

AN EOQ MODEL FOR DETERIORATION ITEMS UNDER TRADE CREDIT POLICY IN A SUPPLY CHAIN SYSTEM

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Abstract This paper attempts to determine economic order quantity for deteriorating items under the conditions of permissible delay in payments, in which the supplier offers the retailer a permissible delay period and the retailer in turn provides a maximal trade credit period to their customers in a supply chain system. A theorem is developed to determine the optimal ordering policies for the retailer under above conditions. These results help the retailer's decision makers to determine accurately the optimal cost. A numerical example demonstrates the applicability of the proposed method. Moreover, sensitivity analysis of the optimal solution with respect to major parameters is carried out. Finally, the results in this paper generalize some already published results.

Keywords: Inventory, trade credit, deterioration, permissible delay in payments

1. Introduction

A supply chain (Chopra and Meindl [3]) consists of all stages involved, directly or indirectly, in fulfilling a customer request. The supply chain not only includes the manufacturers and suppliers, but also transporters, warehouses, retailers and customers themselves. So, the supply chain management (Levi et al. [23]) is a set of approaches utilized to efficiently integrate them so that merchandiser is produced and distributed at the right quantities, to the right location and at the right time, in order to minimize systemwide costs.

Goyal [11] is the first to establish an economic order quantity model under the condition of permissible delay in payments. He assumes that the supplier would offer the retailer a fixed delay period and the retailer could sell the goods and accumulate revenue and earn interest within the trade credit period. Goyal [11] implicitly assumes that the customer would pay for the items as soon as the items are received from the retailer. That is, Goyal [11] assumed that the supplier would offer the retailer a delay period but the retailer would not offer the trade credit period to customers. In most business transactions, this assumption is debatable. Huang [15] defines this situation as one level of trade credit. Huang [13] extends Goyal [11] to provide a fixed trade credit period M between the supplier and the retailer and a maximal trade credit period N ($M > N$) between the retailer and the customer. Basically, the inventory model of Goyal [11] is a supply chain of two stages (the supplier and the retailer). Huang [13] generalizes Goyal [11] to the supply chain of three stages [the supplier, the retailer and customers]. Huang [15] names the above situation two levels of trade credit.

On the other hand, Ghare and Schrader [10] are pioneers to develop an EOQ model by negative exponential distribution which investigation assumes that the instantaneous deterioration rate is constant. Combining Ghare and Schrader [10] and Goyal [11], numerous

studies dealing with the inventory models for deteriorating items under the trade credit can be found in Jaggi and Aggarwal [16], Aggarwal and Jaggi [1], Arcelus et al. [2], Chu et al. [4], Chung and Liao [8, 9], Chung and Huang [7], Liao [18, 19], Tsao and Shreen [25] and their references.

All the above papers mentioned do not consider deteriorating items and two levels of trade credit together. So, the main purpose of this paper is to extend Huang [13] into the inventory model under two levels of trade credit to more match a real life situation.

2. Model Formulation

The following notations are used throughout the whole paper:

Notations:

- A : ordering cost per order
- p : unit selling price per item
- c : unit purchasing price per item.
- D : demand rate per year
- I_e : interest earned per \$ per year
- I_k : interest charged per \$ in stock per year by the supplier
- M : the retailer's trade credit period offered by supplier in years
- N : the maximal trade credit period for customers offered by retailer in years
- h : unit stock holding cost per unit per year excluding interest charges
- T : the cycle time in years
- $TVC(T)$: the annual total relevant cost
- T^* : the optimal cycle time of $TVC(T)$

In addition, the following assumptions are used throughout:

Assumptions:

1. Demand rate is known and constant.
2. The shortages are not allowed.
3. Time period is infinite and replenishment lead time is zero.
4. The distribution of time to deterioration of the items follows exponential distribution with parameter θ (constant rate of deterioration).
5. $I_e \leq I_k$, $M \geq N$ and $p \geq c$.
6. A supplier allows a fixed period, M , to settle the account. During this fixed period no interest is charged by the supplier but beyond this period, interest I_k is charged by the supplier under the terms and conditions agreed upon. The account is settled completely either at the end of the credit period or at the end of the cycle.
7. A retailer allows a maximal trade credit period N for customers to settle the account. If a customer buys one item from the retailer at time t belonging to $(0, N]$, then the customer will have a trade credit period $N - t$ and make the payment at time N . Furthermore, the retailer can accumulate revenue and earn interest after the customer pays for the amount of purchasing cost until the end of the trade credit period offered by the supplier. That is, the retailer can accumulate revenue and earn interest during the period N to M with rate I_e under the condition of trade credit.

Recently, Assumption (7) is rather prevalent. It has been adopted in a lot of papers such as Huang [13–15], Teng and Chang [24], Liao [20], Ho et al. [12], Ouyang et al. [21], Jaggi et al. [17] and their references.

Let $Q(t)$ denote the on-hand inventory level at time t , which is depleted by the effects of demand and deterioration, then the differential equation which describes the instantaneous

states of $Q(t)$ over $(0, T)$ is given as:

$$\frac{dQ(t)}{dt} + \theta Q(t) = -D, \quad 0 \leq t \leq T \quad (2.1)$$

then, with boundary condition $Q(T) = 0$. The solution of equation (2.1) is given by

$$Q(t) = \frac{D}{\theta}(e^{\theta(T-t)} - 1), \quad 0 \leq t \leq T \quad (2.2)$$

Noting that $Q(0) = Q$, then quantity ordered each replenishment cycle is

$$Q = \frac{D}{\theta}(e^{\theta T} - 1) \quad (2.3)$$

Furthermore, the total relevant cost function per cycle is the sum of the ordering cost, inventory holding cost, cost of deteriorated units and interest payable on stock held beyond the permissible period, less the interest earned during the period of (N, M) . From now on, the individual cost is evaluated before they are grouped together.

1. Annual ordering cost = $\frac{A}{T}$
2. Annual inventory holding cost(excluding interest charges)

$$= \frac{h}{T} \int_0^T Q(t)dt = \frac{hD}{\theta^2 T}(e^{\theta T} - \theta T - 1)$$

3. Annual cost of deteriorated units = $\frac{c(Q-DT)}{T} = \frac{cD}{\theta T}(e^{\theta T} - \theta T - 1)$
4. Regarding interests charged and earned, we have the following three cases to discuss:

Case(I): $T \geq M$, shown in Figure 1.

In this case, the sales revenue is utilized to earn interest I_e during the period of (N, M) . When the account is settled, the item still in inventory has to be financed with annual rate I_k . Therefore, the annual interest payable is

$$\frac{cI_k \int_M^T Q(t)dt}{T} = \frac{cI_k D}{\theta^2 T}(e^{\theta(T-M)} - \theta(T-M) - 1)$$

From Figure 1, it implied that the retailer sells products and deposits the revenue into an account during period $(0, N]$, but getting money at time N . Therefore, sales revenue, pDN , is continuously accumulated from period (N, M) and the interest earned of this part is pI_e , multiplied by the area of $NMYZ$. In addition, the sales revenue from period (N, M) is continuously accumulated, so the interest earned of this part is pI_e multiplied by the area of XYZ . Combining the above argument, the annual interest earned is

$$\frac{pI_e \int_N^M Dtdt}{T} = \frac{pI_e D(M^2 - N^2)}{2T} = \frac{pI_e D(M+N)(M-N)}{2T}$$

Case(II): $N \leq T < M$, shown in Figure 2.

In this case, all the sales revenue is utilized to earn interest with annual rate I_e during the period of (N, M) and pays no interest for the items kept in stock. Therefore, the annual interest payable is 0, and the annual interest earned is

$$\frac{pI_e [\int_N^T Dtdt + DT(M-T)]}{T} = \frac{pI_e D}{2T} [2MT - N^2 - T^2]$$

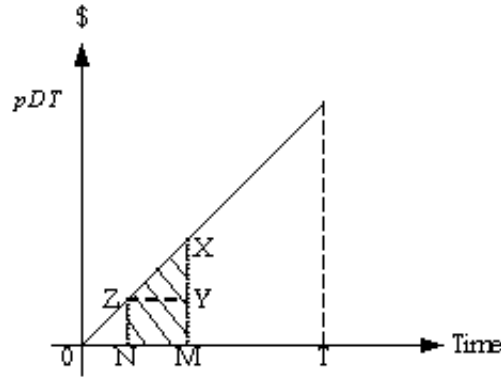


Figure 1: The total accumulation of interest earned when $M \leq T$

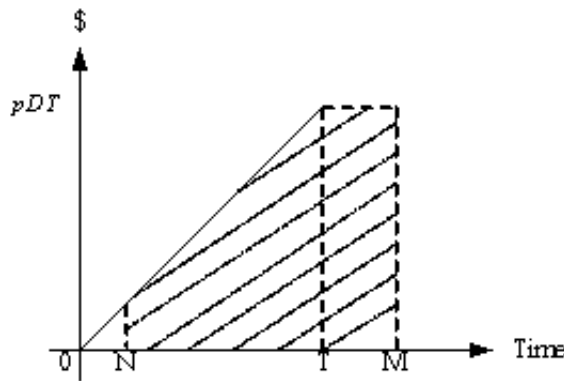


Figure 2: The total accumulation of interest earned when $N \leq T \leq M$

Case(III): $T \leq N$, shown in Figure 3.

In this case, all the sales revenue is utilized to earn interest I_e during the period of (N, M) and pays no interest for the items kept in stock as well. Therefore, the annual interest payable is 0, and the annual interest earned is

$$\frac{pI_e \int_N^M DT dt}{T} = pI_e D(M - N)$$

Combining the above arguments, we obtain that the annual total relevant cost per unit time is given by

$$TVC(T) = \begin{cases} TVC_1(T) & \text{if } M < T \\ TVC_2(T) & \text{if } N < T \leq M \\ TVC_3(T) & \text{if } 0 < T \leq N \end{cases} \quad (2.4)$$

where

$$TVC_1(T) = \frac{A}{T} + \frac{D(c\theta + h)}{\theta^2 T} (e^{\theta T} - \theta T - 1) + \frac{cI_k D}{\theta^2 T} (e^{\theta(T-M)} - \theta(T-M) - 1) - \frac{pI_e D(M^2 - N^2)}{2T} \quad (2.5)$$

$$TVC_2(T) = \frac{A}{T} + \frac{D(c\theta + h)}{\theta^2 T} (e^{\theta T} - \theta T - 1) - \frac{pI_e D}{2T} (2MT - N^2 - T^2) \quad (2.6)$$

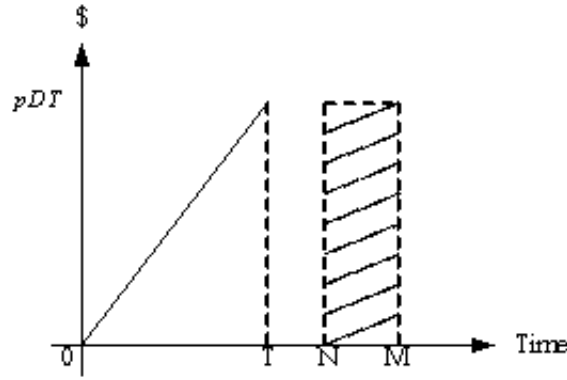


Figure 3: The total accumulation of interest earned when $T \leq N$

and

$$TVC_3(T) = \frac{A}{T} + \frac{D(c\theta + h)}{\theta^2 T} (e^{\theta T} - \theta T - 1) - pI_e D(M - N) \quad (2.7)$$

For convenience to discuss, we extend the domain of $TVC_i(T)$ ($i = 1, 2, 3$) and treat the domain of $TVC_i(T)$ ($i = 1, 2, 3$) as $(0, \infty)$. Then, equation (2.5) yields

$$\begin{aligned} TVC'_1(T) &= -\frac{A}{T^2} + \frac{D(c\theta + h)}{\theta^2 T^2} (\theta T e^{\theta T} - e^{\theta T} + 1) \\ &\quad + \frac{cI_k D}{\theta^2 T^2} (\theta T e^{\theta(T-M)} - e^{\theta(T-M)} + 1 - \theta M) + \frac{pI_e D(M^2 - N^2)}{2T^2} \end{aligned} \quad (2.8)$$

After rearrangement,

$$\begin{aligned} TVC'_1(T) &= \frac{1}{\theta^2 T^2} \left\{ -A\theta^2 + D(c\theta + h)(\theta T e^{\theta T} - e^{\theta T} + 1) \right. \\ &\quad \left. + cI_k D(\theta T e^{\theta(T-M)} - e^{\theta(T-M)} + 1 - \theta M) + \frac{pI_e D(M^2 - N^2)\theta^2}{2} \right\} \\ &= \frac{1}{\theta^2 T^2} f(T) \end{aligned}$$

where

$$\begin{aligned} f(T) &= -A\theta^2 + D(c\theta + h)(\theta T e^{\theta T} - e^{\theta T} + 1) \\ &\quad + cI_k D(\theta T e^{\theta(T-M)} - e^{\theta(T-M)} + 1 - \theta M) + \frac{pI_e D(M^2 - N^2)\theta^2}{2} \end{aligned}$$

Since $f'(T) = D\theta^2 T[(c\theta + h)e^{\theta T} + cI_k e^{\theta(T-M)}] > 0$, so $f(T)$ is increasing on $T > 0$. Let T_1^* denote the root of $TVC'_1(T) = 0$. Since $\lim_{T \rightarrow \infty} f(T) = \infty > 0$, the following results hold.

(i) If $f(M) < 0$, then $T_1^* > M$.

(ii) If $f(M) \geq 0$, then $T_1^* = M$.

On the other hand, equations (2.6) and (2.7) yield

$$TVC'_2(T) = -\frac{A}{T^2} + \frac{D(c\theta + h)}{\theta^2 T^2} (\theta T e^{\theta T} - e^{\theta T} + 1) - \frac{pI_e D}{2T^2} (N^2 - T^2), \quad (2.9)$$

$$\begin{aligned}
 TVC_2''(T) &= \frac{2A}{T^3} + \frac{2D(c\theta + h)}{\theta^2 T^3} \left[e^{\theta T} \left(1 - \theta T + \frac{1}{2} \theta^2 T^2 \right) - 1 \right] + \frac{pI_e D N^2}{T^3} \\
 &> \frac{2A}{T^3} + \frac{2D(c\theta + h)}{\theta^2 T^3} \left[e^{\theta T} \cdot e^{-\theta T} - 1 \right] + \frac{pI_e D N^2}{T^3} \\
 &= \frac{2A}{T^3} + \frac{pI_e D N^2}{T^3} > 0,
 \end{aligned} \tag{2.10}$$

$$TVC_3'(T) = -\frac{A}{T^2} + \frac{D(c\theta + h)}{\theta^2 T^2} (\theta T e^{\theta T} - e^{\theta T} + 1), \tag{2.11}$$

and

$$\begin{aligned}
 TVC_3''(T) &= \frac{2A}{T^3} + \frac{2D(c\theta + h)}{\theta^2 T^3} \left[e^{\theta T} \left(1 - \theta T + \frac{\theta^2 T^2}{2} \right) - 1 \right] \\
 &> \frac{2A}{T^3} + \frac{2D(c\theta + h)}{\theta^2 T^3} \left[e^{\theta T} \cdot e^{-\theta T} - 1 \right] \\
 &= \frac{2A}{T^3} > 0
 \end{aligned} \tag{2.12}$$

Therefore, $TVC_2(T)$ and $TVC_3(T)$ is convex on $(0, \infty)$, respectively. Since $TVC_1(M) = TVC_2(M)$ and $TVC_2(N) = TVC_3(N)$, $TVC(T)$ is continuous and well-defined.

3. Decision Rule Of The Optimal Cycle Time T^*

Consider the following equations:

$$TVC_i'(T) = 0 \quad (i = 1, 2, 3) \tag{3.1}$$

If the root of equation (3.1) exists, then it is unique. Let T_i^* ($i = 1, 2, 3$) denote the root of equation (3.1). Further, equations (2.8), (2.9) and (2.11) yield that

$$\begin{aligned}
 TVC_1'(M) &= TVC_2'(M) \\
 &= -\frac{A}{M^2} + \frac{D(c\theta + h)}{\theta^2 M^2} (\theta M e^{\theta M} - e^{\theta M} + 1) + \frac{pI_e D (M^2 - N^2)}{2M^2}
 \end{aligned} \tag{3.2}$$

and

$$\begin{aligned}
 TVC_2'(N) &= TVC_3'(N) \\
 &= -\frac{A}{N^2} + \frac{D(c\theta + h)}{\theta^2 N^2} (\theta N e^{\theta N} - e^{\theta N} + 1)
 \end{aligned} \tag{3.3}$$

Since $TVC_2(T)$ is convex on $T > 0$ which implies that $TVC_2'(M) > TVC_2'(N)$.

For convenience, let

$$\Delta_1 = -\frac{A}{M^2} + \frac{D(c\theta + h)}{\theta^2 M^2} (\theta M e^{\theta M} - e^{\theta M} + 1) + \frac{pI_e D (M^2 - N^2)}{2M^2} \tag{3.4}$$

and

$$\Delta_2 = -\frac{A}{N^2} + \frac{D(c\theta + h)}{\theta^2 N^2} (\theta N e^{\theta N} - e^{\theta N} + 1) \tag{3.5}$$

Then $\Delta_1 > \Delta_2$. Moreover, equations (3.4) and (3.5) yield

$\Delta_1 < 0$ if and only if $TVC'_1(M) < 0$ if and only if $T_1^* > M$.

$\Delta_1 < 0$ if and only if $TVC'_2(M) < 0$ if and only if $T_2^* > M$.

$\Delta_2 < 0$ if and only if $TVC'_2(N) < 0$ if and only if $T_2^* > N$.

$\Delta_2 < 0$ if and only if $TVC'_3(N) < 0$ if and only if $T_3^* > N$.

Furthermore, if $\Delta_1 \geq 0$, then $TVC_1(T)$ is increasing on $[M, \infty)$. The above arguments lead to the following results.

Theorem 1

1. If $\Delta_1 < 0$, then $TVC(T^*) = TVC(T_1^*)$. Hence T^* is T_1^* .
2. If $\Delta_2 > 0$, then $TVC(T^*) = TVC(T_3^*)$. Hence T^* is T_3^* .
3. If $\Delta_1 \geq 0$ and $\Delta_2 < 0$, then $TVC(T^*) = TVC(T_2^*)$. Hence T^* is T_2^* .

Proof:

1. If $\Delta_1 < 0$, then $\Delta_2 < 0$ which implies that $T_1^* > M$, $T_2^* > M$, $T_2^* > N$ and $T_3^* > N$, respectively. Furthermore, $TVC(T)$ has the minimum value at $T = N$ when $T \leq N$, $TVC(T)$ has the minimum value at $T = M$ when $N \leq T \leq M$ and $TVC(T)$ has the minimum value at $T = T_1^*$ when $T \geq M$. Since $TVC_3(N) = TVC_2(N) > TVC_2(M)$ and $TVC_2(M) = TVC_1(M) > TVC_1(T_1^*)$, $TVC(T)$ has the minimum value at T_1^* for $T > 0$. Hence, we conclude that $TVC(T^*) = TVC(T_1^*)$. Consequently, T^* is T_1^* .
2. If $\Delta_2 > 0$, then $\Delta_1 > 0$ which implies that $T_2^* < M$, $T_2^* < N$, $T_3^* < N$ and $TVC_1(T)$ is increasing on $[M, \infty)$. Furthermore, $TVC(T)$ has the minimum value at $T = T_3^*$ when $T \leq N$, $TVC(T)$ has the minimum value at $T = N$ when $N \leq T \leq M$ and $TVC(T)$ has the minimum value at $T = M$ when $T \geq M$. Since $TVC_3(T_3^*) < TVC_3(N) = TVC_2(N) < TVC_2(M)$ and $TVC_2(M) = TVC_1(M)$, $TVC(T)$ has the minimum value at T_3^* for $T > 0$. Hence, we conclude that $TVC(T^*) = TVC(T_3^*)$. Consequently, T^* is T_3^* .
3. If $\Delta_1 \geq 0$ and $\Delta_2 < 0$ which implies that $T_1^* < M$, $T_2^* < M$, $T_2^* > N$ and $T_3^* > N$. Furthermore, $TVC(T)$ has the minimum value at $T = N$ when $T \leq N$, $TVC(T)$ has the minimum value at $T = T_2^*$ when $N \leq T \leq M$ and $TVC(T)$ has the minimum value at $T = M$ when $T \geq M$. Since $TVC_3(N) = TVC_2(N) > TVC_2(T_2^*)$ and $TVC_2(T_2^*) < TVC_2(M) = TVC_3(M)$. Hence, we conclude that $TVC(T^*) = TVC(T_2^*)$. Consequently, T^* is T_2^* .

Combining the above arguments, we have completed the proof.

4. Numerical Examples

In order to illustrate the above solution procedure, let us consider an inventory system with the following data:

$A = \$200/\text{order}$, $h = \$5/\text{unit}/\text{year}$, $c = \$60/\text{unit}/\text{year}$, $p = \$70/\text{unit}/\text{year}$, $I_k = 20\%$, $I_e = 12\%$, $\theta = 0.01$, $M = 0.3\text{year}$ and $N = 0.2\text{year}$.

Example 1: When $D = 400 \text{ units}/\text{year}$, then $\Delta_1 = -166.6464 < 0$ and $\Delta_2 = -3878.5 < 0$. Using Theorem 1(1), we get $T^* = T_1^* = 0.307$, the optimal order quantity is $Q^* = 122.9887$ and $TVC(T^*) = TVC(T_1^*) = 722.4254$.

Example 2: When $D = 1800 \text{ units}/\text{year}$, then $\Delta_1 = 7027.9 > 0$ and $\Delta_2 = 46.7250 > 0$. Using Theorem 1(2), we get $T^* = T_3^* = 0.1991$, the optimal order quantity is $Q^* = 358.7370$ and $TVC(T^*) = TVC(T_3^*) = 496.6506$.

Example 3: When $D = 500 \text{ units}/\text{year}$, then $\Delta_1 = 347.2476 > 0$ and $\Delta_2 = -3591.8 < 0$. Using Theorem 1(3), we get $T^* = T_2^* = 0.2847$, the optimal order quantity is $Q^* = 142.5528$ and $TVC(T^*) = TVC(T_2^*) = 734.3698$.

From now on, we study the effects of changes in the system parameters A , c , θ , and N

on the optimal length of order cycle T^* , the optimal order quantity per cycle Q^* and the minimum total relevant cost per unit time $TVC(T^*)$ of the following data:

$A = \$200/order$, $h = \$5/unit/year$, $c = \$60/unit/year$, $p = \$70/unit/year$, $I_k = 20\%$, $I_e = 12\%$, $D = 1000 units/year$, $\theta = 0.01$, $M = 0.3year$ and $N = 0.2year$. The sensitivity analysis is performed by changing each of the parameters by -25% and $+25\%$, taking one parameter at a time and keeping the remaining parameters unchanged. The results are summary in Table 1.

Table 1: Sensitivity analysis

Parameter	T^*	Q^*	$TVC(T^*)$
A	150	0.2131	213.3272
	200	0.2292	229.4629
	250	0.2443	244.5987
c	45	0.2305	230.7659
	60	0.2292	229.4629
	75	0.2280	228.2601
θ	0.0075	0.2305	230.6994
	0.0100	0.2292	229.4629
	0.0125	0.2280	228.3252
N	0.150	0.2051	205.3105
	0.200	0.2292	229.4629
	0.250	0.2570	257.3305
D	300	0.3364	101.0899
	400	0.3070	122.9887
	500	0.2847	142.5528

Based on the results of Table 1, the following observation can be made.

1. A higher value of ordering cost A results in higher values of T^* , Q^* and $TVC(T^*)$. Additionally, we find that $TVC(T^*)$ are highly sensitive to changes in A .
2. A higher value of retailer's trade credit N results in higher values of T^* , Q^* and $TVC(T^*)$. Additionally, we find that T^* , Q^* and $TVC(T^*)$ are highly sensitive to the changes in N .
3. A higher value of purchasing price c results in a higher value of $TVC(T^*)$, but lower values of T^* and Q^* . It indicates that if we increase the purchasing price, then the optimal length of ordering cycle and the optimal ordering quantity will be decreased.
4. A higher value of deteriorating rate θ results in a higher value of $TVC(T^*)$, but lower values of T^* and Q^* . It tells us that when the deteriorating rate increases, the optimal length of ordering cycle and the optimal ordering quantity will be decreased.
5. A higher value of demand rate D results in higher values of $TVC(T^*)$ and Q^* , but a lower value of T^* .

5. Special Cases

In this section, there are the following cases to occur:

(i) Shah' model

When $N = 0$ and $p = c$, we have

$$TVC(T) = \begin{cases} TVC_1^S(T) & \text{if } M \leq T, \\ TVC_2^S(T) & \text{if } M > T. \end{cases} \tag{5.1}$$

where

$$TVC_1^S(T) = \frac{A}{T} + \frac{D(c\theta + h)}{\theta^2 T} (e^{\theta T} - \theta T - 1) + \frac{cI_k D}{\theta^2 T} (e^{\theta(T-M)} - \theta(T-M) - 1) - \frac{cI_e D M^2}{2T} \quad (5.2)$$

and

$$TVC_2^S(T) = \frac{A}{T} + \frac{D(c\theta + h)}{\theta^2 T} (e^{\theta T} - \theta T - 1) - \frac{cI_e D}{2T} (2MT - T^2) \quad (5.3)$$

Equations (5.2) and (5.3) are consistent with equations (13) and (10) in Shah [22], respectively. Furthermore, we let

$$\Delta = \frac{D(c\theta + h)(\theta M e^{\theta M} - e^{\theta M} + 1)}{\theta^2 M^2} + \frac{cI_e D}{2} - \frac{A}{M^2}$$

Then, we have the following results.

Corollary 2

1. $TVC_1^S(T)$ is convex on $[M, \infty)$.
2. $TVC_2^S(T)$ is convex on $(0, \infty)$.
3. $TVC(T)$ is convex on $(0, \infty)$.

Corollary 3

1. If $\Delta > 0$, then T^* is T_2^* .
2. If $\Delta < 0$, then T^* is T_3^* .
3. If $\Delta = 0$, then $T^* = T_2^* = T_3^* = M$.

The results of Corollaries 2 and 3 have been discussed in Chung [6].

(ii) Goyal's model

When $\theta \rightarrow 0^+$, $N = 0$ and $p = c$, we have

$$TVC(T) = \begin{cases} \overline{TVC}_1(T) & \text{if } M \leq T, \\ \overline{TVC}_2(T) & \text{if } M > T. \end{cases} \quad (5.4)$$

where

$$\overline{TVC}_1(T) = \frac{A}{T} + \frac{DTh}{2} + \frac{cI_k D(T-M)^2}{2T} - \frac{cI_e D M^2}{2T} \quad (5.5)$$

and

$$\overline{TVC}_2(T) = \frac{A}{T} + \frac{DTh}{2} - \frac{cI_e D(2MT - T^2)}{2T} \quad (5.6)$$

Equations (5.5), (5.6) are consistent with equations (1) and (4) in Goyal [11], respectively. Furthermore, we let

$$\Delta^* = \frac{DM^2(h + cI_e) - 2A}{2}$$

Then, we have the following results.

Corollary 4

1. If $\Delta^* > 0$, then T^* is T_2^* .
2. If $\Delta^* < 0$, then T^* is T_3^* .
3. If $\Delta^* = 0$, then $T^* = T_2^* = T_3^* = M$.

The results of Corollary 4 have been discussed in Chung [5].

(iii) Huang's model

When $\theta \rightarrow 0^+$ and $p = c$, we have

$$TVC(T) = \begin{cases} TVC_1^*(T) & \text{if } M < T \\ TVC_2^*(T) & \text{if } N < T \leq M \\ TVC_3^*(T) & \text{if } 0 < T \leq N \end{cases} \quad (5.7)$$

where

$$TVC_1^*(T) = \frac{A}{T} + \frac{DT h}{2} + \frac{cI_k D(T - M)^2}{2T} - \frac{cI_e D(M^2 - N^2)}{2T} \quad (5.8)$$

$$TVC_2^*(T) = \frac{A}{T} + \frac{DT h}{2} - \frac{cI_e D(2MT - N^2 - T^2)}{2T} \quad (5.9)$$

and

$$TVC_3^*(T) = \frac{A}{T} + \frac{DT h}{2} - cI_e D(M - N) \quad (5.10)$$

Equations (5.8), (5.9) and (5.10) are consistent with equations (2), (3) and (4) in Huang [13], respectively. Furthermore, we let

$$\bar{\Delta}_1 = -2A + DM^2(h + cI_e) - cDN^2I_e$$

and

$$\bar{\Delta}_2 = -2A + DN^2h$$

Then, we have the following results.

Corollary 5

1. If $\bar{\Delta}_1 < 0$, then T^* is T_1^* .
2. If $\bar{\Delta}_2 > 0$, then T^* is T_3^* .
3. If $\bar{\Delta}_1 > 0$ and $\bar{\Delta}_2 < 0$, then T^* is T_2^* .

The results of Corollary 5 have been discussed in Huang [13].

6. Summary

This paper considers a supply chain system consisting of one supplier, one retailer and multiple customers to explore the optimal retailer's replenishment decisions under the conditions of permissible delay in payments, in which the supplier offers the retailer a fixed delay period and the retailer in turn provides a maximal trade credit period to their customers. Theorem 1 gives the solution procedure to find T^* . Numerical examples are given to illustrate Theorem 1. In addition, sensitivity analysis represents the following results: first, T^* , Q^* and $TVC(T^*)$ increase with increase in the values of parameters A and N . Second, T^* and Q^* decrease while $TVC(T^*)$ increases with increase in the values of parameters c and θ . Third, T^* decreases while Q^* and $TVC(T^*)$ increase with increase in the value of parameter D . Additionally, although the optimal cycle time cannot be expressed in a closed form, it can be obtained through the use of the Intermediate Value Theorem [26]. Moreover, if $N = 0$ and $p = c$, Shah [22] can be treated as a special case of this paper. Finally, if the deterioration is ignored, Equations (2.4) are reduced to Goyal [11] and Huang [13], respectively.

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