technology. In such cases, a firm often attempts to develop a higher-standard technology because it fears the invention of superior technologies by its competitors. As a result, in the innovative case, a higher-standard technology tends to appear in a market.

The analysis in the two cases gives a good account of the characteristics mentioned above. In the de facto standard case, the competition increases the incentive to develop the lower-standard technology, which is easy to complete, while in the innovative case, the competition increases the incentive to develop the higher-standard technology, which is difficult to complete. The increase comes from both the direct and indirect effect of the completion. In particular, we show that in the de facto standard case the competition is

[2]Most studies, such as [2, 20, 25, 34], model technical innovation as a Poisson arrival; we also follow this convention.

[3]Gresham's law is the economic principle that in the circulation of money "bad money drives out good," i.e., when depreciated, mutilated, or debased coinage (or currency) is in concurrent circulation with money of high value in terms of precious metals, the good money is withdrawn from circulation by hoarders.

likely to lead the firms to invest in the lower-standard technology, which is never chosen in the single firm situation. This result explains a real problem caused by too bitter R&D competition. It is possible that the competition spoils the higher-standard technology that consumers would prefer[4], while the development hastened by the competition increases consumers' profits compared with that of the monopoly. That is, the result accounts for both positive and negative sides of the R&D competition for consumers. Of course, as described in [31], practical R&D management is often much more flexible and complex (e.g., growth and sequential options studied in [23, 24]) than the simple model in this paper. However, it is likely that the essence of the results remains unchanged in more practical setups.

In addition to the implications about the R&D competition given above, we also mention theoretical contribution in relation to existing streams of the studies on real options with strategic interactions. In fact, there are enormous number of papers that analyze strategic real options models between two firms. While there is a stream of literature concentrating on incomplete and asymmetric information[5], our model is built on complete information. In literature under complete information, Grenadier [11] proposed the basic model, and it has been extended to several directions (e.g., involving the research term in [34], the exit decision in [27], the entry and exit decision in [10]). Among those studies, a distinctive feature of our model is that the firms have the option to choose the project type, which is properly defined in connection with the research term. Technically, we combine the model by [34] with that of [3]. By doing so, we capture the simultaneous changes of the investment timing and the choice of the project type due to the competition. In particular, there is an interaction between the timing and project type choices (recall the indirect effect).

In terms of treating high and low standard technologies, this paper is related to [19, 20]. Their models give the technological innovation exogenously and assume that the firms can receive revenue flows immediately after the investment using the available technology. Then, they show how the possibility that a higher-standard technology will emerge in the future influences the investment decision. In our model, on the other hand, the endogenous factor (i.e., the firm's choice of the type and the timing of the investment projects) in addition to the exogenous one (i.e., randomness in the research term) causes the technological innovation. That is, the firm itself can trigger the innovation generating the patent value. Thus, this paper, unlike [19, 20], investigates the investment decision of R&D which will provoke future technological innovation.

The paper is organized as follows. After Section 2 derives the optimal investment timing for the single firm, Section 3 formulates the problem of the R&D competition between two firms. Section 4 derives the equilibrium strategies in the two typical cases, namely, the de facto standard case and the innovative case. Section 5 gives numerical examples, and finally Section 6 concludes the paper.

2. Single Firm Situation

Throughout the paper, we assume all stochastic processes and random variables are defined on the filtered probability space $(\Omega, \mathcal{F}, P; \mathcal{F}_t)$. This paper is based on the model by [34]. This section considers the investment decision of the single firm without fear of preemption. The firm can set up a research project for developing a new technology i (we denote technologies

[4]It is reasonable to suppose that consumers benefit from the invention of higher-standard technologies, though, strictly speaking, we need to incorporate consumers' value functions into the model.
[5]See, for example, [13, 16, 22, 28, 29].

1 and 2 for the lower-standard and higher-standard technologies, respectively) by paying an indivisible investment cost k_i. As in [34], for analytical advantage we assume that the firm has neither option to suspend nor option to switch the projects, though practical R&D investment often allows more managerial flexibility, such as to abandon, expand and switch (see [31]).

In developing technology i, from the time of the investment the invention takes place randomly according to a Poisson distribution with constant hazard rate h_i. The firm must pay the research expense l_i per unit of time during the research term and can receive the profit flow $D_iY(t)$ from the discovery. Here, $Y(t)$ represents a market demand of the technologies at time t and influence cash flows which the technologies generate. It must be noted that the firm's R&D investment is affected by two different types of uncertainty (i.e., technological uncertainty and product market uncertainty). For simplicity, $Y(t)$ obeys the following geometric Brownian motion, which is independent of the Poisson processes representing technological uncertainty.

$$\mathrm{d}Y(t) = \mu Y(t)\mathrm{d}t + \sigma Y(t)\mathrm{d}B(t) \quad (t > 0),\ Y(0) = y, \tag{1}$$

where $\mu \geq 0, \sigma > 0$ and $y > 0$ are given constants and $B(t)$ denotes the one-dimensional $\mathcal{F}_t$ standard Brownian motion. Quantities k_i, h_i, D_i and l_i are given constants satisfying

$$0 \leq k_1 \leq k_2,\ 0 < h_2 < h_1,\ 0 < D_1 < D_2,\ 0 < l_1 \leq l_2, \tag{2}$$

so that technology 2 is more difficult to develop and generates a higher profit flow from its completion than technology 1.

Let us now comment upon the model. For analysis in later sections, we modified the original setup by [34] at the two following points, but there are no essential difference. In [34], the completed technology generates not a profit flow but a momentary profit as the value of the patent at its completion, and there is no research expense during the research term (i.e, $l_i = 0$). In [20] the Poisson process determining technological innovation is exogenous to the firms as in [14], but we assume that a firm's investment initiates the Poisson process determining the completion of the technology. This is the main difference from the model studied in [20] that also treats two technologies.

The firm that monitors the state of the market can set up development of either technologies 1 or 2 at the optimal timing maximizing the expected payoff under discount rate $r\ (> \mu)$. Then, the firm's problem is expressed as the following optimal stopping problem:

$$V_0(y) = \sup_{\tau \in \mathcal{T}} E\left[\max_{i=1,2} E\left[\int_{\tau+t_i}^{\infty} \mathrm{e}^{-rt} D_i Y(t)\mathrm{d}t - \mathrm{e}^{-r\tau}k_i - \int_{\tau}^{\tau+t_i} \mathrm{e}^{-rt} l_i \mathrm{d}t \mid \mathcal{F}_\tau\right]\right], \tag{3}$$

where $\mathcal{T}$ is a set of all $\mathcal{F}_t$ stopping times and t_i denotes a random variable following the exponential distribution with hazard rate h_i. In problem (3), $\max_{i=1,2} E[\cdot \mid \mathcal{F}_\tau]$ means that the firm can choose the optimal technology at the investment time τ.

By the calculation we obtain

$$\begin{aligned} & E\left[\int_{\tau+t_i}^{\infty} \mathrm{e}^{-rt} D_i Y(t)\mathrm{d}t - \mathrm{e}^{-r\tau}k_i - \int_{\tau}^{\tau+t_i} \mathrm{e}^{-rt} l_i \mathrm{d}t \mid \mathcal{F}_\tau\right] && (4) \\ = \ & \mathrm{e}^{-r\tau} E^{Y(\tau)}\left[\int_{t_i}^{\infty} \mathrm{e}^{-rt} D_i Y(t)\mathrm{d}t - k_i - \int_0^{t_i} \mathrm{e}^{-rt} l_i \mathrm{d}t\right] && (5) \\ = \ & \mathrm{e}^{-r\tau} \int_0^{\infty} \left(\int_s^{\infty} \mathrm{e}^{-rt} D_i E^{Y(\tau)}[Y(t)]\mathrm{d}t - k_i - \int_0^s \mathrm{e}^{-rt} l_i \mathrm{d}t\right) h_i \mathrm{e}^{-h_i s}\mathrm{d}s && (6) \\ = \ & \mathrm{e}^{-r\tau}(a_{i0}Y(\tau) - I_i), && (7) \end{aligned}$$

where we use the strong Markov property of $Y(t)$ in (5) and independence between t_i and $Y(t)$ in (6). Here, we need to explain the notation $E^{Y(\tau)}[\cdot]$ in (5) and (6). For a real number x, the notation $E^x[\cdot]$ denotes the expectation operator given that $Y(0) = x$, which can be changed from the original initial value y. When the initial value is unchanged from the original value y, we omit the superscript y, that is, $E[\cdot] = E^y[\cdot]$. The notation $E^{Y(\tau)}[\cdot]$ represents the random variable $\psi(Y(\tau))$, where $\psi(x) = E^x[\cdot]$. For example, $E^x[Y(t)]$ is $x\mathrm{e}^{\mu t}$, and therefore $E^{Y(\tau)}[Y(t)]$ in (6) becomes $Y(\tau)\mathrm{e}^{\mu t}$. Thus, problem (3) can be reduced to

$$V_0(y) = \sup_{\tau \in \mathcal{T}} E[\mathrm{e}^{-r\tau} \max_{i=1,2}(a_{i0}Y(\tau) - I_i)], \tag{8}$$

where a_{i0} and I_i are defined by

$$a_{i0} = \frac{D_i h_i}{(r-\mu)(r+h_i-\mu)} \tag{9}$$

$$I_i = k_i + \frac{l_i}{r+h_i}. \tag{10}$$

Here, $a_{i0}Y(\tau)$ represents the expected discounted value of the future profit generated by technology i at the investment time τ, and I_i represents its total expected discounted cost at time τ.[6] (2) and (10) imply $I_1 < I_2$, but the inequality $a_{10} < a_{20}$ does not necessarily hold depending upon a trade-off between h_i and D_i. Note that (8) is essentially the same as the problem examined in [3].

We make a brief explanation as to the difference between (8) and

$$\tilde{V}(y) = \max_{i=1,2} \left\{ \sup_{\tau \in \mathcal{T}} E[\mathrm{e}^{-r\tau}(a_{i0}Y(\tau) - I_i)] \right\}. \tag{11}$$

(11) is a problem in which at time 0 the firm must decide which technology it develops. That is, in problem (11), the firm cannot switch the technology even before the investment time τ once the firm choose the technology at initial time. Since fewer cases of R&D investment applies to the setting (11), we consider the setting (8) in which the firm can determine the technology standard at the investment time τ. In addition, it holds that $V_0(y) \geq \tilde{V}(y)$ because the firm has more managerial flexibility in problem (8) than in problem (11).

Let $V_0(y)$ and τ_0^* denote the value function and the optimal stopping time of problem (8), respectively. Note that τ_0^* is expressed in a form independent of the initial value y. As in most real options literature (e.g., [6]), we define

$$\beta_{10} = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1, \tag{12}$$

$$\beta_{20} = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0. \tag{13}$$

Proposition 2.1 The value function $V_0(y)$ and the optimal stopping time τ_0^* of the single firm's problem (8) are given as follows:

[6] Our setup is essentially the same as the setup by [34] that assumes $l_i = 0$, because we do not allow suspension in the research term. That is, I_i defined by (10) can be regarded as a sunk investment cost. However, in this paper we consider l_i plus k_i in order to relate the cost I_i with the hazard rate h_i.

Case 1: $0 < a_{20}/a_{10} \le 1$

$$V_0(y) = \begin{cases} A_0 y^{\beta_{10}} & (0 < y < y_{10}^*) \\ a_{10}y - I_1 & (y \ge y_{10}^*), \end{cases} \tag{14}$$

$$\tau_0^* = \inf\{t \ge 0 \mid Y(t) \ge y_{10}^*\}. \tag{15}$$

Case 2: $1 < (a_{20}/a_{10})^{\beta_{10}/(\beta_{10}-1)} < I_2/I_1$

$$V_0(y) = \begin{cases} A_0 y^{\beta_{10}} & (0 < y < y_{10}^*) \\ a_{10}y - I_1 & (y_{10}^* \le y \le y_{20}^*) \\ B_0 y^{\beta_{10}} + C_0 y^{\beta_{20}} & (y_{20}^* < y < y_{30}^*) \\ a_{20}y - I_2 & (y \ge y_{30}^*), \end{cases} \tag{16}$$

$$\tau_0^* = \inf\{t \ge 0 \mid Y(t) \in [y_{10}^*, y_{20}^*] \cup [y_{30}^*, +\infty)\}. \tag{17}$$

Case 3: $(a_{20}/a_{10})^{\beta_{10}/(\beta_{10}-1)} \ge I_2/I_1$

$$V_0(y) = \begin{cases} B_0 y^{\beta_{10}} & (0 < y < y_{30}^*) \\ a_{20}y - I_2 & (y \ge y_{30}^*), \end{cases} \tag{18}$$

$$\tau_0^* = \inf\{t \ge 0 \mid Y(t) \ge y_{30}^*\}. \tag{19}$$

Here, constants A_0, B_0, C_0 and thresholds $y_{10}^*, y_{20}^*, y_{30}^*$ are determined by imposing value matching and smooth pasting conditions (see [6]). Quantities B_0 and y_{30}^* take different values in Cases 2 and 3, though we use the same notations. Note that $I_1 < I_2$ and $\beta_{10} > 1$.
Proof See Appendix A.

In Proposition 2.1, $A_0 y^{\beta_{10}}, B_0 y^{\beta_{10}}$ and $C_0 y^{\beta_{20}}$ correspond to the values of the option to invest in technology 1 at the trigger y_{10}^*, the option to invest in technology 2 at the trigger y_{30}^* and the option to invest in technology 1 at the trigger y_{20}^*, respectively. In Case 1, where the expected discounted profit of technology 1 is higher than that of technology 2, the firm initiates development of technology 1 at time (15) independently of the initial market demand y. In Case 3, where technology 2 is much superior to technology 1, on the contrary, the firm invests in technology 2 at time (19) regardless of y. In Case 2, where both projects has similar values by the trade-off between the profitability and the research term and cost, the firm's optimal investment strategy has three thresholds y_{10}^*, y_{20}^* and y_{30}^*, and therefore the project chosen by the firm depends on the initial value y. Above all, if $y \in (y_{20}^*, y_{30}^*)$, the firm defers not only investment, but also choice among the two projects (i.e, whether the firm invests in technology 2 when the market demand $Y(t)$ increases to the upper trigger y_{30}^* or invests in technology 1 when $Y(t)$ decreases to the lower trigger y_{20}^*). Hence, $V_0(y) > \tilde{V}_0(y)$ holds only for $y \in (y_{20}^*, y_{30}^*)$ in Case 2, while $V_0(y)$ equals to $\tilde{V}_0(y)$ in other regions in Case 2 and other cases.

By letting volatility $\sigma \uparrow +\infty$ with other parameters fixed, we have $\beta_{10} \downarrow 1$ by definition (12) and therefore $(a_{20}/a_{10})^{\beta_{10}/(\beta_{10}-1)} \uparrow +\infty$ if $a_{10} < a_{20}$. As a result, with high product market uncertainty, instead of Case 2, Case 3 holds and the firm chooses the higher-standard technology 2 rather than the lower-standard technology 1, unless the expected discounted profit generated by technology 1 exceeds that of technology 2. The similar result has also been mentioned in [3].

3. Situation of Two Noncooperative Firms

We turn now to a problem of two symmetric firms. This paper considers a symmetric setting to avoid unnecessary confusion, but the results in this paper could remain true to some extent in an asymmetric case. For a standard discussion of an asymmetric situation, see [18]. We assume that two Poisson processes modeling the two firms' innovation are independent of each other, which means that the progress of the research project by one of the firms does not affect that of its rival. The scenarios of the cash flows into the firms can be classified into four cases. Figure 1 illustrates the cash flows into the firm that has completed a technology first (denoted Firm 1) and the other (denoted Firm 2). In the period

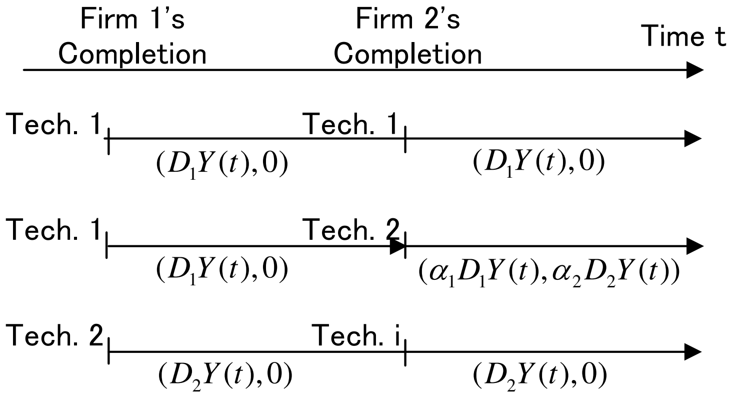

Figure 1: (Firm 1's cash flow, Firm 2's cash flow)

when a single firm has succeeded in the development of technology i, the firm obtains the monopoly cash flow $D_iY(t)$. If both firms develop the same technology i, the one that has completed first receives the profit flow $D_iY(t)$ resulting from the patent perpetually and the other obtains nothing, according to the setup by [34]. Of course, the firm that has completed the lower-standard technology 1 after the competitor's completion of the higher-standard technology 2 obtains no cash flow. When the firm has completed technology 2 after the competitor's completion of technology 1, from the point technology 2 generates the profit flow $\alpha_2D_2Y(t)$, and technology 1 generates $\alpha_1D_1Y(t)$, where α_i are constants satisfying $0 \leq \alpha_1, \alpha_2 \leq 1$. It is considered that the technology's share in the product market determines α_1 and α_2.

As usual (see the books [6, 18]), we solve the game between two firms backward. We begin by supposing that one of the firms has already invested, and find the optimal decision of the other. In the remainder of this paper, we call the one who has already invested *leader* and call the other *follower*, though we consider two symmetrical firms. Thereafter, in the next section, we look at the situation where neither firms has invested, and consider the decision of either as it contemplates whether to go first, knowing that the other will react in the way just calculated as the follower's optimal response. The main difference from the existing literature such as [6, 11, 18, 27, 34] is that the follower's optimal response depends on the technology i chosen by the leader. Let $F_i(Y)$ and $\tau^*_{F_i}$ denote the expected discounted payoff (at time t) and the investment time of the follower responding optimally to the leader who has invested in technology i at time t satisfying $Y(t) = Y$. We denote by $L_i(Y)$ the expected discounted payoff (at time t) of the leader who has invested in technology i at $Y(t) = Y$.

3.1. Case where the leader has invested in technology 2

This subsection derives $F_2(Y), \tau^*_{F_2}$ and $L_2(Y)$. Given that the leader has invested in technology 2 at $Y(t) = Y$, the follower solves the following optimal stopping problem:

$$\begin{aligned} F_2(Y) = \mathrm{e}^{rt} \sup_{\tau \in \mathcal{T}, \tau \geq t} E \Bigg[\mathrm{e}^{-h_2 \tau} \max \Bigg\{ E \Bigg[1_{\{t_1 < s_2\}} \Bigg(\int_{\tau+t_1}^{\tau+s_2} \mathrm{e}^{-rs} D_1 Y(s) \mathrm{d}s \\ + \int_{\tau+s_2}^{\infty} \mathrm{e}^{-rs} \alpha_1 D_1 Y(s) \mathrm{d}s \Bigg) - \mathrm{e}^{-r\tau} k_1 - \int_{\tau}^{\tau+t_1} \mathrm{e}^{-rs} l_1 \mathrm{d}s \mid \mathcal{F}_\tau \Bigg], \\ E \Bigg[1_{\{t_2 < s_2\}} \int_{\tau+t_2}^{+\infty} \mathrm{e}^{-rs} D_2 Y(s) \mathrm{d}s - \mathrm{e}^{-r\tau} k_2 - \int_{\tau}^{\tau+t_2} \mathrm{e}^{-rs} l_2 \mathrm{d}s \mid \mathcal{F}_\tau \Bigg] \Bigg\} \mid Y(t) = Y \Bigg], \end{aligned} \tag{20}$$

where $E[\cdot \mid Y(t) = Y]$ represents the expectation conditioned that $Y(t) = Y$. Recall t_i represents a random variable following the exponential distribution with hazard rate h_i. The random variable s_i is independent of t_i and also follows the exponential distribution with hazard rate h_i. Note that the research term of the follower choosing technology i is expressed as t_i in (20). The interval between τ and the discovery time of the leader follows the exponential distribution with hazard rate h_2 (hence, it is expressed as s_2 in (20)) *under the condition that the leader has yet to complete technology* 2 *at time* τ. The reason is that the discovery occurs according to the Poisson process which is Markovian. What has to be noticed is that the follower's problem (21) is discounted by $\mathrm{e}^{-h_2\tau}$ differently from the single firm's problem (3). This is because the leader's completion of technology 2 deprives the follower of the future option to invest. As in the single firm's problem (3), $\max_{i=1,2} E[\cdot \mid \mathcal{F}_\tau]$ means that the follower chooses the better project at the investment time τ. Furthermore, $1_{\{t_i<s_2\}}$ denotes a defining function and means that the follower's payoff becomes nothing if the leader completes technology 2 first. In order to derive $F_2(Y)$ and $\tau^*_{F_2}$, we rewrite problem (20) as the following problem with initial value $Y(0) = Y$, using the Markov property,

$$\begin{aligned} F_2(Y) = \sup_{\tau \in \mathcal{T}} E^Y \Bigg[\mathrm{e}^{-h_2 \tau} \max \Bigg\{ E^Y \Bigg[1_{\{t_1 < s_2\}} \Bigg(\int_{\tau+t_1}^{\tau+s_2} \mathrm{e}^{-rs} D_1 Y(s) \mathrm{d}s \\ + \int_{\tau+s_2}^{\infty} \mathrm{e}^{-rs} \alpha_1 D_1 Y(s) \mathrm{d}s \Bigg) - \mathrm{e}^{-r\tau} k_1 - \int_{\tau}^{\tau+t_1} \mathrm{e}^{-rs} l_1 \mathrm{d}s \mid \mathcal{F}_\tau \Bigg], \\ E^Y \Bigg[1_{\{t_2 < s_2\}} \int_{\tau+t_2}^{+\infty} \mathrm{e}^{-rs} D_2 Y(s) \mathrm{d}s - \mathrm{e}^{-r\tau} k_2 - \int_{\tau}^{\tau+t_2} \mathrm{e}^{-rs} l_2 \mathrm{d}s \mid \mathcal{F}_\tau \Bigg] \Bigg\} \Bigg], \end{aligned} \tag{21}$$

where $E^Y[\cdot]$ means the (conditional) expectation operator given that the initial value $Y(0)$ is Y instead of y, as explained in before. Then, τ and s in problem (21), unlike those in (20), represents how long it has passed since the leader's investment time t. Strictly speaking, the optimal stopping time in problem (21) is different from that in problem (20), $\tau^*_{F_2}$, since the initial time in problem (21) corresponds to time t in problem (20). However, it is easy to derive $\tau^*_{F_2}$ from the solution in problem (21), and hence we hereafter identify problem (21) with problem (20).

Via the similar calculation to (4)–(7) we can rewrite problem (21) as

$$F_2(Y) = \sup_{\tau \in \mathcal{T}} E^Y[\mathrm{e}^{-(r+h_2)\tau} \max_{i=1,2}(a_{i2}Y(\tau) - I_i)], \tag{22}$$

where a_{ij} are defined by

$$a_{11} = \frac{D_1 h_1}{(r-\mu)(r+2h_1-\mu)}, \tag{23}$$

$$a_{12} = \frac{D_1 h_1}{(r+h_1+h_2-\mu)(r+h_2-\mu)}\left(1+\frac{\alpha_1 h_2}{r-\mu}\right), \tag{24}$$

$$a_{21} = \frac{D_2 h_2}{(r-\mu)(r+h_1+h_2-\mu)}\left(1+\frac{\alpha_2 h_1}{r+h_2-\mu}\right), \tag{25}$$

$$a_{22} = \frac{D_2 h_2}{(r-\mu)(r+2h_2-\mu)}. \tag{26}$$

Quantity $a_{ij}Y(\tau)$ represents the expected discounted value of the future cash flow of the firm that invests in technology i at time τ when its opponent is on the way to development of technology j. From the expression (22), we can show the following proposition.

Proposition 3.1 The follower's payoff $F_2(Y)$, investment time $\tau^*_{F_2}$ and the leader's payoff $L_2(Y)$ are given as follows:

Case 1: $0 < a_{22}/a_{12} \le 1$

$$\begin{aligned} F_2(Y) &= \begin{cases} A_2 Y^{\beta_{12}} & (0 < Y < y^*_{12}) \\ a_{12}Y - I_1 & (Y \ge y^*_{12}), \end{cases} \\ \tau^*_{F_2} &= \inf\{s \ge t \mid Y(s) \ge y^*_{12}\}, \\ L_2(Y) &= \begin{cases} a_{20}Y - I_2 - \tilde{A}_2 Y^{\beta_{12}} & (0 < Y < y^*_{12}) \\ a_{21}Y - I_2 & (Y \ge y^*_{12}). \end{cases} \end{aligned}$$

Case 2: $1 < (a_{22}/a_{12})^{\beta_{12}/(\beta_{12}-1)} < I_2/I_1$

$$\begin{aligned} F_2(Y) &= \begin{cases} A_2 Y^{\beta_{12}} & (0 < Y < y^*_{12}) \\ a_{12}Y - I_1 & (y^*_{12} \le Y \le y^*_{22}) \\ B_2 Y^{\beta_{12}} + C_2 Y^{\beta_{22}} & (y^*_{22} < Y < y^*_{32}) \\ a_{22}Y - I_2 & (Y \ge y^*_{32}), \end{cases} \\ \tau^*_{F_2} &= \inf\{s \ge t \mid Y(s) \in [y^*_{12}, y^*_{22}] \cup [y^*_{32}, +\infty)\}, \\ L_2(Y) &= \begin{cases} a_{20}Y - I_2 - \tilde{A}_2 Y^{\beta_{12}} & (0 < Y < y^*_{12}) \\ a_{21}Y - I_2 & (y^*_{12} \le Y \le y^*_{22}) \\ a_{20}Y - I_2 - \tilde{B}_2 Y^{\beta_{12}} - \tilde{C}_2 Y^{\beta_{22}} & (y^*_{22} < Y < y^*_{32}) \\ a_{22}Y - I_2 & (Y \ge y^*_{32}). \end{cases} \end{aligned}$$

Case 3: $(a_{22}/a_{12})^{\beta_{12}/(\beta_{12}-1)} \ge I_2/I_1$

$$\begin{aligned} F_2(Y) &= \begin{cases} B_2 Y^{\beta_{12}} & (0 < Y < y^*_{32}) \\ a_{22}Y - I_2 & (Y \ge y^*_{32}), \end{cases} \\ \tau^*_{F_2} &= \inf\{s \ge t \mid Y(s) \ge y^*_{32}\}, \\ L_2(Y) &= \begin{cases} a_{20}Y - I_2 - \tilde{B}_2 Y^{\beta_{12}} & (0 < Y < y^*_{32}) \\ a_{22}Y - I_2 & (Y \ge y^*_{32}). \end{cases} \end{aligned}$$

Here, β_{12} and β_{22} denote (12) and (13) replaced r by $r + h_2$, respectively. Here, $r + h_2$ is the discount factor taking account of the possibility that the option is vanished with intensity h_2. After constants A_2, B_2, C_2 and thresholds $y^*_{12}, y^*_{22}, y^*_{32}$ are determined by both value matching and smooth pasting conditions in the follower's value function $F_2(Y)$, constants $\tilde{A}_2, \tilde{B}_2$ and $\tilde{C}_2$ are determined by the value matching condition alone in the leader's payoff function $L_2(Y)$. As in Proposition 2.1, the same notations in different cases do not necessarily mean the same values. Note that $I_1 < I_2$ and $\beta_{12} > 1$.

Proof See Appendix B.

Constants A_2, B_2, C_2 and thresholds $y^*_{12}, y^*_{22}, y^*_{32}$ in Proposition 3.1 correspond to constants A_0, B_0, C_0 and thresholds $y^*_{10}, y^*_{20}, y^*_{30}$ in Proposition 2.1, respectively. Let us explain the leader's payoff briefly. Constants $\tilde{A}_2, \tilde{B}_2$ and $\tilde{C}_2$ value the possibility that Y rises above y^*_{12} prior to the leader's completion, the possibility that Y rises above y^*_{32} prior to the leader's completion, and the possibility that Y falls bellow y^*_{22} prior to the leader's completion, respectively. Since these situations cause the follower's investment, the leader's payoff is reduced from the monopoly profit $a_{20}Y - I_2$ (see $Y \in (0, y^*_{12})$ in Case 1, $Y \in (0, y^*_{12}) \cup (y^*_{22}, y^*_{32})$ in Case 2, and $Y \in (0, y^*_{32})$ in Case 3).

3.2. Case where the leader has invested in technology 1

We now consider $F_1(Y), \tau^*_{F_1}$ and $L_1(Y)$. In the previous subsection, i.e., in the case where the leader has chosen technology 2, the follower's opportunity to invest is completely lost at the leader's completion of technology 2. However, in the case where the leader has invested in technology 1, there remains the inactive follower's option after the leader's invention of technology 1. That is, the follower can invest in technology 2 even after the leader's discovery if the follower has not invested in any technology yet. Due to this option value, we need more complicated discussion in this subsection.

Let $f_1(Y)$ and $\tau^*_{f_1}$ be the expected discounted payoff and the optimal stopping time of the follower responding optimally to the leader who has already succeeded in development of technology 1 at $Y(t) = Y$. In other words, $f_1(Y)$ represents the remaining option value to invest in technology 2 after the leader's completion of technology 1. We need to derive $f_1(Y)$ and $\tau^*_{f_1}$ before analyzing $F_1(Y)$ and $\tau^*_{F_1}$. Given that the leader has already completed technology 1 at $Y(t) = Y$, the follower's problem becomes

$$f_1(Y) = \sup_{\tau \in \mathcal{T}} E^Y \left[\int_{\tau+t_2}^{\infty} \mathrm{e}^{-rt} \alpha_2 D_2 Y(t) \mathrm{d}t - \mathrm{e}^{-r\tau} k_2 - \int_{\tau}^{\tau+t_2} \mathrm{e}^{-rt} l_2 \mathrm{d}t \right], \tag{27}$$

which is equal to a problem of a firm that can develop only technology 2. In this subsection, we omit a description of a problem which corresponds to (20), and describe only a problem (which corresponds to (21)) with initial value $Y(0) = Y$. In the same way as calculation (4)–(7), we can rewrite problem (27) as

$$f_1(Y) = \sup_{\tau \in \mathcal{T}} E^Y [\mathrm{e}^{-r\tau} (\alpha_2 a_{20} Y(t) - I_2)]. \tag{28}$$

It is easy to obtain the value function $f_1(Y)$ and the optimal stopping time $\tau^*_{f_1}$ of the follower. If $\alpha_2 > 0$, then

$$f_1(Y) = \begin{cases} B' Y^{\beta_{10}} & (0 < Y < y') \\ \alpha_2 a_{20} Y - I_2 & (Y \geq y'), \end{cases} \tag{29}$$

$$\tau^*_{f_1} = \inf\{s \geq t \mid Y(s) \geq y'\}, \tag{30}$$

where B' and y' are constants determined by the value matching and smooth pasting conditions (we omit the explicit solutions to avoid cluttering). If $\alpha_2 = 0$, we have $f_1(Y) = 0$ and $\tau^*_{f_1} = +\infty$.

Assuming that the leader has begun developing technology 1 at $Y(t) = Y$, the follower's problem is expressed as follows:

$$F_1(Y) = \sup_{\tau\in\mathcal{T}} E^Y\left[\mathrm{e}^{-h_1\tau}\max\left\{E^Y\left[1_{\{t_1<s_1\}}\int_{\tau+t_1}^{+\infty}\mathrm{e}^{-rs}D_1Y(s)\mathrm{d}s - \mathrm{e}^{-r\tau}k_1 - \int_\tau^{\tau+t_1}\mathrm{e}^{-rs}l_1\mathrm{d}s \mid \mathcal{F}_\tau\right],\right.\right.$$
$$E^Y\left[1_{\{t_2<s_1\}}\int_{\tau+t_2}^{+\infty}\mathrm{e}^{-rs}D_2Y(s)\mathrm{d}s + 1_{\{t_2\ge s_1\}}\int_{\tau+t_2}^{+\infty}\mathrm{e}^{-rs}\alpha_2D_2Y(s)\mathrm{d}s - \mathrm{e}^{-r\tau}k_2\right.$$
$$\left.\left.\left.-\int_\tau^{\tau+t_2}\mathrm{e}^{-rs}l_2\mathrm{d}s \mid \mathcal{F}_\tau\right]\right\} + 1_{\{\tau\ge s_1'\}}\mathrm{e}^{-rs_1'}f_1(Y(s_1'))\right], \tag{31}$$

where s_1' represents another random variable following the exponential distribution with hazard rate h_2. In (31), the interval between the leader's investment time t and completion time is expressed as s_1'. By the Markov property, the interval between t and the completion time of the non-conditional leader has the same distribution as the interval between τ and the completion time of the leader *who is conditioned to be yet to complete technology* 1 *at* τ. Compared with the follower's problem (21) in the previous subsection, problem (31) has the additional term $E^Y[1_{\{\tau\ge s_1'\}}\mathrm{e}^{-rs_1'}f_1(Y(s_1'))]$. This term corresponds to the remaining option value of the inactive follower. As in (4)–(7), problem (31) can be reduced to

$$F_1(Y) = \sup_{\tau\in\mathcal{T}} E^Y[\mathrm{e}^{-(r+h_1)\tau}\max_{i=1,2}(a_{i1}Y(\tau) - I_i) + 1_{\{\tau\ge s_1'\}}\mathrm{e}^{-rs_1'}f_1(Y(s_1'))], \tag{32}$$

where a_{11} and a_{21} are defined by (23) and (25), respectively. Generally, problem (32), unlike (22), is difficult to solve analytically because of the additional term. In the next section, we overcome the difficulty by focusing on two typical cases, namely, the de fact standard case, where $(\alpha_1, \alpha_2) = (1, 0)$, and the innovative case, where $(\alpha_1, \alpha_2) = (0, 1)$.

4. Analysis in Two Typical Cases

This section examines the firms' behaviour in the de fact standard case, where $(\alpha_1, \alpha_2) = (1, 0)$, and the innovative case, where $(\alpha_1, \alpha_2) = (0, 1)$. In real life, both $\alpha_1 > 0$ and $\alpha_2 > 0$ are usually hold and the two cases are extreme. However, such a real case approximates to one of the two cases or has an intermediate property, depending on the relationship between α_1 and α_2, and therefore analysis in the two cases helps us to understand the essence of the problem. In order to exclude a situation where both firms mistakenly invest simultaneously[7], we assume that the initial value y is small enough, that is,

Assumption A

$$\max_{i=1,2}(a_{i0}y - I_i) < 0,$$

as in [34] when we discuss the firms' equilibrium strategies. Assumption A is likely to hold in the context of R&D. A firm tends to delay its investment decision of R&D (rarely invest immediately), because the R&D investment decision is carefully made taking account of the distant future.

We moreover restrict our attention to the case where the firm always chooses the higher-standard technology 2 in the single firm situation, for the purpose of contrasting the competitive situation with the single firm situation. To put it more concretely, we assume

[7] We must distinguish between mistaken simultaneous investment and joint investment which is examined in Subsection 4.3. For the details of the stopping time game, see Appendix C.

Assumption B

$$\left(\frac{a_{20}}{a_{10}}\right)^{\frac{\beta_{10}}{\beta_{10}-1}} \geq \frac{I_2}{I_1},$$

so that Case 3 follows in Proposition 2.1.

In the first place, we analytically derive the follower's payoff $F_1(Y)$ and the leader's payoff $L_1(Y)$ in both de fact standard and innovative cases. Note that the results on $F_2(Y)$ and $L_2(Y)$ in Proposition 3.1 hold true by substituting $(\alpha_1, \alpha_2) = (1, 0)$ and $(\alpha_1, \alpha_2) = (0, 1)$ into (24) and (25). Then, we compare the leader's payoff $L(Y)$ with the follower's payoff $F(Y)$, where $L(Y)$ and $F(Y)$ are defined by

$$\begin{aligned} L(Y) &= \max_{i=1,2} L_i(Y), \\ F(Y) &= \begin{cases} F_1(Y) & (L_1(Y) > L_2(Y)) \\ F_2(Y) & (L_1(Y) \leq L_2(Y)). \end{cases} \end{aligned}$$

By the comparison, we see the situation where both firms try to preempt each other.

4.1. De facto standard case

Since $\alpha_2 = 0$ holds in this case, the follower's option value $f_1(Y(s_1'))$ vanishes just like in Subsection 3.2. Thus, we can solve the follower's problem (32) in the same way as problem (22). Indeed, $F_1(Y)$ and $\tau^*_{F_1}$ agree with $F_2(Y)$ and $\tau^*_{F_2}$ replaced a_{i2}, β_{i2} with a_{i1}, β_{i1}, respectively in Proposition 3.1, where β_{11} (> 1) and β_{21} (< 0) denote (12) and (13) replaced discount rate r with $r + h_1$, respectively. Recall that a_{11} and a_{21} were defined by (23) and (25). In this case, we denote three thresholds corresponding to y^*_{12}, y^*_{22} and y^*_{32} in Proposition 3.1 by y^*_{11}, y^*_{21} and y^*_{31}, respectively. Then, the payoff $L_1(Y)$ of the leader who has invested in technology 1 at $Y(t) = Y$ coincides with $L_2(Y)$ replaced a_{2i}, I_2, β_{i2} and y^*_{i2} by a_{1i}, I_1, β_{i1} and y^*_{i1}, respectively in Proposition 3.1.

Let us compare the follower's decision in the de facto standard case with the single firm's decision derived in Section 2. Using

$$\begin{aligned} \frac{a_{20}}{a_{10}} &= \frac{D_2 h_2 (r + h_1 - \mu)}{D_1 h_1 (r + h_2 - \mu)} \\ &> \frac{D_2 h_2 (r + h_1 + h_2 - \mu)}{D_1 h_1 (r + h_2 + h_2 - \mu)} = \frac{a_{22}}{a_{12}} \\ &> \frac{D_2 h_2 (r + h_1 + h_1 - \mu)}{D_1 h_1 (r + h_2 + h_1 - \mu)} = \frac{a_{21}}{a_{11}}, \end{aligned}$$

which result from $r - \mu > 0$ and $h_1 > h_2 > 0$, we have

$$\frac{a_{21}}{a_{11}} < \frac{a_{22}}{a_{12}} < \frac{a_{20}}{a_{10}}. \tag{33}$$

Equation (33) states that the relative expected profit of technology 2 to technology 1 is smaller than that of the single firm case. Using $1 < \beta_{10} < \beta_{12} < \beta_{11}$, we also obtain

$$1 < \frac{\beta_{11}}{\beta_{11} - 1} < \frac{\beta_{12}}{\beta_{12} - 1} < \frac{\beta_{10}}{\beta_{10} - 1}. \tag{34}$$

(33) and (34) suggest a possibility that $(a_{2i}/a_{1i})^{\beta_{1i}/(\beta_{1i}-1)}$ is smaller than I_2/I_1 and 1 even under Assumption B, and then the follower's optimal choice could be technology 1. In

consequence, the presence of the leader increases the follower's incentive to choose the lower-standard technology 1, which is easy to complete, compared with in the single firm situation (the direct effect).

From $a_{i1} < a_{i2},\ r + h_2 < r + h_1$, problem formulations (22) and (32) (note that $f_1 = 0$ in the de facto standard case), it follows that

$$F_1(Y) < F_2(Y) \quad (Y > 0).$$

That is, from the follower's viewpoint, the case where the leader has chosen technology 2 is preferable to the case where the leader has chosen technology 1. This is due to that the leader who has invested in technology 1 is more likely to preempt the follower because of its short research term.

Finally, we take a look at the situation where neither firm has invested. Let us see that there exists a possibility that technology 1 can be developed owing to the competition even if technology 2 generates much more profit than technology 1 at its completion. Although, as has been pointed out, $(a_{2i}/a_{1i})^{\beta_{1i}/(\beta_{1i}-1)}$ could be smaller than I_2/I_1 and 1 under Assumption B, we now consider the case where

$$\left(\frac{a_{2i}}{a_{1i}}\right)^{\frac{\beta_{1i}}{\beta_{1i}-1}} \geq \frac{I_2}{I_1} \tag{35}$$

holds, which means that a cash flow resulting from technology 2 is expected to be much greater than that of technology 1.

Since the initial value $Y(0) = y$ is small enough (Assumption A), in the single firm situation the firm invests in technology 2 (Assumption B) as soon as the market demand $Y(t)$ rises to the level y^*_{30} (Figure 2). Development of technology 1 is meaningless because the firm without fear of preemption can defer the investment sufficiently. However, the firm with the fear of preemption by its rival will try to obtain the leader's payoff by investing a slight bit earlier than its rival when the leader's payoff $L(Y)$ is larger than the follower's payoff $F(Y)$. Repeating this process causes the investment trigger to fall to the point where $L(Y)$ is equal to $F(Y)$ (y_P in Figure 3). At the point the firms are indifferent between the two roles, and then one of the firms invests at time $\inf\{t \geq 0 \mid Y(t) \geq y_P\}$ as leader, while the other invests at time $\tau^*_{F_i}$ (if there remains the option to invest) as follower. This phenomenon is *rent equalization* explained in [9, 34]. This asymmetric outcome where one of the firms becomes a leader and the other becomes a follower is called *preemption equilibrium.* For further details of the stopping time game and the equilibrium, see Appendix C. If the fear of preemption hastens the investment time sufficiently (e.g., threshold y_P becomes smaller than $\tilde{y}$ in Figure 2), then threshold y_P becomes the intersection of $L_1(Y)$ and $F_1(Y)$ rather than the intersection of $L_2(Y)$ and $F_2(Y)$ (Figure 3). It suggests a possibility that in the preemption equilibrium the leader invests in technology 1 (the indirect effect). Needless to say, the leader is more likely to choose technology 1 if (35) is not satisfied. The above discussion gives a good account of the phenomenon observed frequently in de facto standard wars.

4.2. Innovative case

This subsection examines the innovative case, where $(\alpha_1, \alpha_2) = (0, 1)$ is satisfied. We now consider the follower's optimal response assuming that the leader has invested in technology 1 at $Y(t) = Y$. Let $\tilde{F}_1(Y)$ denote the payoff (strictly speaking, the expected discounted payoff at time t) of the follower who initiate developing technology 2 at time $\tau^*_{f_1}$ defined

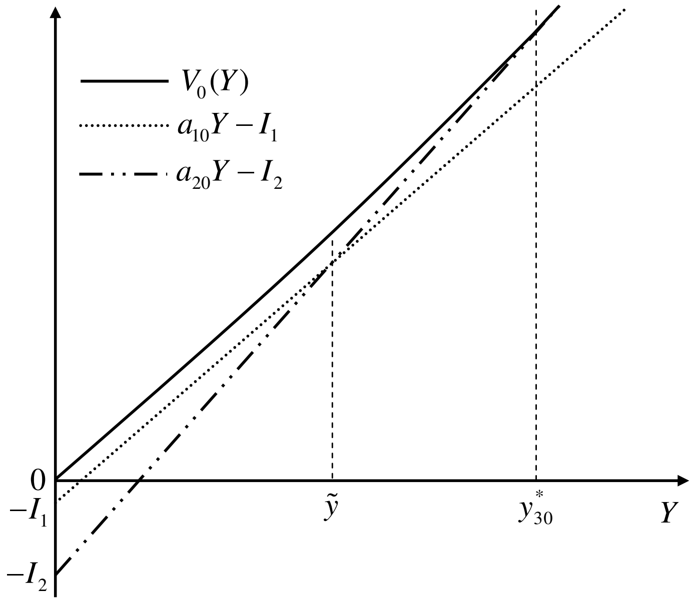

Figure 2: The value function $V_0(Y)$ in the single firm case

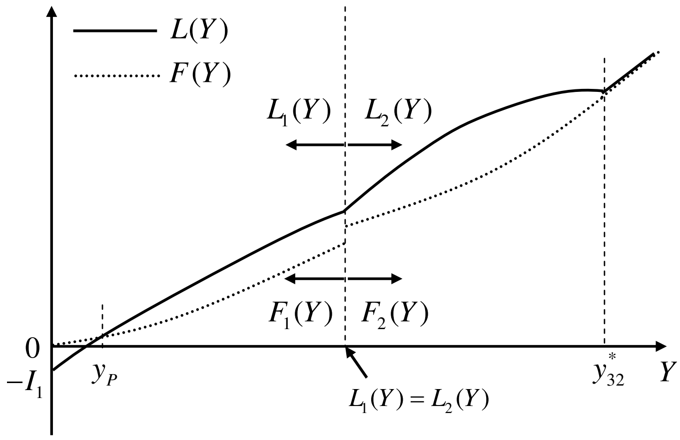

Figure 3: The leader's payoff $L(Y)$ and the follower's payoff $F(Y)$

by (30). We can show that in the innovative case the follower's best response $\tau^*_{F_1}$ coincides with $\tau^*_{f_1}$ and also show $\tilde{F}_1(Y) = f_1(Y) = F_1(Y) = V_0(Y)$ as follows.

By $\alpha_2 = 1$, the payoff of the follower who invests in technology 2 at time s ($\geq t$) is $a_{20}Y(s) - I_2$, whether the leader has completed technology 1 or not. Then we have $\tilde{F}_1(Y) = f_1(Y)$. Under Assumption B the single firm's value function $V_0(Y)$ is expressed as that of Case 3 in Proposition 2.1. Using $\alpha_2 = 1$, we have

$$V_0(Y) = f_1(Y) = \tilde{F}_1(Y). \tag{36}$$

On the other hand, by definition of the follower's problem (32), it can readily be seen that the relationship

$$F_1(Y) \leq V_0(Y) \tag{37}$$

holds between $F_1(Y)$ and $V_0(Y)$. Note that the follower's option value to invest in technology 2 is the same as that of the single firm case. In contrast, the follower's option value to invest in technology 1 is lower than that of the single firm case. The reason is that the follower's option value to invest in technology 1 vanishes completely at the leader's invention of technology 1. (36) and (37) suggest $F_1(Y) \leq \tilde{F}_1(Y)$. Thus, we have $\tilde{F}_1(Y) = F_1(Y)$, taking account of $F_1(Y) \geq \tilde{F}_1(Y)$ resulting from the optimality of $F_1(Y)$. Consequently, the follower's optimal response $\tau^*_{F_1}$ coincides with $\tau^*_{f_1}$ and $\tilde{F}_1(Y) = f_1(Y) = F_1(Y) = V_0(Y)$ holds. We should notice that the follower behaves as if there were no leader.

Using the follower's investment time $\tau^*_{F_2} = \tau^*_{f_1}$ derived above (note that $y' = y^*_{30}$ in (30) by $\alpha_2 = 1$), we have the leader's payoff $L_1(Y)$ as $L_2(Y)$ replaced a_{2i}, I_2, β_{12} and y^*_{32} by a_{1i}, I_1, β_{11} and y^*_{30}, respectively in Case 3 in Proposition 3.1.

Next, we compare the follower's decision in the innovative case with the single firm's decision. Using

$$\begin{aligned} \frac{a_{20}}{a_{10}} &= \frac{a_{22}}{a_{12}} \times \frac{r + h_1 - \mu}{r + h_1 + h_2 - \mu} \times \frac{(r-\mu)(r + 2h_2 - \mu)}{(r + h_2 - \mu)^2} \\ &< \frac{a_{22}}{a_{12}}, \end{aligned}$$

$a_{21} = a_{20}$ and $a_{11} < a_{10}$, we have

$$1 < \frac{a_{20}}{a_{10}} < \frac{a_{2i}}{a_{1i}} \quad (i = 1, 2). \tag{38}$$

Equation (38) means that the relative expected profit of technology 2 to technology 1 is greater than that of the single firm case, contrary to (33) in the de facto standard case. Since (34) remains true, the relationship between $(a_{22}/a_{12})^{\beta_{12}/(\beta_{12}-1)}$ and I_2/I_1 depends on the parameters even under Assumption B. This suggests a slight possibility that the follower chooses technology 1 in the case where the leader has chosen technology 2, while as we showed in the beginning of this subsection the follower's best response to the leader who has invested in technology 1 is choosing technology 2 regardless of Y. However, in most cases the effect of (38) dominates the effect of (34), that is,

$$\frac{I_2}{I_1} < \left(\frac{a_{20}}{a_{10}}\right)^{\frac{\beta_{10}}{\beta_{10}-1}} < \left(\frac{a_{22}}{a_{12}}\right)^{\frac{\beta_{12}}{\beta_{12}-1}}$$

hold. To sum up, the presence of the leader, unlike in the de facto standard case, tends to decreases the incentive of the lower-standard technology 1, which is easy to complete (the direct effect).

By definition of the follower's problem (22) we can immediately show

$$F_2(Y) < V_0(Y) = F_1(Y) \quad (Y > 0).$$

In other words, contrary to the de facto standard case, the follower prefers the leader developing technology 1 to the leader developing technology 2. This is because the follower can deprive the leader who has chosen technology 1 of the profit by completing technology 2.

Finally, let us examine the situation where neither firm has taken action. We obtain the following proposition with respect to the preemption equilibrium.

Proposition 4.1 The inequality

$$L_1(Y) < F_1(Y) \quad (Y > 0) \tag{39}$$

holds, and therefore in the preemption equilibrium the leader always chooses technology 2. Furthermore, if

$$\left(\frac{a_{22}}{a_{12}}\right)^{\frac{\beta_{12}}{\beta_{12}-1}} > \frac{I_2}{I_1} \tag{40}$$

((40) is satisfied for reasonable parameter values as mentioned earlier), then in the preemption equilibrium the follower, also, always chooses technology 2.

Proof See Appendix D.

Table 1 summarizes the comparison results between the de facto standard and innovative cases.

4.3. Case of joint investment

The joint investment equilibria, which are, unlike the preemption equilibria, symmetric outcomes, may also occur even if the two firms are noncooperative. The results on the joint investment equilibria in our setup is similar to that in [34] and therefore they are briefly described below.

Assuming that the two firms are constrained to invest in the same technology at the same timing, the firm's problem can be reduced to

$$\sup_{\tau \in \mathcal{T}} E[\mathrm{e}^{-r\tau} \max_{i=1,2}(a_{ii} Y(\tau) - I_i)], \tag{41}$$

in the same procedure as (4)–(7). Recall that a_{11} and a_{22} were defined by (23) and (26), respectively. It is worth noting that the expression (41) does not depend on whether the de facto standard case or the innovative case. Using

$$\begin{aligned} \frac{a_{20}}{a_{10}} &= \frac{D_2 h_2 (r + h_1 - \mu)}{D_1 h_1 (r + h_2 - \mu)} \\ &< \frac{D_2 h_2 (r + 2h_1 - \mu)}{D_1 h_1 (r + 2h_2 - \mu)} = \frac{a_{22}}{a_{11}} \end{aligned}$$

and Assumption B, we have

$$\frac{I_2}{I_1} < \left(\frac{a_{20}}{a_{10}}\right)^{\frac{\beta_{10}}{\beta_{10}-1}} < \left(\frac{a_{22}}{a_{11}}\right)^{\frac{\beta_{10}}{\beta_{10}-1}}.$$

Thus, the value function (denoted by $J(y)$) and the optimal stopping time (denoted by τ_J^*) of problem (41) coincide with $V_0(Y)$ and τ_0^* replaced a_{20} with a_{22} in Case 3 in Proposition 2.1, that is, the two firms set up the development of technology 2 at the same time

$$\tau_J^* = \inf\{t \geq 0 \mid Y(t) \geq y_{33}^*\}, \tag{42}$$

where y_{33} denotes the joint investment trigger corresponding to y_{30} in Proposition 2.1. As in the single firm case, in joint investment both firms always choose technology 2.

If there exists any Y satisfying $L(Y) > J(Y)$, then the only preemption equilibria (not necessarily unique), which are asymmetric outcomes, occur. Otherwise, there arises the joint investment equilibria (not necessarily unique) in addition to the preemption equilibria. In this case, the joint investment equilibrium attained by the optimal joint investment rule (42) Pareto-dominates the other equilibria. For further details of the joint investment equilibria, see [18, 34].

5. Numerical Examples

This section presents some examples in which the single firm's payoff $V_0(Y)$, the leader's payoff $F(Y)$, the joint investment payoff $J(Y)$ and the equilibrium strategies are numerically computed. We set the parameter values as Table 2 in order that Assumption B is satisfied and the single firm case corresponds a standard example in [6] (note $a_{20} = I_2 = 1$). Table 3 shows β_{ij}, and Tables 4 and 5 indicate a_{ij}, I_i and y_{ij}^*. To begin with, we compute the single

Table 1: Comparison between the de facto standard and innovative cases

	De facto standard	Innovative
Relative expected profit	$a_{2i}/a_{1i} < a_{20}/a_{10}$	$a_{2i}/a_{1i} > a_{20}/a_{10}$
Follower's value function	$F_1(Y) < F_2(Y)$	$F_1(Y) > F_2(Y)$
Preemption equilibrium	Both firms: likely to choose Tech. 1	Leader: Tech. 2, Follower: Tech. 2 (in most cases)

Table 2: Parameter setting

r	μ	σ	D_1	D_2	h_1	h_2	k_1	k_2	l_1	l_2
0.04	0	0.2	0.025	0.05	0.32	0.16	0	0	0.18	0.2

Table 3: Values of β_{ij}

β_{10}	β_{20}	β_{11}	β_{21}	β_{12}	β_{22}
2	1	4.77	−3.77	3.7	−2.7

Table 4: Values common to both cases

a_{10}	a_{20}	a_{11}	a_{22}	I_1	I_2	y_{30}^*	y_{33}^*
0.56	1	0.29	0.56	0.5	1	2	3.6

Table 5: Values dependent on the cases

	a_{12}	a_{21}	y_{11}^*	y_{21}^*	y_{31}^*	y_{12}^*	y_{22}^*	y_{32}^*
De facto standard	0.38	0.38	2.15	5.46	5.59	1.78	2.81	3.04
Innovative	0.08	1	N/A	N/A	2	N/A	N/A	2.47

firm's problem. Figure 4 illustrates its value function $V_0(Y)$ corresponding to Case 3 in Proposition 2.1, where the investment time τ_0^* is

$$\tau_0^* = \inf\{t \geq 0 \mid Y(t) \geq y_{30}^* = 2\}. \quad (43)$$

Second, let us turn to the de facto standard case. Because the inequalities

$$1 < \left(\frac{a_{2i}}{a_{1i}}\right)^{\frac{\beta_{1i}}{\beta_{1i}-1}} < \frac{I_2}{I_1} \quad (i = 1, 2)$$

hold, the follower's optimal response $\tau_{F_i}^*$ has three triggers (see Table 5), that is, which technology the follower chooses depends on the initial value Y. Figure 5 illustrates the leader's payoff $L_i(Y)$ and the follower's payoff $F_i(Y)$. In Figure 5, $F_i(Y)$ is smooth while $L_i(Y)$ changes drastically at the follower's triggers y_{1i}^*, y_{2i}^* and y_{3i}^*. This means that the leader is greatly affected by the technology chosen by the follower. Particularly, a sharp rise of $L_i(Y)$ in the interval $[y_{2i}^*,\ y_{3i}^*]$ in Figure 5 states that the leader prefers the follower choosing technology 2 to the follower choosing technology 1.

The payoffs $L(Y)$, $F(Y)$, and $J(Y)$ appear in Figure 6. Let us consider the firms' equilibrium strategies under Assumption A, i.e., the condition that the initial market demand y is small enough. Note that as mentioned in Section 4.3 the optimal joint investment strategy has the unique trigger y_{33}^* and both firms always choose technology 2. We see from Figure 6 that the preemption equilibrium is a unique outcome in the completion between the two firms, since there exists Y satisfying $J(Y) < L(Y)$. By assumption A, in the preemption equilibrium one of the firms becomes a leader investing in technology 1 at

$$\inf\{t \geq 0 \mid Y(t) \geq y_P = 0.93\} \quad (44)$$

(y_P is the intersection of $L(Y)$ and $F(Y)$ in Figure 6) and the other invests in technology 1 as follower at

$$\tau_{F_1}^* = \inf\{t \geq 0 \mid Y(t) \geq y_{11}^* = 2.15\}$$

if the leader has not succeeded in the development until this point. We observe that the leader's investment time (44) becomes earlier than the single firm's investment time (43). Furthermore, we see that the preemption trigger y_P in Figure 6 is the intersection of $L_1(Y)$ and $F_1(Y)$ instead of that of $L_2(Y)$ and $F_2(Y)$ and see that both firms switch the target from technology 2 chosen in the single firm situation to technology 1. Thus, consumers could suffer disadvantage that the only lower-standard technology emerges due to the competition.

It is obvious from Figure 6 that in the case where the roles of the firms are exogenously given, i.e., in the leader-follower game

$$\sup_{\tau \in \mathcal{T}} E[\mathrm{e}^{-r\tau} L(Y(\tau))],$$

the leader invests in technology 1. Therefore, in this instance, rather than the fear of preemption by the competitor, the presence of the competitor causes development of the lower-standard technology 1, which is never developed in the single firm situation. That is, the direct effect is strong enough to change the technology standard chosen by the firms.

Let us now replace $\sigma = 0.2$ by $\sigma = 0.8$ with other parameters fixed in Table 2 and consider the firms' strategic behavior under Assumption A. Notice that the higher product market uncertainty σ becomes the greater the advantage of technology 2 over technology

1 becomes. Figure 7 illustrates $L(Y), F(Y)$ and $J(Y)$. Since $J(Y) > L(Y)$ in Figure 7, the joint investment equilibria arise together with the preemption equilibria. There are two preemption equilibria corresponding the two leader's triggers y_{P_1} and y_{P_2}. It is reasonable to suppose that which type of equilibria occurs depends on the firms' inclination to the preemption behavior. In this instance, it can be readily seen from Figure 7 that in the corresponding leader-follower game the leader invests in technology 2 at the joint investment trigger y_{33}^*. This suggests that relative to the case in Figure 6, the fear of preemption by the competitor could drive the leader to develop the lower-standard technology 1, which never emerges in the noncompetitive situation, at the trigger y_{P_1}. That is, the direct effect is not strong enough to change the technology standard chosen by the firms, and the indirect effect together with the direct effect changes the firms' investment strategies.

Finally, we examine the innovative case. It can be deduced from the inequality

$$\left(\frac{a_{22}}{a_{12}}\right)^{\frac{\beta_{12}}{\beta_{12}-1}} > \frac{I_2}{I_1}$$

that the follower always chooses technology 1 (Table 5). The leader's payoff $L_i(Y)$ and the follower's payoff $F_i(Y)$ appear in Figure 8. The payoff $F_1(Y)$ dominates the others since it is equal to $V_0(Y)$ as shown in Section 4.2. Figure 9 illustrates $L(Y), F(Y)$ and $J(Y)$. We examine the firms' strategic behaviour under Assumption A. There occurs no joint investment outcome as there exists Y satisfying $J(Y) < L(Y)$. In the preemption equilibrium, as shown in Proposition 4.1, both firms invest in the same technology 2 but the different timings. Indeed, in equilibrium one of the firms invests in technology 2 at

$$\inf\{t \geq 0 \mid Y(t) \geq y_P = 1.06\} \tag{45}$$

(y_P denotes the intersection of $L(Y)$ and $F(Y)$ in Figure 9) as leader, while the other invests in the same technology at

$$\tau_{F_2}^* = \inf\{t \geq 0 \mid Y(t) \geq y_{32}^* = 2.47\}$$

as follower if the leader has yet to complete the technology at this point. We see that the leader's investment time (44) is earlier than the single firm's investment time (43) but is later than (44) in the de facto standard case. The preemption trigger y_P is the intersection of $L_2(Y)$ and $F_2(Y)$, and therefore the technology developed by firms remains unchanged by the competition. It is worth noting that y_P agrees with the preemption trigger in the case where the firms has no option to choose technology 1, that is, the preemption trigger derived in [34].

We make an additional comment on Assumption A. As assumed in the beginning of Section 4, this paper have investigated the equilibrium strategy under Assumption A. However, Figures 6, 7, and 9 also show $L(Y), F(Y)$, and $J(Y)$ for Y larger than $\max_{i=1,2}\{I_i/a_{i0}\}$. Thus, from the figures, we could examine the firm's equilibrium strategy in cases where the initial value is too large to satisfy Assumption A. It must be noted that the results in those cases may depend on the parameter values; for this reason, we have limited the discussion to the case where Assumption A holds.

6. Conclusion

This paper extended the R&D model in [34] to the case where a firm has the freedom to choose the timing and the standard of the research project, where the higher-standard

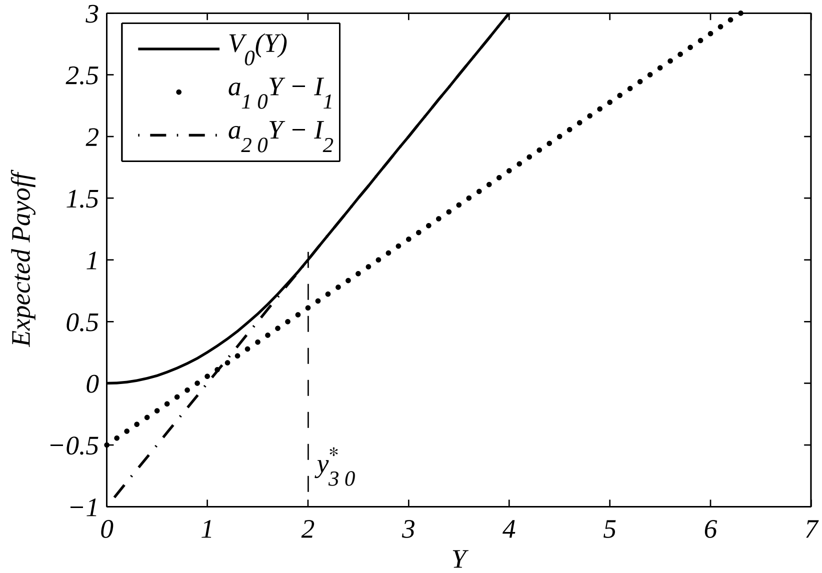

Figure 4: The single firm's value function $V_0(Y)$

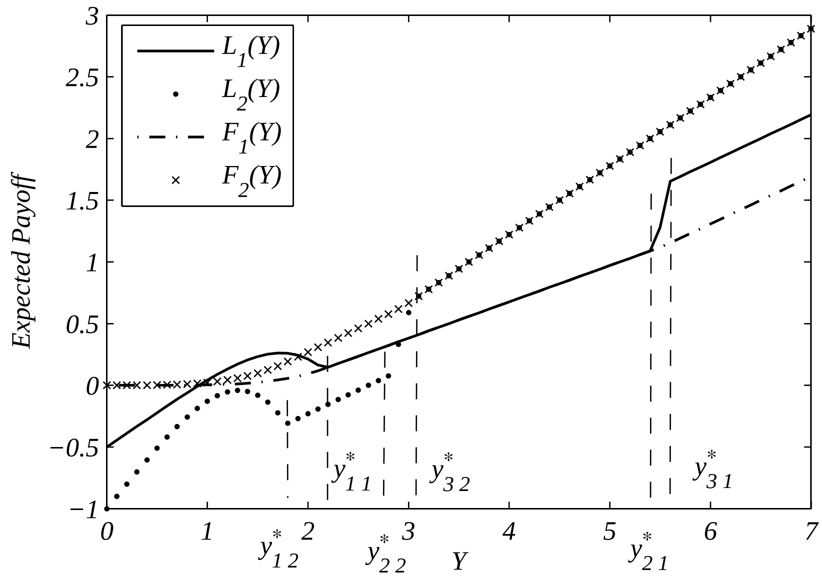

Figure 5: $L_i(Y)$ and $F_i(Y)$ in the de facto standard case

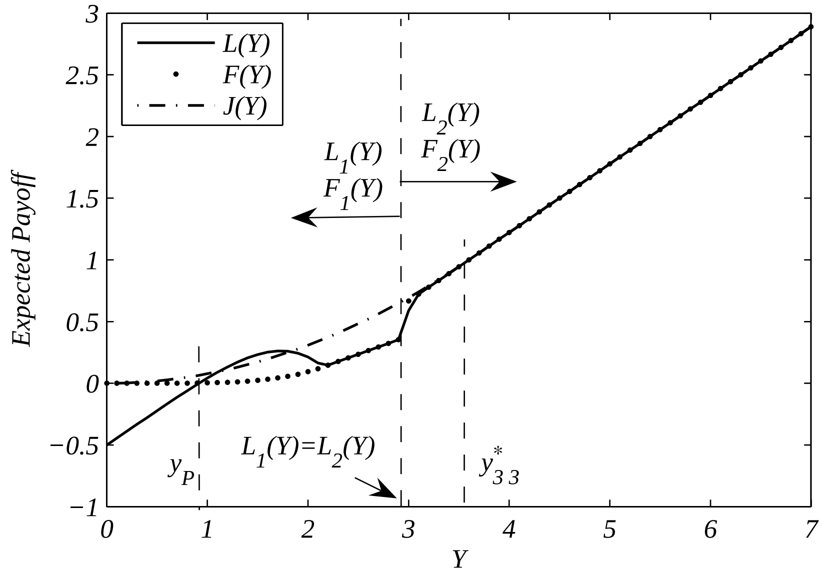

Figure 6: $L(Y), F(Y)$ and $J(Y)$ in the de facto standard case

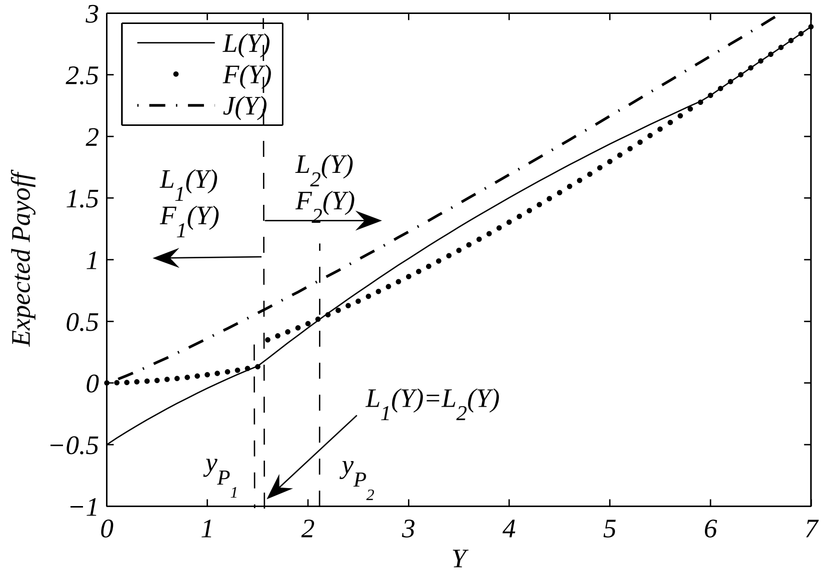

Figure 7: $L(Y), F(Y)$ and $J(Y)$ for $\sigma = 0.8$ in the de facto standard case

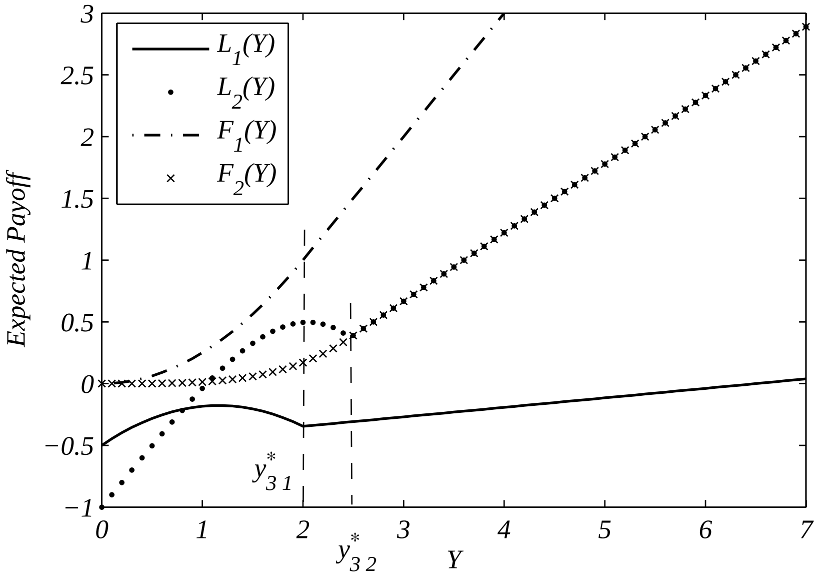

Figure 8: $L_i(Y)$ and $F_i(Y)$ in the innovative case

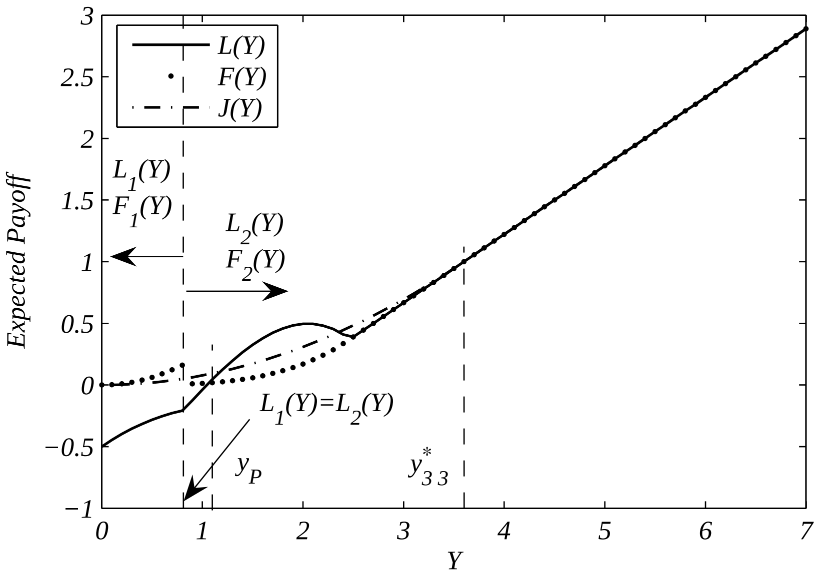

Figure 9: $L(Y), F(Y)$ and $J(Y)$ in the innovative case

technology is difficult to complete and generates a greater cash flow. First, we derived the firm's optimal decision in the single firm situation. We thereafter extended the model to the situation of two firms and examined in full detail two typical cases, i.e., the de facto standard case and the innovative case. The results obtained in this paper can be summarized as follows.

The competition between the two firms affects not only the firms' investment timing decision, but also their choice of the technology standard directly and indirectly. The choice of the project standard is indirectly affected by the hastened investment timing in the stopping time game between the two firms, as well as by the direct change of the project value by the presence of the competitor. In the de facto standard case, the competition increases the incentive to choose the lower-standard technology, which is easy to complete; in the innovative case, on the contrary, the competition increases the incentive to choose the higher-standard technology, which is difficult to complete. The main contribution of this paper is showing that in the de facto standard case a lower-standard technology could emerge than is developed in the single firm situation. This implies the possibility that too bitter competition among firms adversely affects not only the firms but also consumers.

Finally, we mention potential extensions of this research. One of the remaining problems is to find a system in which noncooperative firms conduct more efficient R&D investment from the viewpoint of social welfare including consumers. A tax and a subsidy investigated in [15, 21] could provide viable solutions to the problem. Although this paper considers a simple model with two types of uncertainty, namely technological uncertainty and market uncertainty, other types of uncertainty (see [17]) and other options, such as options to abandon and expand, could be involved with practical R&D investment (see [31]). It also remains as an interesting issue for future research to incorporate incomplete information (for example, uncertainty as to rivals' behavior as investigated in [22, 28]) in the model.

A. Proof of Proposition 2.1

In Case 2, (16) and (17) immediately follow from the discussion in [3]. In Case 1, using relationships $a_{10} \geq a_{20}, I_1 < I_2$ and $Y(t) > 0$, we have

$$\sup_{\tau \in \mathcal{T}} E[\mathrm{e}^{-r\tau} \max_{i=1,2}(a_{i0}Y(\tau) - I_i)] = \sup_{\tau \in \mathcal{T}} E[\mathrm{e}^{-r\tau}(a_{10}Y(\tau) - I_1)],$$

which implies (14) and (15). In Case 3, by taking into consideration that the right-hand side of (18) dominates $a_{10}y - I_1$, we can show (18) and (19) by a standard technique to solve an optimal stopping problem (see [30]). □

B. Proof of Proposition 3.1

Problem (22) coincides with problem (8) replaced r and a_{i0} by $r + h_2$ and a_{i2}, respectively. Thus, we easily obtain the follower's payoff $F_2(Y)$ and investment time $\tau^*_{F_2}$ in the same way as Proposition 2.1. We next consider the leader's payoff $L_2(Y)$. In Case 1 and 3, we readily have the same expression as that of [34] since the follower's trigger is single. In Case 2, we obtain the similar expression, though the calculation becomes more complicated due to the existence of three triggers. □

C. The stopping time game and its equilibrium

In this paper, we adopt the concept of the stopping time game introduced in [7] because of its intuitive simplicity. We make a brief explanation of the concept by [7] below. See [7] for further details.

The stopping time game proceeds as follows. In the absence of an action by either player, the game environment evolves according to the stochastic process (1). If a firm has not invested until time t, its action set is $A_t = \{0, 1, 2\}$, where 0 stands for *no entry* and $1, 2$ for *invest* in technology $1, 2$ respectively. If a firm has already invested before time t, then the action set A_t is the null. Investment by one of the firms (called leader) terminates the game and determines the (expected) payoff of both firms because the other (called follower) necessarily takes the optimal response. We assume that the simultaneous action yields the expected payoff $(L(Y) + F(Y))/2$ to both firms (we take $\alpha = 1/2$ in p. 746 in [7]). That is, one of the firms prove to invest infinitesimally earlier than the other even if both attempt to invest at the same timing. The remaining one must take the optimal response as a follower. The probability that a firm is chosen as a leader is fair, i.e., 1/2.

A strategy for a firm is generally defined as a mapping from the history of the game H_t to the action set A_t. Here, at time t the history H_t has two components: the sample path of the stochastic process (1) and the actions of two players up to time t. Since the stochastic process (1) is Markovian, we restrict attention to Markovian strategies and a Markovian perfect equilibrium. Then, in equilibrium, at time $\inf\{t \geq 0 \mid Y(t) \geq y_P\}$ both firms attempt to invest in technology i satisfying $L_i(y_P) \geq L_j(y_P)$ for $j \neq i$. Under the assumption only one of the firms is actually allowed to invest at time $\inf\{t \geq 0 \mid Y(t) \geq y_P\}$ and the other invests at time $\tau^*_{F_i}$ as a follower.

Several studies such as [11, 34] use the above concept by [7], but there is another stream [18, 32, 33] that has tried to elucidate the possibility of mistaken simultaneous investment. They introduces a more complex strategy space of the firms, instead of the technical assumption that the simultaneous action yields the payoff $(L(Y) + F(Y))/2$. Even in their approach, the outcome still holds true under Assumption A due to rent equalization. They suggest that, without Assumption A both firms may make simultaneous investment mistakenly, and of course the analysis without Assumption A is an interesting issue in future research direction.

D. Proof of Proposition 4.1

The leader's payoff $L_1(Y)$ is equal to

$$L_1(Y) = \begin{cases} a_{10}Y - I_1 - \tilde{B}_1 Y^{\beta_{12}} & (0 < Y < y^*_{30}) \\ a_{12}Y - I_1 & (Y \geq y^*_{30}), \end{cases}$$

where the constant $\tilde{B}_1 > 0$ is determined by the value matching condition at the trigger y^*_{30}. Using $a_{10} > a_{12}$ and $\tilde{B}_1 > 0$, we have

$$L_1(Y) < a_{10}Y - I_1 \leq V_0(Y) = F_1(Y) \quad (Y > 0),$$

which implies that there is no incentive to invest in technology 1 earlier than the competitor. Therefore, there arises no preemption equilibrium where the leader invests in technology 1. Next, we assume (40). In this case, the follower's decision corresponds to that of Case 3 in Proposition 2.1. In consequence, in the preemption equilibrium, the follower, also, always chooses technology 2. □

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Michi Nishihara
Center for the Study of Finance and Insurance
Osaka University
1-3 Machikaneyama-cho, Toyonaka
Osaka 560-8531, Japan
E-mail: nishihara@sigmath.es.osaka-u.ac.jp