Journal of the Operations Research
Society of Japan
2009, Vol. 52, No. 2, 147-162

DYNAMIC NETWORK DEA: AN ILLUSTRATION

Peter Bogetoft, *Copenhagen Business School*
Rolf Färe, Shawna Grosskopf, *Oregon State University*
Kathy Hayes, *Southern Methodist University*
Lori Taylor, *Texas A&M University*

(Received March 29, 2008; Revised October 24, 2008)

Abstract Network DEA provides a flexible way to customize DEA problems to specific applications. It provides 'links' across DMU's, or alternatively allows us to look inside a complex DMU with multiple nodes. In this short paper, we provide an illustration of a dynamic network DEA model. It is a network in the sense that it links the behavior of DMU's across time. Our particular illustration is to an 'old' issue: the impact of public capital on technology and productivity (TFP) at the state level in the U.S. We use a dynamic network production model developed by Färe and Grosskopf [8,9] to measure dynamic efficiency. As a byproduct of the model, we solve for optimal public and private investment paths, which we then can compare to realized investment. We apply this to U.S. manufacturing data over the 1978-1999 period using state level data.

Keywords: DEA, economics, linear programming

1. Introduction

Researchers have long been interested in the impact of public capital on private productivity. About two decades ago the issue was revived by two papers: Aschauer's [2] provocative analysis suggesting that public capital is grossly under provided in the United States and Munnell's [25, 26, 27] analyses suggesting that state and local public capital is under provided in the United States. Munnell's analyses have been particularly influential because she generated a panel data set on public and private capital for the U.S. states that has been used by many subsequent researchers (e.g. Morrison and Schwartz [24, 22, 23], Kelejian and Robinson [20], Holtz-Eakin [16], Domazlicky and Weber [6], Boisso, Grosskopf and Hayes [3]).

Interest in the effects of public capital is not limited to the United States, however. Researchers have recently examined the impact of public capital investments in Japan (e.g. Kataoka [19], Thangavelu and Owyong [34], Shioji [32] and Ihori and Kondo [18]), Mexico (e.g. Rodriguez-Oreggia [31], and Ramirez [29]) India (Lall [21]) and Spain (Herranz-Loncan [17]).

The most recent work on the United States does not suggest that public capital is underprovided. Holtz-Eakin [16] and Garcia-Mila, McGuire and Porter [11] find little evidence that public capital contributes to private sector productivity. Morrison and Schwartz [22,23,24] find evidence of positive direct productivity impacts of public capital but conclude that these direct effects are typically offset by indirect effects on factor accumulation. Brown, Hayes and Taylor [4] find that not only does growth in public capital tend to discourage the accumulation of private capital and labor, it may also directly discourage output growth. Henderson and Kumbhakar [15] find evidence that the returns to public capital are similar to those of private capital, suggesting that public capital is optimally provided.

A common characteristic of this literature has been that productivity is measured indirectly from an estimated production or cost function. A recent trend has been to use more direct measures of productivity based on nonparametric programming techniques. Domazlicky and Weber [6,7] calculate Malmquist productivity indexes for each of the 48 contiguous states and use them to examine the impact of agglomeration economies and education levels on productivity. They find no relationship between public capital and private productivity. Boisso et al. [3] also calculate Malmquist productivity indices and then examine the impact of business cycles and various measures of public capital. In contrast to Domazlicky and Weber, Boisso et al. find that the ratio of public capital to private capital has a positive impact on productivity. Boisso et al. also find evidence of spillover effects with respect to highway capital. Grosskopf, Hayes and Taylor [13] estimate Malmquist productivity for both manufacturing as a whole and the subset of high tech industries within manufacturing, and estimate impacts of state and local fiscal policy. Their results suggest that high tech manufacturing is more productive than manufacturing as a whole and that state and local policy can explain variations in performance.

In this paper we take a somewhat different approach. We maintain the frontier approach used in our earlier work, but generalize the technology to allow for intertemporal, dynamic effects. This dynamic model allows us to solve for maximum potential output over our time period (1977-1999) as well as the optimal public and private investment paths over this period. As before we use state level manufacturing sector data. This approach allows us to compare optimal public investment with observed investment which provides us with an alternative test of whether more public sector investment is the path to improved manufacturing growth. We too find little evidence that public capital is underprovided.

2. The Basic Dynamic Technology

In this section we introduce the dynamic technology which is our basic model for estimation of optimal private and public investment.* This model resembles standard income-expenditure models, i.e., total output is allocated to final output (consumption) and investment; here private and public investment. The dynamics of our technology are modeled as the choice of consuming total output in the period of production or instead diverting some current production toward adding to the next period capital stock. Essentially we are formulating a discrete Ramsey [30] model, which we specify as an activity analysis or data envelopment analysis (DEA) variation of the Ramsey model. This allows us to solve the model using simple linear programming problems.

In his 1928 paper, F.P. Ramsey studied the trade-off between consumption and investment in a continuous time model. He used a general production function to model the production of a homogeneous output that could either be consumed or invested. In our model we use a discrete formulation of time and employ an activity analysis (DEA) model as our production technology.

To provide some intuition for our model, assume that we have three time periods $t-1, t, t+1$ and that there is a technology P^τ, $\tau = t-1, t, t+1$. In addition at each τ there are some exogenous inputs x^τ and final outputs fy^τ. Final output is that part of total production y^τ that is not allocated to private iy^τ or public gy^τ investments, i.e.,

$$y^\tau = fy^\tau + iy^\tau + gy^\tau. \tag{2.1}$$

*This section is based on Färe and Grosskopf [8,9]. See C. Kao 'Network Data Envelopment Analysis: Current Development and Future Research,' presented at the 2008 Asian-Pacific Productivity Conference, for a recent survey of network DEA including dynamic DEA.

We may now sketch our model as a network, using the above notation.

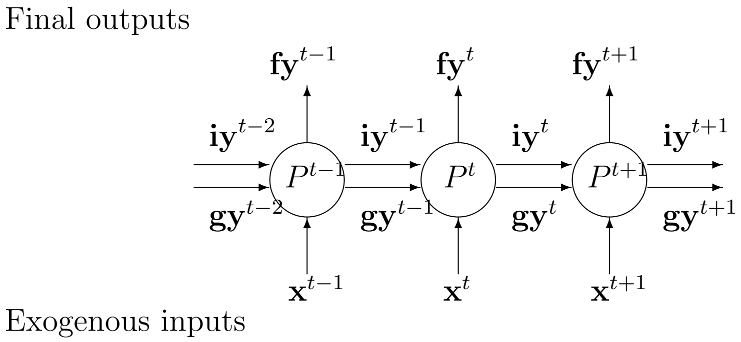

Figure 1: The Dynamic Technology

The bottom vertical arrows indicate the exogenous inputs into the respective technologies P^{τ}. The top vertical arrows indicate the final output from each technology. We eventually define our objective function over these final outputs, just as in the Ramsey model. The two horizontal arrows entering each technology represent the private and public investment from the previous period. Thus decisions concerning consumption versus investment in one period have consequences in the ensuing periods.

One attractive feature of the dynamic model illustrated in Figure 1 is that it can be implemented as a dynamic activity analysis or DEA model. Given that we have $k = 1, \ldots, K$ observations (in our case states) of $m = 1, \ldots, M$ outputs $(y_1, \ldots, y_M)$ and $n = 1, \ldots, N$ inputs $(x_1, \ldots, x_N)$ in each period t, the model may be written for the three period case as

$$
\begin{aligned}
P(x^{t-1}, x^t, x^{t+1}, iy^{t-2}, gy^{t-2}) &= \{(fy^{t-1}, fy^t, (fy^{t+1} + iy^{t+1} + gy^{t+1})) : \\
fy_m^{t-1} + iy_m^{t-1} + gy_m^{t-1} &\leqq \sum_{k=1}^{K} z_k^{t-1}(fy_{km}^{t-1} + iy_{km}^{t-1} + gy_{km}^{t-1}), \forall m, && (2.2) \\
& \sum_{k=1}^{K} z_k^{t-1} x_{kn}^{t-1} \leqq x_n^{t-1}, n = 1, \ldots, N, && (2.3) \\
& \sum_{k=1}^{K} z_k^{t-1} iy_{km}^{t-2} \leqq iy_m^{t-2}, m = 1, \ldots, M, && (2.4) \\
& \sum_{k=1}^{K} z_k^{t-1} gy_{km}^{t-2} \leqq gy_m^{t-2}, m = 1, \ldots, M, && (2.5) \\
& z_k^{t-1} \geqq 0, k = 1, \ldots, K, && (2.6) \\
fy_m^{t} + iy_m^{t} + gy_m^{t} &\leqq \sum_{k=1}^{K} z_k^{t}(fy_{km}^{t} + iy_{km}^{t} + gy_{km}^{t}), \forall m, && (2.7) \\
& \sum_{k=1}^{K} z_k^{t} x_{kn}^{t} \leqq x_n^{t}, n = 1, \ldots, N, && (2.8) \\
& \sum_{k=1}^{K} z_k^{t} iy_{km}^{t-1} \leqq iy_m^{t-1}, m = 1, \ldots, M, && (2.9)
\end{aligned}
$$

$$\sum_{k=1}^{K} z_k^t gy_{km}^{t-1} \leqq gy_m^{t-1}, m = 1,\ldots,M, \tag{2.10}$$

$$z_k^t \geqq 0, k = 1,\ldots,K, \tag{2.11}$$

$$fy_m^{t+1} + iy_m^{t+1} + gy_m^{t+1} \leqq \sum_{k=1}^{K} z_k^{t+1}(fy_{km}^{t+1} + iy_{km}^{t+1} + gy_{km}^{t+1}), \forall m, \tag{2.12}$$

$$\sum_{k=1}^{K} z_k^{t+1} x_{kn}^{t+1} \leqq x_n^{t+1}, n = 1,\ldots,N, \tag{2.13}$$

$$\sum_{k=1}^{K} z_k^{t+1} iy_{km}^{t} \leqq iy_m^{t}, m = 1,\ldots,M, \tag{2.14}$$

$$\sum_{k=1}^{K} z_k^{t+1} gy_{km}^{t} \leqq gy_m^{t}, m = 1,\ldots,M, \tag{2.15}$$

$$z_k^{t+1} \geqq 0, k = 1,\ldots,K\}, \tag{2.16}$$

where the z_k^τ are intensity variables for $k = 1,\ldots,K, \tau = t-1,\ldots,t+1$ in our example. The P technology for period $t-1$ is modeled by the (2.2)-(2.6), where (2.2)is the output constraint, (2.3) is the input constraint, (2.4)-(2.5) are the intertemporal investment constraints and (2.6) is the constraint on the intensity variables. Similarly, the period t technology is modeled by (2.7)-(2.11) and the $t+1$ technology by (2.12)-(2.16). Note that each period's technology has its own intensity variables $z_k^\tau, \tau = t-1, t, t+1, k = 1,\ldots,K$. Färe and Grosskopf [8] have shown that the dynamic technology inherits its properties from the single period subtechnologies, so in this case the dynamic technology exhibits constant returns to scale and strong disposability of inputs and outputs.

For each observation k' we can estimate its dynamic efficiency by solving the following problem which generalizes the model to many periods. Here to make the problem more concrete, we use the time periods from our data set, namely 1977-1999. As our objective, we choose to maximize the stream of final outputs over time, allowing for each period to be scaled individually, namely $\theta^\tau, \tau = 1977,\ldots,1999$. Thus, we have

$$(D_o(x^{k',1977},\ldots,x^{k',1999}, ic^{k',1977}, gc^{k',1977}, iy^{k',1999}, gy^{k',1999}))^{-1} = \max \sum_{\tau=1978}^{1999} \theta^\tau \text{ s.t.:} \tag{2.17}$$

$$\theta^\tau fy^{k',\tau} + iy^{k',\tau} + gy^{k',\tau} \in P(x^{k',1977},\ldots,x^{k',1999}, ic^{k',1977}, gc^{k',1977}, iy^{k',1999}, gy^{k',1999}), \quad \tau = 1978,\ldots,1999. \tag{2.18}$$

Next we need to be a little bit more specific about how we are formulating our problem. In our case, we define state output (GSP) to be the sum of final output and intermediate output, $y_k^\tau = fy_k^\tau + iy_k^\tau + gy_k^\tau, k = 1,\ldots,K; \tau = 1977,\ldots,1999$. We treat $(iy^\tau + gy^\tau)$ as investment, which is related to our public and private capital stock measures, which we call ic_k^τ and gc_k^τ and are defined in the following way,

$$\begin{aligned} ic_k^\tau &= ic_k^{\tau-1}(1-\delta) + iy_k^{\tau-1}, k = 1,\ldots,K; \tau = 1978,\ldots,1999 \\ gc_k^\tau &= gc_k^{\tau-1}(1-\delta) + gy_k^{\tau-1}, k = 1,\ldots,K; \tau = 1978,\ldots,1999, \end{aligned} \tag{2.19}$$

where δ is the depreciation rate and iy^τ and gy^τ are endogenous.† We can now write the following linear programming problem for each observation $k' = 1, \dots, K$

$$(D_o(x^{k',1977}, \dots, x^{k',1999}, ic^{k',1977}, gc^{k',1977}, iy^{k',1999}, gy^{k',1999}))^{-1} = \max_{\theta^\tau, z^\tau} \sum_{\tau=1978}^{1999} \theta^\tau \tag{2.20}$$

subject to:

First Period: $\tau = 1978$

$$\begin{aligned}
\theta^\tau f y^\tau_{k'} + iy^\tau + gy^\tau &\leqq \sum_{k=1}^{K} z^\tau_k y^\tau_k \\
\sum_{k=1}^{K} z^\tau_k ic^\tau_k &\leqq ic^{\tau-1}_{k'}(1-\delta) + iy^{\tau-1}_{k'} \\
\sum_{k=1}^{K} z^\tau_k gc^\tau_k &\leqq gc^{\tau-1}_{k'}(1-\delta) + gy^{\tau-1}_{k'} \\
\sum_{k=1}^{K} z^\tau_k x^\tau_{kn} &\leqq x^\tau_{k'n}, n = 1, \dots, N, \\
\theta^\tau &\geqq 1 \\
z^\tau_k &\geqq 0, k = 1, \dots, K,
\end{aligned}$$

Middle Periods: $\tau = 1979, \dots, 1998$

$$\begin{aligned}
\theta^\tau f y^\tau_{k'} + iy^\tau + gy^\tau &\leqq \sum_{k=1}^{K} z^\tau_k y^\tau_k \\
\sum_{k=1}^{K} z^\tau_k ic^\tau_k &\leqq ic^{\tau-1}_{k'}(1-\delta) + iy^{\tau-1} \\
\sum_{k=1}^{K} z^\tau_k gc^\tau_k &\leqq gc^{\tau-1}_{k'}(1-\delta) + gy^{\tau-1} \\
\sum_{k=1}^{K} z^\tau_k x^\tau_{kn} &\leqq x^\tau_{k'n}, n = 1, \dots, N, \\
\theta^\tau &\geqq 1 \\
z^\tau_k &\geqq 0, k = 1, \dots, K,
\end{aligned}$$

End Period: $\tau = 1999$

$$\begin{aligned}
\theta^\tau y^\tau_{k'} + iy^\tau + gy^\tau &\leqq \sum_{k=1}^{K} z^\tau_k y^\tau_k \\
\sum_{k=1}^{K} z^\tau_k ic^\tau_k &\leqq ic^{\tau-1}_{k'}(1-\delta) + iy^{\tau-1} \\
\sum_{k=1}^{K} z^\tau_k gc^\tau_k &\leqq gc^{\tau-1}_{k'}(1-\delta) + gy^{\tau-1}
\end{aligned}$$

†We use the year-to-year changes in real capital stock as our measures of net investment, $iy^\tau - \delta ic^{\tau-1}_k$ and $gy^\tau - \delta gc^{\tau-1}_k$.

$$\begin{aligned}\sum_{k=1}^{K} z_k^\tau x_{kn}^\tau &\leqq x_{k'n}^\tau, n = 1, \ldots, N,\\ \theta^\tau &\geqq 1\\ z_k^\tau &\geqq 0, k = 1, \ldots, K,\end{aligned}$$

This problem yields an annual efficiency score θ^τ, plus solution values for the net investments in public and private capital, $iy^\tau - \delta ic_k^{\tau-1}, gy^\tau - \delta gc_k^{\tau-1}$ for each period and each state. Here we have restricted the annual efficiency score to be greater than or equal to one, which means that final consumption cannot be reduced in any given period below the observed level. Starting conditions are set up to be consistent with what was observed in 1977; that is, in 1978, private and public capital are restricted to sum to less than or equal to the sum of observed investment between 1977 and 1978 (the observed change in the capital stock between 1978 and 1977) and the 1977 observed capital stock. Our transversality or end point constraint requires that 'optimal' investment sum to observed investment in 1999.

We also restrict the sum of optimal public and private investment to be no larger than the sum of observed public and private investment in every period. This allows for reallocation between public and private investment, holding total investment at observed levels, allowing us to compare the optimal to observed levels.‡

Finally, we also calculate static efficiency and Malmquist productivity as points of comparison. The static and Malmquist models do not allow for reallocation across time periods. The static model looks at each period as a separate technology. The Malmquist productivity model also assumes that each period has its own technology, but compares the data across adjacent periods by estimating technical change (compares frontiers in adjacent periods) and efficiency change (compares how far a state is from the frontier in adjacent periods). It is in effect a comparative static model.

3. Data

Any analysis of optimal public investment requires high quality estimates of public and private capital stocks by state. We focus on the manufacturing sector because the data support a particularly attractive strategy for estimating private capital at the state level.

This state-level analysis uses one aggregate output measure total, real manufacturing output and three inputs manufacturing employment, private manufacturing capital and state and local public capital. Annual estimates of manufacturing employment and real gross manufacturing product in each state come from the U.S. Bureau of Economic Analysis (BEA).

State-level estimates of public and private capital stocks are not available from official sources, and must be imputed. Grosskopf, Hayes and Taylor [14] describes our method for estimating net manufacturing capital stocks in each state, while Brown, Hayes and Taylor [4] describes the method for estimating public capital stocks. In each case, we use data on real investment flows by industry and state to calculate perpetual-inventory estimates of the capital stock in each state. We then sum the perpetual-inventory estimates across the states each year, and assign each state a share of the BEA's national capital stock estimates according to the state's share of the sum-of-states estimate.

The key to our approach is the calculation of the perpetual inventory estimates. For each U.S. state, we transform nominal data on manufacturing investments and public capital outlays into real investment flows using the implicit deflators BEA used when constructing

‡As noted by an anonymous referee this imposes 'perfect substitutability' which could be relaxed.

the corresponding national capital stock estimates.[§] These real investment flows are then used to estimate perpetual inventory estimates of capital stock following the same geometric depreciation strategy used by BEA to generate the national estimates of capital stocks.[¶] Thus, we calculate our perpetual-inventory estimates of net capital stocks in each state for period t as

$$N_{tj} = \sum_{i=1}^{t} I_{ij}(1-\delta_j/2)(1-\delta_j)^{t-i} \tag{3.1}$$

where $t \geq i$, N_{tj} is the net capital stock of asset type j, I_{ij} is real investment in year i, and δ_j is the annual geometric rate of depreciation for asset of type j. We assume that the geometric rates of depreciation for each state equal BEA's implicit national rate of depreciation for that type of asset. State-level estimates of gross capital stock in manufacturing are available directly from BEA for 1977. We use these estimates as the starting point for our perpetual inventory estimates of state-level capital stocks in manufacturing. State-level estimates of public capital stocks are not available for any year, so we start our perpetual inventory estimates for that sector with capital outlays for 1958, and build up from there.

4. Results

The solution to our dynamic network DEA programming problem yields state specific estimates of technical efficiency (θ^τ) for each year 1978-1999, and optimal public and private investment for 1978-1999. As in Grosskopf, Hayes and Taylor [14] we exclude data from Alaska, Hawaii and New Mexico.

We begin with a summary table of the results for several of our models. Included are mean results by year for the static model, and the dynamic model wherein we restricted the sum of optimal investments to equal the total observed in each state each year. These means represent the average over states of the annual efficiency score; a value of unity would signal efficient performance, i.e., no increase in output is feasible in that period. Values greater than unity give the factor by which output could be expanded. Table 1 shows that, as we would expect, we typically see that the potential gains increase (i.e., the efficiency score increases) as the model becomes more flexible, i.e, the static scores are typically lower than the dynamic results.

We also include box plots for these results to give a better picture of the distribution of annual efficiency across states in each year. Figure 2 presents a box-plot for the static efficiency estimates; figure 3 is a box plot for the dynamic efficiency estimates.[‖] The line in each figure indicates mean efficiency while the boxes indicate two-standard deviations around the mean in each period.

The static results are estimated from simple DEA output-oriented technical efficiency measures, where each period is a separate technology. These results indicate that there is variation over time, i.e., the boxes drift up and down over time with the worst average performance occurring in the late 1980s, and the best average performance during the 1992-93 period. The dispersion, i.e., the size of the box, appears to narrow slightly over time.

[§]For the period 1979-81, there are no data on manufacturing investment at the state level, although there are state-level estimates of gross capital stocks for total manufacturing in 1978 and 1981. We use the change in gross stocks between 1978 and 1981 to calculate each state's share of U.S. manufacturing investment for 1979, 1980 and 1981.

[¶]For additional details, see Brown, Hayes and Taylor [4] or Grosskopf, Hayes and Taylor [14].

[‖]Dynamic efficiency estimates could not be calculated for eight states. To make comparisons easier, we exclude efficiency estimates for those states from the data used to generate the static efficiency figure and all subsequent figures.

Table 1: Annual Efficiency Scores

Year	Static Mean	Dynamic Sum
SL78	1.698916	2.104077
SL79	1.667558	1.805718
SL80	1.579109	1.744667
SL81	1.600924	1.664872
SL82	1.582455	1.607180
SL83	1.670477	1.694359
SL84	1.686515	1.737564
SL85	1.661577	1.789180
SL86	1.604034	1.675641
SL87	1.783116	1.667590
SL88	1.865452	1.579128
SL89	1.869062	1.602333
SL90	1.839512	1.612180
SL91	1.700252	1.596692
SL92	1.470092	1.395256
SL93	1.395098	1.363205
SL94	1.470092	1.482282
SL95	1.695226	1.507590
SL96	1.471549	1.492974
SL97	1.481656	1.537667
SL98	1.512435	1.684769
SL99	1.638515	1.835718
mean	1.747149	1.644575
N	47	39

Next we turn to the box plots for the dynamic model with the sum restriction. These results also show variation over time, but with more dramatic dispersion and a slightly different pattern. Again, the best performance occurs during the 1992-93 period. The worst performance occurs at the very beginning and end of the period, which may have something to do with our transversality conditions, but may also reflect generally improving efficiency throughout the 1980s followed by a surge in investment during the 1997-1999 period as firms and governments prepared for the Y2K transition. The dynamic model provides no evidence for an increase in inefficiency during the late 1980s, as is suggested by the static results.

Our Malmquist productivity results are included here, although they are not strictly comparable to the other results presented here. In contrast to the static results which are simple technical efficiency estimates in which each period is a separate technology, these are indexes constructed as ratios of static technical efficiency measures, and represent (comparative static) changes between adjacent periods. We have included the overall productivity change indexes (Malm) as well as the efficiency change and technical change components, i.e.,[**]

$$\text{Malmquist Prod Change} = \text{Effic Change} * \text{Technical Change}, \tag{4.1}$$

where efficiency change captures technology diffusion effects (catching up) and technical

[**]For a more detailed discussion of the Malmquist productivity index and its history see Grosskopf [12].

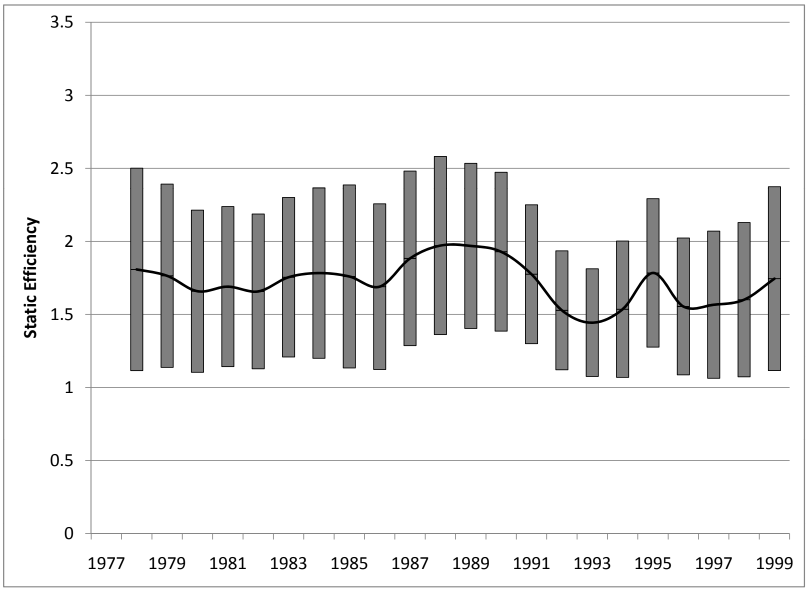

Figure 2: The Static Efficiency Results

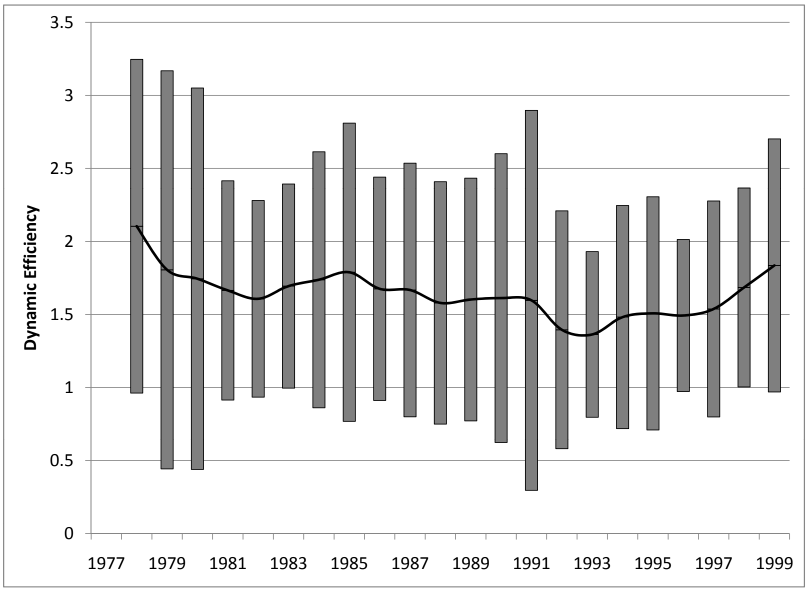

Figure 3: The Dynamic Efficiency Results

change captures shifts in the frontier (innovation).[††] Some annual averages are included in Table 2. Values greater than one signal improvement, unity signals no change and values less than one represent declines. Generally, we see small average productivity improvements, which are typically driven by technical change on average. More helpful are the box plots of the cumulated productivity, efficiency and technical changes. The cumulation allows us to see the pattern of productivity growth and its components over time.[‡‡]

Table 2: Malmquist Productivity Results (means)

Year	Malmquist	Effic Change	Tech Change
1977	1.0257223	1.0122874	1.0134867
1978	1.0192453	1.0482472	0.9724438
1979	0.9573172	1.0043623	0.9532068
1980	1.0289158	0.9973489	1.0314578
1981	0.9589016	0.9896046	0.9691295
1982	1.0656830	0.9691147	1.1004399
1983	1.0764641	1.0109431	1.0651937
1984	1.0075351	0.9884120	1.0194811
1985	0.9744766	1.0316940	0.9449407
1986	1.0841769	1.0171014	1.0662512
1987	1.0635439	1.0560443	1.0076651
1988	0.9853773	0.9920655	0.9936263
1989	0.9868143	1.0234196	0.9642906
1990	0.9709036	1.0013942	0.9698878
1991	1.0147240	1.0413488	0.9752474
1992	1.0300160	0.9547135	1.0883056
1993	1.0465093	0.8722177	1.2069045
1994	1.0604067	1.0599244	1.0037283
1995	1.0266856	0.9385990	1.0945686
1996	1.0209451	0.9032156	1.1353526
1997	1.0157342	0.9630518	1.0638369
1998	1.0363814	0.9813200	1.0574749

The Malmquist box plot shows a general pattern of productivity improvement over this time period, with a modest acceleration in productivity growth at the end of the period. Another feature is the increasing dispersion over the time period; the boxes are much longer at the end of our sample period than at the beginning.

The pattern of cumulated efficiency change is more variable, and does not indicate an upward trend. Instead, there is a decline at the end of the time period. Again, we see increasing dispersion across states over time.

Finally, the cumulated technical change results show us that the improvement in the 1990's is due to technical progress. The offsetting efficiency change results from states that are left behind during this period of innovation. Again we see evidence of increasing dispersion over time.

[††]We do not include a scale change component here; instead it has been subsumed in the efficiency change term.
[‡‡]Cumulated changes are calculated by multiplying consecutive period changes.

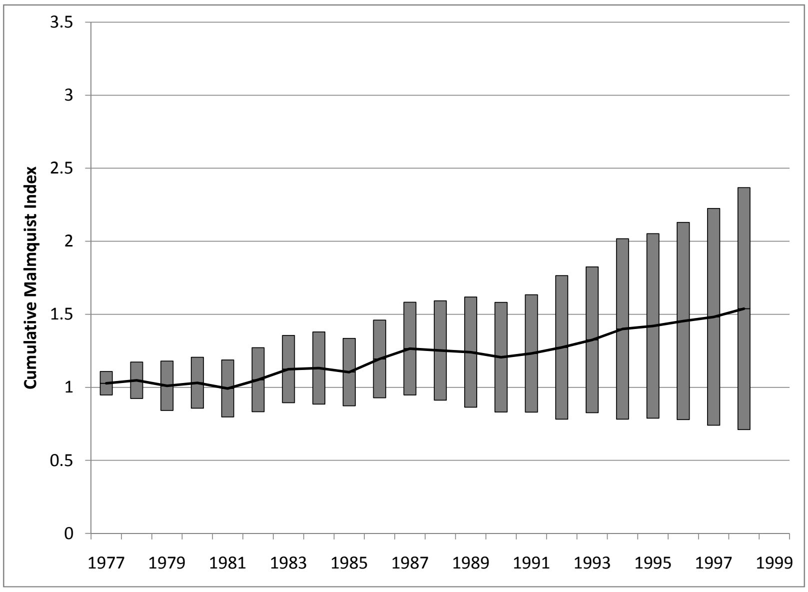

Figure 4: Cumulative Malmquist Productivity

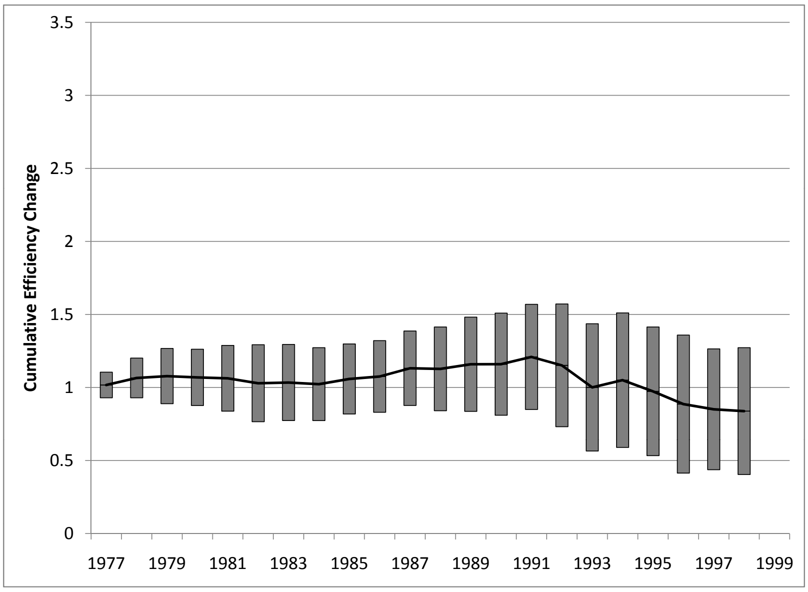

Figure 5: Cumulative Efficiency Change

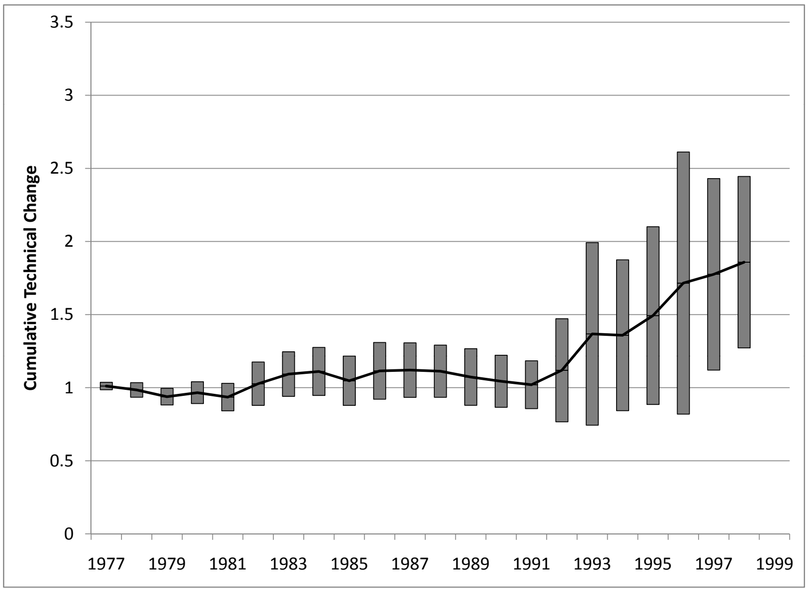

Figure 6: Cumulative Technical Change

The difference in pattern between the Malmquist boxplots and the dynamic efficiency boxplots is striking. The Malmquist boxplots indicate that productivity was generally improving and becoming more diffused across the U.S. states between 1978 and 1999. The dynamic efficiency results do not indicate any trend toward higher productivity, nor do they indicate greater dispersion. Clearly, the dynamic efficiency model provides an interesting new perspective on productivity change in the United States.

We would like to point out that the Malmquist results, since they are comparative static (i.e., they do not include dynamic interaction across time,) are not strictly comparable to our dynamic results which is our focus here. An obvious next step would be to use the dynamic efficiency results to construct a dynamic Malmquist productivity index.

We now return to our original research question, i.e., would increases in public sector investment improve dynamic efficiency? Recall that in our dynamic model, the sum of optimal investment is constrained to equal observed in each state each period, but reallocation of investment between the public and private sectors. In order to see whether there has been over or under public and private sector investment we take the percentage difference between optimal and observed investment:

$$gdiff_{kt} = (gy_{k^*t} - gy_{k^*t})/|gy_{k^*t}|, t = 1, \ldots, T, k = 1, \ldots, K, \tag{4.2}$$

$$idiff_{kt} = (iy_{k^*t} - iy_{k^*t})/|iy_{k^*t}|, t = 1, \ldots, T, k = 1, \ldots, K, \tag{4.3}$$

where gdiff is the difference for public sector investment and idiff is the difference between observed and optimal private sector investment.

If gdiff (idiff) is greater than zero, then optimal public (private) sector investment exceeds the observed investment. Positive values of gdiff (idiff) would be consistent with the hypothesis that there has been under investment in public (private) capital. We begin with the box plots for gdiff.

The figure plots the distribution of gdiff across states for each year in the sample; time is on the horizontal axis and gdiff is on the vertical axis. For the most part the mean (the line)

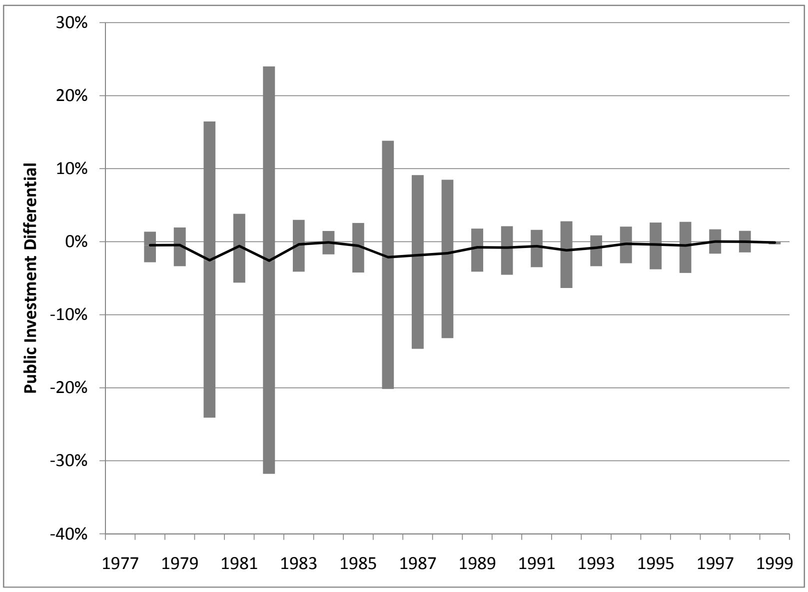

Figure 7: Comparing Public Sector Investment with Optimal

is below zero, i.e., observed public investment is greater than optimal public investment, although the percentage deviation is generally modest. There is variation over time, however, and there are clearly observations in the tail for which there is underinvestment in public infrastructure relative to private manufacturing investment in that state. Furthermore, public capital might be productive for sectors other than manufacturing, so overinvestment from the perspective of the manufacturing sector need not mean overinvestment from the perspective of the economy as a whole.

Turning to the box plot for idiff, we see that relative to public investment, mean idiff lies generally above zero, i.e., optimal manufacturing investment is often greater than observed. Again we see variation over the time period, and across states within given time periods. The underinvestment in private manufacturing capital is much bigger in percentage terms than the overinvestment in public capital, largely because the stock of public capital is so much larger than the stock of manufacturing capital.

Our evidence suggests the taxes needed to finance public sector capital investments may have discouraged the accumulation of private manufacturing capital, leading to overinvestment in public capital and underinvestment in private manufacturing capital, at least from the perspective of the manufacturing sector. This finding is consistent with Taylor and Brown (2006), who found evidence that public sector spending on infrastructure crowded out private sector capital and discouraged private sector job growth during the late 1980s and most of the 1990s.

Our results suggest that there is no strong evidence that there is too little investment in public relative to private manufacturing capital in the U.S. over the 1978-1999 time period. Of course these preliminary results are only suggestive; the dynamic model used here is very simplistic. A much more detailed specification of the dynamics, as for example in dynamic DEA model of firms in Nemoto and Goto [28] would be worth investigating. Disaggregation of capital and inclusion of human capital would also be desirable, as would an analysis of nonmanufacturing private capital. We have not included discounting in the model, and

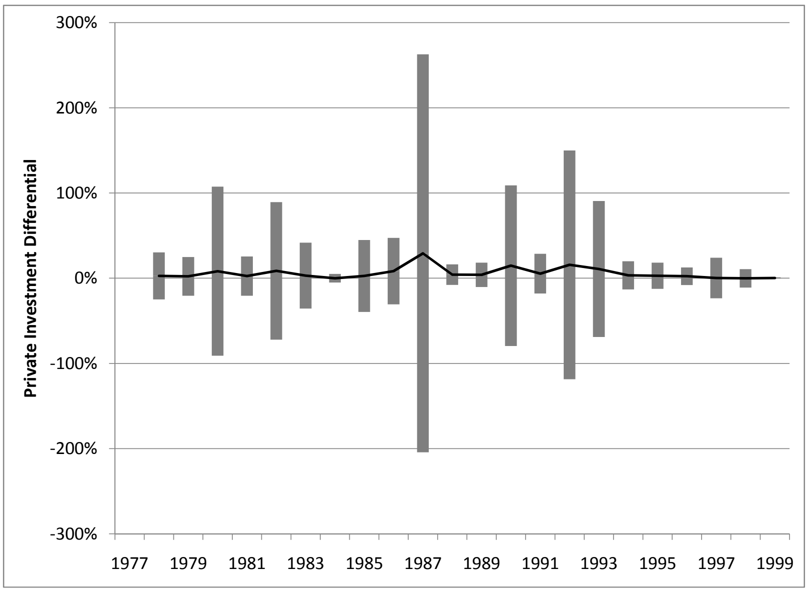

Figure 8: Comparing Private Sector Investment with Optimal

we have made no attempt to pursue statistical inference. Bootstrapping methods would be useful for the latter. Also interesting would be application of this general approach to Japanese data.

References

[1] J.L. Andersen and P. Bogetoft: *Quota trading and profitability: Theoretical models and applications to Danish fisheries* (Working Paper from Dept. of Economics and Natural Resources, KVL, 2003).

[2] D.A. Aschauer: Is public expenditure productive? *Journal of Monetary Economics*, **23** (1989), 177–200.

[3] D. Boisso, S. Grosskopf and K. Hayes: Productivity and efficiency in the US: Effects of business cycles and public capital. *Regional Science and Urban Economics*, **30** (2000), 663–681.

[4] S. Brown, K. Hayes and L. Taylor: State and local policy, factor markets and regional growth. *Review of Regional Studies*, **33(1)** (2003), 40–60.

[5] B.R. Domazlicky and W.L. Weber: Determinants of total factor productivity, technological change and efficiency differentials among states, 1977-86. *Review of Regional Studies*, **28(2)** (1998), 19–33.

[6] B.R. Domazlicky and W.L. Weber: Productivity growth and pollution in state manufacturing. *Review of Economics and Statistics*, **83** (2001), 195–99.

[7] B.R. Domazlicky and W.L. Weber: Total factor productivity in the contiguous United States, 1977-86. *Journal of Regional Science*, **37** (1987), 213–33.

[8] R. Färe and S. Grosskopf: *Intertemporal Production Frontiers* (Kluwer-Nijhoff, 1996).

[9] R. Färe and S. Grosskopf: Efficiency and productivity in rich and poor countries. In B.S. Jensen and K. Wong (eds.): *Dynamics, Economic Growth, and International Trade* (The University of Michigan Press, Ann Arbor, 1997), 243–264.

[10] M.J. Farrell: The measurement of productive efficiency. *Journal of the Royal Statistical Society*, Series A, General, **120(3)** (1957), 253–81.

[11] T. Garcia-Mila, T.J. McGuire and R.H. Porter: The effect of public capital in state-level production functions reconsidered. *Review of Economics and Statistics*, **78** (1996), 177–180.

[12] S. Grosskopf: Some remarks on the Malmquist productivity index and its decomposition. *Journal of Productivity Analysis*, **20** (2003), 459–74.

[13] S. Grosskopf, K. Hayes and L. Taylor: *Productivity, innovation and diffusion in the United States: Can policy put the states on the technology frontier?* (mimeo 2002).

[14] S. Grosskopf, K. Hayes and L. Taylor: Sources of manufacturing productivity growth: U.S. states 1990-1999. In R. Färe, S. Grosskopf and D. Primont (eds.): *Aggregation, Efficiency, and Measurement* (Springer Science+Business Media, LLC, New York, 2007), 97–116.

[15] D. Henderson and S. Kumbhakar: Public and private capital productivity puzzle: A nonparametric Aaproach. *Southern Economic Journal*, **73(1)** (2006), 219–232.

[16] D. Holtz-Eakin: Public-sector capital and the productivity puzzle. *Review of Economics and Statistics*, **76** (1994), 12–21.

[17] A. Herranz-Loncan: Infrastructure investment and spanish economic growth, 1850-1935. *Explorations in Economic History*, **44(3)** (2007), 452–468.

[18] T. Ihori and H. Kondo: The efficiency of disaggregate public capital provision in Japan. *Public Finance and Management*, **1(2)** (2001), 161–182.

[19] M. Kataoka: Effect of public investment on the regional economies in postwar japan. *Review of Urban and Regional Development Studies*, **17(2)** (2005), 115–139.

[20] H.H. Kelejian and D.P. Robinson: Infrastructure productivity estimation and its underlying econometric specifications: A sensitivity analysis. *Papers in Regional Science*, **76** (1997), 115–131.

[21] S.V. Lall: Infrastructure and regional growth, growth dynamics and policy relevance for India. *Annals of Regional Science*, **41(3)** (2007), 581–599.

[22] C.J. Morrison and A.E. Schwartz: State infrastructure and productive performance. *American Economic Review*, **86(5)** (1996), 1095–1111.

[23] C.J. Morrison and A.E. Schwartz: Public infrastructure, private input demand, and economic performance in New England manufacturing. *Journal of Business and Economic Statistics*, **14** (1996), 91–101.

[24] C.J. Morrison and A.E. Schwartz: Distinguishing external from internal scale effects: the case of public infrastructure. *Journal of Productivity Analysis*, **5** (1994), 249–70.

[25] A.H. Munnell: Why has productivity declined? productivity and public investment. *New England Economic Review*, 1990, January/February, 3–22.

[26] A.H. Munnell and L.M. Cook: How does public infrastructure affect regional economic performance? *New England Economic Review*, 1990, September/October, 11–33.

[27] A.H. Munnell and L.M. Cook: How does public infrastructure affect regional economic performance? In A.H. Munnell (ed.): *Is There a Shortfall in Public Capital Investment?* (Federal Reserve Bank of Boston, Boston, 1990), 69–112.

[28] J. Nemoto and M. Goto: Dynamic data envelopment analysis: modeling intertemporal behavior of a birm in the presence of productive inefficiencies. *Economic Letters*, **64** (1999), 51–56.

[29] M.D. Ramirez: A panel unit root and panel cointegration test of the complementarity hypothesis in the Mexican case: 1960-2001. *Atlantic Economic Journal*, **35(3)** (2007), 343–356.

[30] F.P. Ramsey: A mathematical theory of saving. *Economic Journal*, **38** (1928), 543–559.

[31] E. Rodriguez-Oreggia: Winners and losers of regional growth in Mexico and their dynamics. *Investigacion Economica*, **66(259)** (2007), 43–62.

[32] E. Shioji: Public capital and economic growth: a convergence approach. *Journal of Economic Growth*, **6(3)** (2001), 205–227.

[33] L.L. Taylor and S.P.A. Brown: The private sector impact of state and local government: has more become bad? *Contemporary Economic Policy*, **24(4)** (2006), 548–62.

[34] S.M. Thangavelu and D.T. Owyong: Impact of public capital on the manufacturing productive performance of Japanese industries. *Applied Economics*, **32(12)** (2000), 1515–1520.

[35] U.S. Department of Commerce: *Fixed Reproducible Tangible Wealth in the United States, 1925-94* (US Government Printing Office, Washington, DC, 1999).

Shawna Grosskopf
303 Ballard
Department of Economics
Oregon State University
Corvallis, OR 97331, USA
E-mail: `shawna.grosskopf@orst.edu`