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OPTIMAL COOPERATIVE HARVESTING PATTERNS OF AGRICULTURAL FRESH PRODUCTS IN CASE OF MULTIPLE FARMERS AND MULTIPLE MARKETS UNDER PERIODICAL FLOWERING

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Abstract How to make a coordination policy between suppliers is one of key issues in Supply Chain Management. This paper deals with a case where multiple farmers harvest and deliver agricultural fresh products to multiple markets in proportion to each market size provided that the plants related to the fresh products get flowering periodically like tropical fruits such as papaya. A cooperative model is formulated in a mathematical form to obtain the optimal harvesting patterns for multiple farmers who harvest the fresh products cooperatively to maximize the consumption level of fresh products daily used in multiple markets. Although this model becomes a kind of mixed integer linear programming problem hard to solve in general, this paper reduces it into a simple LP problem easy to solve, exploiting some properties of optimal harvesting patterns analytically obtained in an individual un-cooperative model. Numerical analyses provide optimal harvesting patterns for cooperative multiple farmers, and make it clear that the cooperation effect depends on the delivery lead times from multiple farmers to multiple markets and the shift periods between flowering cycles among farmers. In a two-farm, two-market model, it is also shown that the increment of the consumption level of fresh products in the cooperative model compared with that in the individual un-cooperative model becomes largest when the shift period between flowering cycles among two farmers is just half a flowering cycle.

Keywords: Mathematical modeling, linear programming, supply chain management

1. Introduction

Supply chain of agricultural fresh products has the following properties which are distinguished from the usual industrial products: (1) Plant flowering and maturing process depends on the climate and the other natural phenomena which are hardly controlled artificially. The amount of fresh products harvestable at any time depends on this natural factors. (2) Deterioration process of fresh products starts just after harvested, and the deterioration rate depends on the circumstances where the fresh products are dealt with. Transportation from farms to markets and carrying inventory in markets are the major causes of such deterioration of the fresh products before consumption.

There are a lot of papers dealing with perishable products. Wee[10] classified the types of deterioration into decay, damage, spoilage, evaporation, obsolescence, pilferage and loss of utility (or value) of commodity in lot size modeling. Misra[5], Mak[4], Raafat et al.[6], Yang and Wee[14, 15], Skouri and Papachristos[7] and Lin and Lin[3] discussed production-inventory policies for perishable items. Dye et al.[2] developed a lot size model with varying rate of deterioration. These papers assume that deterioration process is independent of manufacturing history of each item and that the deterioration amount is proportional to the current amount of each item, which is suitable only to radioactive material. Deterioration process of agricultural products does not follow this kind of deterioration but depends on the time when it is harvested.

Cheng et al.[1], Wang[8] and Wang and Cheng[9] provided some scheduling models with deterioration jobs caused by frequent product changes. The deterioration process is assumed to be proportional to historical time duration, which is independent of manufacturing history of each item.

Widodo et al.[11–13] formulated a basic model for harvesting-delivering agricultural fresh products under periodical flowering, and derived an optimal harvesting pattern to a single-farm, single-market model and to a single-farm, multiple-market model analytically. In supply chain management for agricultural fresh products, it is crucial to analyze the cooperation effect between multiple farmers, because farms are usually located in suburban area and have several markets with different market-sizes where farmers can supply their fresh agricultural products. If they pursue their own profit only, some markets may get shortage in daily consumption of fresh products and excess inventory of fresh products in some markets could cause a lot of agricultural products disposed after quick deterioration. Recent development of transportation measures such as cheap local air freights and frequent express truck delivery reduce the geographic distance more than ever, and each farm can supply their fresh products to more markets. If the optimal harvesting pattern in the multiple-market and multiple-farmer model is obtained, the coordination methods among farmers can be proposed on the basis of the result. This kind of analysis, however, has not been implemented yet because the model becomes too complicated to solve.

Therefore, we formulate first an individual un-cooperative farm model with multiple-markets, and derive its optimal harvesting pattern. Using properties of this optimal harvesting pattern, we introduce new variables to formulate the cooperative farm model, and reduce it into a simple, tractable linear programming problem. Some analyses are implemented by solving the final LP problem, showing that the cooperation effect depends on the combination of lead times from farms to markets and also on shift periods between flowering cycles among multiple farms.

2. Model Conditions

We construct a periodical flowering-harvesting model of fresh agricultural products with multiple farms and multiple markets under the following conditions:

(1) Fresh products are harvested in multiple farms, denoted by $\mathcal{Q} \equiv \{1, 2, \cdots, Q(= |\mathcal{Q}|)\}$, and delivered to multiple markets, denoted by $\mathcal{M} \equiv \{1, 2, \cdots, M(= |\mathcal{M}|)\}$.

(2) Flowering occurs every F periods in each farm, but the starting time of flowering in farm q is shifted by ΔF_q from the starting time in the first farm. For simplicity, it is assumed that $\Delta F_1 = 0 \leq \Delta F_2 \leq \Delta F_3 \leq \cdots \leq \Delta F_Q < F$.

(3) Maturing curve of the plant in farm q is given by $P_q u_q(i)$, $0 \leq i \leq n_q$, as shown in Figure 1 where flowering occurs at period 0 and the maturing process ends at $n_1 = 23$. The maximum value of end periods n_q, $q \in \mathcal{Q}$, is given by $n \equiv \max_{q\in\mathcal{Q}} n_q$. Each maturing curve is quasi-concave in period i after flowering occurs, and has a peak given by $P_q u_q(\widehat{k})$. The notation P_q stands for the maximum amount (weight × unit) of fresh products and $u_q(i)$ denotes the normalized maturing curve.

(4) Fresh products are harvested at an amount of $X_q(i)$ at period i in farm q, and the sequence, $X_q(i)$, $0 \leq i \leq n_q$, is called "harvesting pattern" and is the same for each flowering in farm q. The harvesting pattern expresses when and how much fresh products

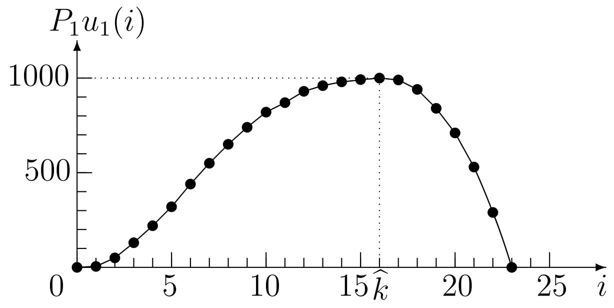


Figure 1: Plant maturing curve $P_1u_1(i)$

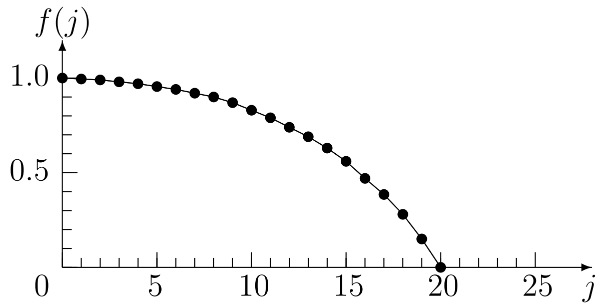


Figure 2: Deterioration curve $f(j)$

should be harvested over the maturing periods in each farm. This harvesting pattern is repeated every F periods in each flowering cycle in farm q. The optimal harvesting pattern, denoted by $X_q^*(i)$, $0 \le i \le n_q$, $q \in \mathcal{Q}$, is unique for each farm and usually different from each other. All fresh products harvested at each period are immediately shipped bound for each market directly from each farm. Letting $X_{qm}(i)$ be the amount of fresh products harvested in farm q and shipped bound for market m at period i, we get $X_q(i) = \sum_{m\in\mathcal{M}} X_{qm}(i)$ and $X_q^*(i) = \sum_{m\in\mathcal{M}} X_{qm}^*(i)$, where $X_{qm}^*(i)$ means the optimal value of $X_{qm}(i)$.

(5) Any requirement for harvesting fresh products should be met through the earliest plant maturing as possible in each farm, called "Earliest Flowering First(EFF) Rule." Any demand should be satisfied through the fresh products supplied by the earliest harvest, called "Earliest Harvest First(EHF) Rule."

(6) Deterioration curve $f(j)$, $0 \le j$, as shown in Figure 2, is monotone decreasing in duration j periods after the related fresh products are harvested, and independent of harvesting period. The deterioration curves are the same for any fresh products harvested in any farm.

(7) Fresh products are supplied from multiple farms for daily consumption in each market, and the amount of fresh products consumed in market m is guaranteed to be the same every period, called "daily consumption level," denoted by D_m, $m \in \mathcal{M}$, in proportion to the relative market size $\gamma_m(> 0)$, $m \in \mathcal{M}$, $\sum_{m\in\mathcal{M}} \gamma_m = 1$.

(8) Lead times L_{qm}, $q \in \mathcal{Q}$, $m \in \mathcal{M}$, for delivering fresh products from farm q to market m are all given.

(9) Fresh products harvested at period k in farm q can be carried for more than one period as on-hand inventory in market m at the end of period $L_{qm} + i(k \le i)$, denoted by $s_{qm}(i,k)$.

The objective is to maximize the daily consumption level to be satisfied in multiple markets every period through periodical flowering in multiple farms under the above conditions.

3. Individual Un-cooperative Farm Model

We first consider an individual un-cooperative farm model. Exploiting the results for a single-farm, multiple-farm model with unrestricted inventory[11, 12], we can derive the following linear programming problem to maximize the total daily consumption level in multiple markets in proportion to each market size. In this model, farms are considered independently without cooperation and the shift period ΔF_q in assumption (2) is not necessary to consider any more.

$$\text{maximize} \ \sum_{q\in\mathcal{Q}} D_{(q)}, \tag{3.1}$$

$$\text{subject to} \ \sum_{i=K_q-F}^{K_q} \frac{X_q(i)}{u_q(i)} = P_q, \ q \in \mathcal{Q}, \tag{3.2}$$

$$X_q(i) = \sum_{m\in\mathcal{M}} X_{qm}(i), \ K_q - F \le i \le K_q, \ q \in \mathcal{Q}, \tag{3.3}$$

$$D_{qm} = \gamma_m D_{(q)}, \ m \in \mathcal{M}, \ q \in \mathcal{Q}, \tag{3.4}$$

$$\begin{aligned} D_{qm} + s_{qm}(K_q - F, K_q - F) &= \{X_{qm}(K_q - F) + X_{qm}(K_q)\} f(L_{qm}) \\ &\quad + \sum_{k=K_q-F}^{K_q-1} s_{qm}(K_q - 1, k) \frac{f(L_{qm} + K_q - k)}{f(L_{qm} + K_q - k - 1)}, \\ &\qquad m \in \mathcal{M}, \ q \in \mathcal{Q}, \end{aligned} \tag{3.5}$$

$$\begin{aligned} D_{qm} + \sum_{k=K_q-F}^{i} s_{qm}(i, k) = X_{qm}(i) f(L_{qm}) + \sum_{k=K_q-F}^{i-1} s_{qm}(i-1, k) \frac{f(L_{qm} + i - k)}{f(L_{qm} + i - k - 1)}, \\ K_q - F < i \le K_q - 1, \ m \in \mathcal{M}, \ q \in \mathcal{Q}, \end{aligned} \tag{3.6}$$

$$\begin{aligned} X_{qm}(k) \ge \frac{s_{qm}(k, k)}{f(L_{qm})} \ge \frac{s_{qm}(k+1, k)}{f(L_{qm} + 1)} \ge \cdots \ge \frac{s_{qm}(K_q - 1, k)}{f(L_{qm} + K_q - k - 1)}, \\ K_q - F \le k \le K_q - 1, \ m \in \mathcal{M}, \ q \in \mathcal{Q}, \end{aligned} \tag{3.7}$$

$$X_q(i) \ge 0, \ X_{qm}(i) \ge 0, \ K_q - F \le i \le K_q, \ m \in \mathcal{M}, \ q \in \mathcal{Q}, \tag{3.8}$$

$$s_{qm}(j, i) \ge 0, \ K_q - F \le i \le j \le K_q - 1, \ m \in \mathcal{M}, \ q \in \mathcal{Q}, \tag{3.9}$$

$$D_{(q)} \ge 0, \ D_{qm} \ge 0, \ m \in \mathcal{M}, \ q \in \mathcal{Q}, \tag{3.10}$$

$$F + 1 \le K_q \le n_q + F - 1, \ q \in \mathcal{Q}. \tag{3.11}$$

Maturing curve given in assumption (3) is introduced in equation (3.2), where the total amount of fresh product harvested in each flowering cycle is restricted by P_q in farm q. The Earliest Harvest First(EHF) Rule and the Earliest Flowering First(EFF) Rule in assumption (5) are considered in equations (3.5) and (3.6). In equation (3.5), $X_{qm}(K_q - F)$ is the amount of fresh product harvested at period $K_q - F$ in the next flowering cycle, and period K in the current flowering cycle and period $K_q - F$ in the next flowering cycle are overlapped in this period. The deterioration curve in assumption (6) is incorporated in equations (3.5)$\sim$(3.7), where $s_{qm}(i-1, k)/f(L_{qm} + i - k - 1)$ denotes the amount of fresh products harvested at period k in farm q and corresponding to on-hand inventory at period $L_{qm} + i - 1$ in market m. Assumption (7) is considered in equation (3.4).

Decision variables are $X_q(\,\cdot\,)$, $X_{qm}(\,\cdot\,)$, $s_{qm}(\,\cdot\,,\,\cdot\,)$, $D_{(q)}$, D_{qm} and K_q, where F, n_q, $f(\,\cdot\,)$, P_q and $u_q(\,\cdot\,)$ are given constants. In this problem, we set $u_q(i) = \varepsilon$ for $i = 0$ and $i \ge n_q$ instead of setting $u_q(i) = 0$ so that equation (3.2) does not become meaningless, where ε is a sufficiently small positive value. The range of K_q is specified so that it becomes possible to harvest whole fresh products at period $n_q - 1$ for satisfying daily consumption in each market, that is, $X_q(n_q - 1) = P_q u_q(n_q - 1)$, and $K_q - F = n_q - 1$(or $K_q = n_q + F - 1$) becomes possible. It is noticeable that since K_q is one of decision variables, the above problem becomes an intractable mixed integer linear programming problem.

Substituting equation (3.3) into equation (3.2), and introducing new variables P_{qm}, we can derive the following two-phase problem equivalent to the above.

$$\text{maximize} \sum_{q\in\mathcal{Q}} D_{(q)}, \tag{3.12}$$

$$\text{subject to} \sum_{m\in\mathcal{M}} P_{qm} = P_q,\ q\in\mathcal{Q}, \tag{3.13}$$

$$D_{(q)} \le D^*_{qm}(P_{qm})/\gamma_m,\ m\in\mathcal{M},\ q\in\mathcal{Q}, \tag{3.14}$$

$$P_{qm} \ge 0,\ m\in\mathcal{M},\ q\in\mathcal{Q}, \tag{3.15}$$

$$D_{(q)} \ge 0,\ q\in\mathcal{Q}, \tag{3.16}$$

$$D^*_{qm}(P_{qm}) \equiv \text{maximize } D_{qm}, \tag{3.17}$$

$$\text{subject to} \sum_{i=K_q-F}^{K_q} \frac{X_{qm}(i)}{u_q(i)} = P_{qm}, \tag{3.18}$$

$$D_{qm} + s_{qm}(K_q-F, K_q-F) = \{X_{qm}(K_q-F) + X_{qm}(K_q)\} f(L_{qm}) + \sum_{k=K_q-F}^{K_q-1} s_{qm}(K_q-1,k)\frac{f(L_{qm}+K_q-k)}{f(L_{qm}+K_q-k-1)}, \tag{3.19}$$

$$D_{qm} + \sum_{k=K_q-F}^{i} s_{qm}(i,k) = X_{qm}(i) f(L_{qm}) + \sum_{k=K_q-F}^{i-1} s_{qm}(i-1,k)\frac{f(L_{qm}+i-k)}{f(L_{qm}+i-k-1)},\quad K_q-F < i \le K_q-1, \tag{3.20}$$

$$X_{qm}(k) \ge \frac{s_{qm}(k,k)}{f(L_{qm})} \ge \frac{s_{qm}(k+1,k)}{f(L_{qm}+1)} \ge \cdots \ge \frac{s_{qm}(K_q-1,k)}{f(L_{qm}+K_q-k-1)},\quad K_q-F \le k \le K_q-1, \tag{3.21}$$

$$X_{qm}(i) \ge 0,\ K_q-F \le i \le K_q, \tag{3.22}$$

$$s_{qm}(j,i) \ge 0,\ K_q-F \le i \le j \le K_q-1, \tag{3.23}$$

$$D_{qm} \ge 0, \tag{3.24}$$

$$F+1 \le K_q \le n_q+F-1. \tag{3.25}$$

This problem consists of two parts: the main problem defined by equations (3.12) through (3.17) and the subproblem defined by equations (3.17) through (3.25). The subproblem provides an optimal harvesting pattern in farm q for market m under the maturing curve with the maximum value P_{qm} specified by the main problem. The main problem finds an optimal decomposition of $\boldsymbol{P}_q \equiv (P_{q1}, P_{q2}, \cdots, P_{qM})$ to maximize the daily consumption level in proportion to each market size.

Widodo et al.[11, 12] derived an optimal harvesting pattern, $X^*_{qm}(K^*_q-F+1)$, $X^*_{qm}(K^*_q-F+2)$, $\cdots$, $X^*_{qm}(K^*_q)$, to the above subproblem as follows.

$$\ell^*_{qm(i)} \equiv \arg\min\left\{ \min_{0\le\ell\le i-K^*_q+F-1} \frac{1}{u_q(i-\ell)f(L_{qm}+\ell)},\ \min_{i-K^*_q+F\le\ell\le F-1} \frac{1}{u_q(i+F-\ell)f(L_{qm}+\ell)} \right\},\quad K^*_q-F+1 \le i \le K^*_q, \tag{3.26}$$

$$
\begin{aligned}
k^*_{qm(i)} &\equiv i - \ell^*_{qm(i)} + F\delta(K^*_q - F + 1 - i + \ell^*_{qm(i)}),\ K^*_q - F + 1 \le i \le K^*_q, && (3.27)\\
\delta(x) &= 1,\ \text{if } x > 0;\ 0,\ \text{otherwise,}\\
I^1_{qm}(\ell) &\equiv \left\{k^*_{qm(i)}\ \middle|\ \ell^*_{qm(i)} = \ell,\ K^*_q - F + 1 \le i \le K^*_q\right\},\ 0 \le \ell \le F - 1, && (3.28)\\
I^2_{qm}(i) &\equiv \left\{\ell^*_{qm(j)}\ \middle|\ k^*_{qm(j)} = i,\ K^*_q - F + 1 \le j \le K^*_q\right\},\\
&\qquad\qquad K^*_q - F + 1 \le i \le K^*_q, && (3.29)\\
D^*_{qm}(P_{qm}) &\equiv P_{qm}\beta_{qm}, && (3.30)\\
\beta_{qm} &\equiv \left\{\sum_{\ell=0}^{F-1} \frac{1}{f(L_{qm}+\ell)} \sum_{i\in I^1_{qm}(\ell)} \frac{1}{u_q(i)}\right\}^{-1}, && (3.31)\\
X^*_{qm}(i) &\equiv \sum_{\ell\in I^2_{qm}(i)} \frac{D^*_{qm}}{f(L_{qm}+\ell)},\ K^*_q - F + 1 \le i \le K^*_q, && (3.32)
\end{aligned}
$$

where K^*_q is obtained by letting $u_q(K^*_q - F + 1)$, $u_q(K^*_q - F + 2)$, $\cdots$, $u_q(K^*_q)$ be the largest F values from among $u_q(i)$, $0 < i < n_q$, for farm q. In equation (3.26), the notation ℓ denotes how long the fresh products is carried as on-hand inventory in market m to satisfy demand at period $L_{qm} + i$. When ℓ exceeds $i - (K^*_q - F + 1)$, daily consumption at period $L_{qm} + i$ corresponding to the current flowering should be satisfied by some harvest in the previous flowering. In other words, some harvest in the current flowering satisfies a part of daily consumption corresponding to the next flowering. In this meaning, the range of $[K^*_q - F + 1,\ K^*_q]$ denotes the range within which an optimal harvesting pattern for farm q should be determined. The optimal harvesting pattern in farm q is repeated such as $X^*_{qm}(K^*_q - F + 1)$, $X^*_{qm}(K^*_q - F + 2)$, $\cdots$, $X^*_{qm}(K^*_q)$, $X^*_{qm}(K^*_q - F + 1)$, $X^*_{qm}(K^*_q - F + 2)$, $\cdots$, and is independent of the optimal harvesting pattern in the other farm. Each optimal harvesting pattern provides the part of an optimal daily consumption level in multiple markets, D^*_{qm}, $m \in \mathcal{M}$, which composes the total optimal daily consumption level in each market, that is, $D^*_m = \sum_{q\in\mathcal{Q}} D^*_{qm}$.

Since the main problem is additive, we can solve it by maximizing $D_{(q)}$ subject to the related constraints with respect to farm q independently. Substituting equation (3.30) into equations (3.13) and (3.14), we get $D_{(q)} \sum_{m\in\mathcal{M}} \gamma_m/\beta_{qm} \le P_q$ and the optimal solution to the main problem as follows.

$$
\begin{aligned}
D^*_{(q)} &\equiv P_q \left\{\sum_{m\in\mathcal{M}} \frac{\gamma_m}{\beta_{qm}}\right\}^{-1},\ q \in \mathcal{Q}, && (3.33)\\
P^*_{qm} &\equiv \frac{\gamma_m D^*_{(q)}}{\beta_{qm}},\ m \in \mathcal{M},\ q \in \mathcal{Q}. && (3.34)
\end{aligned}
$$

Consequently, we can reduce the original problem defined by equations (3.1) to (3.11) into the following problem by introducing new variables $X_{qm}(j, i)$, the amount of fresh products harvested at farm q at period i to satisfy daily consumption in market m at period $L_{qm} + j$, $i \le j$, meaning that fresh products harvested at period i in farm q, transported to market m with lead time L_{qm} and carried as on-hand inventory in the market for $j - i$ periods before consumption.

$$
\text{maximize } \sum_{q\in\mathcal{Q}} D_{(q)}, \qquad (3.35)
$$

$$
\text{subject to } \sum_{i=K^*_q-F+1}^{K^*_q} \frac{X_{qm}(i)}{u_q(i)} = P_{qm},\ m \in \mathcal{M},\ q \in \mathcal{Q}, \qquad (3.36)
$$

$$\sum_{m\in\mathcal{M}} P_{qm} = P_q,\ q\in\mathcal{Q}, \tag{3.37}$$

$$D_{qm} = \gamma_m D_{(q)},\ m\in\mathcal{M},\ q\in\mathcal{Q}, \tag{3.38}$$

$$X_{qm}(i) = \sum_{\ell\in I^2_{qm}(i)} X_{qm}(i+\ell, i),\ K_q^* - F + 1 \le i \le K_q^*,\ m\in\mathcal{M},\ q\in\mathcal{Q}, \tag{3.39}$$

$$X_{qm}(j,i) = X_{qm}(j, i-F),\ K_q^* - F + 1 \le j < i \le K_q^*,\ m\in\mathcal{M},\ q\in\mathcal{Q}, \tag{3.40}$$

$$D_{qm} = X_{qm}(j, j-\ell_{qm(j)}) f(L_{qm} + \ell_{qm(j)}),$$
$$K_q^* - F + 1 \le j \le K_q^*,\ m\in\mathcal{M},\ q\in\mathcal{Q}, \tag{3.41}$$

$$X_{qm}(i) \ge 0,\ K_q^* - F + 1 \le i \le K_q^*,\ m\in\mathcal{M},\ q\in\mathcal{Q}, \tag{3.42}$$

$$X_{qm}(j,i) \ge 0,\ K_q^* - F + 1 \le i \le j \le K_q^*,\ m\in\mathcal{M},\ q\in\mathcal{Q}, \tag{3.43}$$

$$D_{(q)} \ge 0,\ \ D_{qm} \ge 0,\ P_{qm} \ge 0,\ m\in\mathcal{M},\ q\in\mathcal{Q}, \tag{3.44}$$

$$0 \le \ell_{qm(j)} \le F - 1,\ K_q^* - F + 1 \le j \le K_q^*,\ m\in\mathcal{M},\ q\in\mathcal{Q}. \tag{3.45}$$

Decision variables are $X_{qm}(\,\cdot\,)$, $X_{qm}(\,\cdot\,,\,\cdot\,)$, $\ell_{qm(\,\cdot\,)}$, P_{qm}, $D_{(q)}$ and D_{qm}. In the above un-cooperative model, the optimal harvesting pattern is obtained by setting $\ell_{qm(j)} = \ell^*_{qm(j)}$, $K_q^* - F + 1 \le j \le K_q^*$, $m\in\mathcal{M}$, $q\in\mathcal{Q}$, because consumption level is maximized when the fresh products are harvested at period $k^*_{qm(i)}$(or $i - \ell^*_{qm(i)}$), immediately transported to market m and carried as on-hand inventory for $\ell^*_{qm(i)}$ periods before consumption at period $i + L_{qm}$ in market m. If farms are cooperative to maximize the daily consumption level in multiple markets, all these combinations of $(i,\ \ell^*_{qm(i)})$, $K_q^* - F + 1 \le i \le K_q^*$, can be considered to find optimal harvesting patterns for all farms, but we have to determine which pair of $(i,\ \ell^*_{qm(i)})$ should be selected for constructing optimal harvesting pattern.

4. Cooperative Farm Model

According to the property of the optimal solution given in equations (3.26) to (3.32) for the individual un-cooperative farm model, it is obvious that the optimal harvesting pattern is obtained during the duration $[K_q^* - F + 1,\ K_q^*]$ for each flowering in farm q, $q\in\mathcal{Q}$, resulting in the optimal supply cycle of F periods in each farm. Using the similar expression to the above model defined by equations (3.35) to (3.45), we formulate the cooperative farm model as follows.

$$\text{maximize } D, \tag{4.1}$$

$$\text{subject to } \sum_{j=K_q^*-F+1}^{K_q^*} \frac{X_{qm}(j)}{u_q(j)} = P_{qm},\ \ m\in\mathcal{M},\ \ q\in\mathcal{Q}, \tag{4.2}$$

$$\sum_{m\in\mathcal{M}} P_{qm} = P_q,\ \ q\in\mathcal{Q}, \tag{4.3}$$

$$D_m = \gamma_m D,\ m\in\mathcal{M}, \tag{4.4}$$

$$X_{qm}(j) = \sum_{\ell=0}^{F-1} X_{qm}(j+\ell, j),\ K_q^* - F + 1 \le j \le K_q^*,\ m\in\mathcal{M},\ q\in\mathcal{Q}, \tag{4.5}$$

$$D_m = \sum_{q\in\mathcal{Q}} \sum_{\ell=0}^{F-1} X_{qm}(i - L_{qm} - \Delta F_q, i - \ell - L_{qm} - \Delta F_q) f(L_{qm} + \ell),$$
$$K_{(m)} - F + 1 \le i \le K_{(m)},\ m\in\mathcal{M}, \tag{4.6}$$

$$X_{qm}(j + \ell - \tau F, j - \tau F) = X_{qm}(j+\ell, j) \text{ for any integer } \tau,$$
$$K_q^* - F + 1 \le j \le K_q^*,\ 0 \le \ell \le F - 1,\ m\in\mathcal{M},\ q\in\mathcal{Q}, \tag{4.7}$$

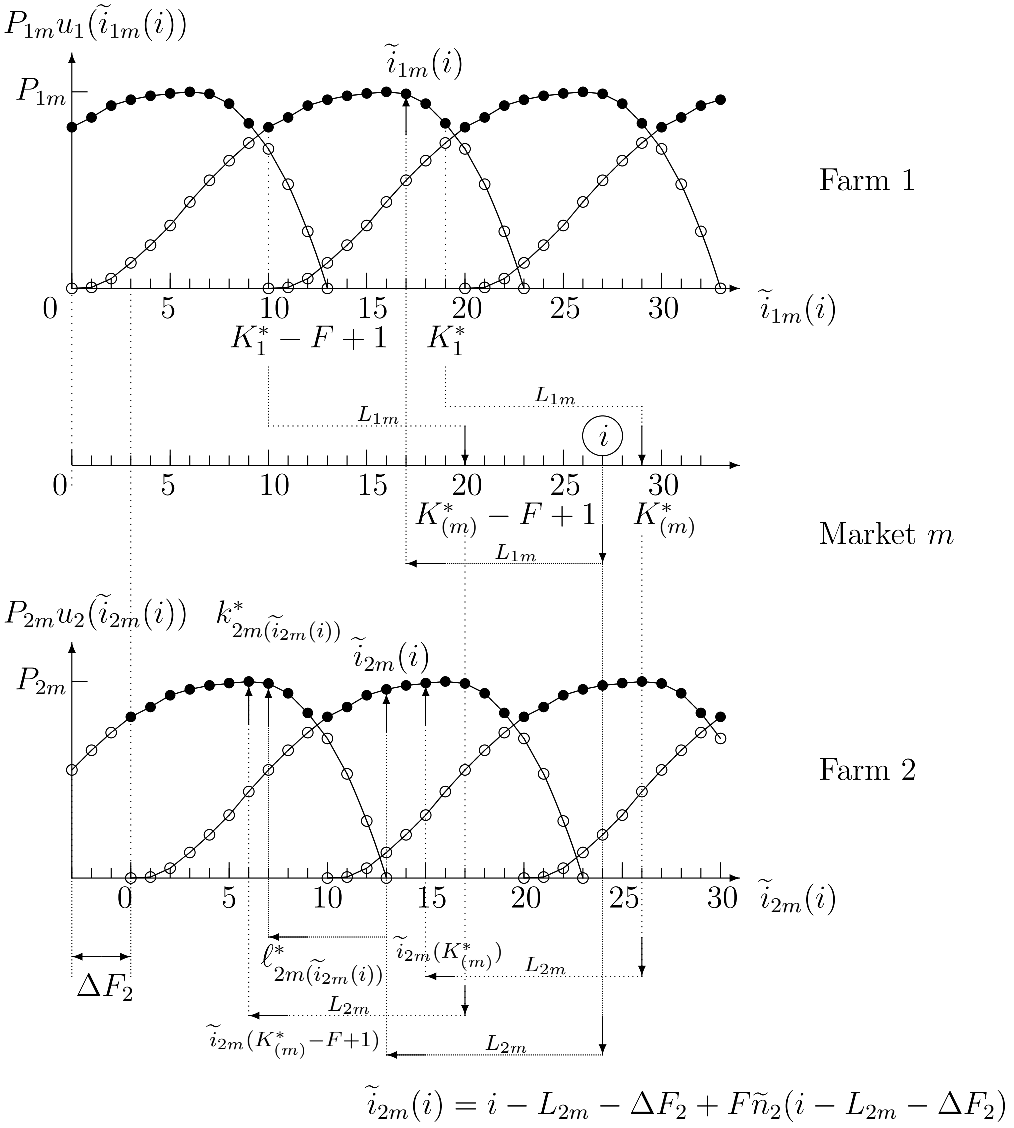


Figure 3: Relationships between periods in the markets and farms

$$X_{qm}(j) \geq 0,\ K_q^* - F \leq j \leq K_q^*,\ m \in \mathcal{M},\ q \in \mathcal{Q}, \tag{4.8}$$

$$X_{qm}(i - L_{mq} - \Delta F_q, i - \ell - L_{qm} - \Delta F_q) \geq 0,$$
$$K_{(m)} - F \leq i \leq K_{(m)},\ 0 \leq \ell \leq F - 1,\ m \in \mathcal{M},\ q \in \mathcal{Q}, \tag{4.9}$$

$$D \geq 0,\ D_m \geq 0,\ P_{qm} \geq 0,\ m \in \mathcal{M},\ q \in \mathcal{Q}, \tag{4.10}$$

$$F + 1 + L_{1m} \leq K_{(m)} \leq n + F - 1 + L_{1m},\ m \in \mathcal{M}, \tag{4.11}$$

where $n \equiv \max_{q \in \mathcal{Q}} n_q$, the maximum value of end periods in all farms.

In this model, assumption (2) is incorporated in equation (4.6) as shown in Figure 3. In equation (4.5), $X_{qm}(j + \ell, j)$ means the fresh products harvested at period j in farm q is immediately transferred to market m with lead time L_{qm} and kept for ℓ periods as on-hand inventory to satisfy demand at period $j + L_{qm} + \ell$ in the market. Since the origin of period in each farm is set at the time when flowering occurs at each farm, the origin can be different from each other if flowering occurs at different time period, as illustrated in Figure 3. Therefore, we set the origin of period in each market as the same origin as in Farm 1, and set $K_{(m)} = K_{(m)}^* \equiv K_1^* + L_{1m}$, $m \in \mathcal{M}$. In the above problem, $K_{(m)}$,

$m \in \mathcal{M}$, are also decision variables, but we can fix the value of $K_{(m)}$ arbitrarily because the consumption pattern in each market is also the same every flowering cycle from assumption (2), and because the consumption pattern does not depend on the end period $K_{(m)}$ unlike the relationships between the harvesting pattern and its end period, K_q^*. The origin of time period in each flowering cycle in farm q is shifted by ΔF_q from the origin in Farm 1 as shown in Figure 3. In equation (4.6), $X_{qm}(i - L_{qm} - \Delta F_q, i - \ell - L_{qm} - \Delta F_q)$ denotes the fresh products harvested at period $i - \ell - L_{qm} - \Delta F_q$ in farm q is immediately transferred to market m with lead time L_{qm} and kept for ℓ periods to be consumed at period i in market m. The term $f(L_{qm} + \ell)$ in equation (4.6) means the available amount of fresh products decreases to this fraction during the time interval of $L_{qm} + \ell$ after harvested. From the property of optimal harvesting pattern in individual un-cooperative farm model, we can say that the best combination of harvesting period and duration for carrying inventory is given by equations (3.26) and (3.27), considering the maturing curve and the deterioration curve. Therefore, we can replace equations (4.5) and (4.6) with the following equations:

$$X_{qm}(j) = \sum_{\ell \in I^2_{qm}(j)} X_{qm}(j + \ell, j),\ K_q^* - F + 1 \leq j \leq K_q^*,\ q \in \mathcal{Q} \tag{4.12}$$

$$D_m = \sum_{q \in \mathcal{Q}} X_{qm}(i - L_{qm} - \Delta F_q, i - \ell^*_{qm(i - L_{qm} - \Delta F_q)} - L_{qm} - \Delta F_q) f(L_{qm} + \ell^*_{qm(i - L_{qm} - \Delta F_q)}),$$
$$K^*_{(m)} - F + 1 \leq i \leq K^*_{(m)},\ m \in \mathcal{M}, \tag{4.13}$$

As illustrated in Figure 3, we introduce the following new variables $\widetilde{i}_{qm}(i)$, $m \in \mathcal{M}$, $q \in \mathcal{Q}$, to find the period in a suitable flowering cycle in farm q corresponding to period i in market m:

$$\widetilde{i}_{qm}(i) \equiv i - L_{qm} - \Delta F_q + F\widetilde{n}_q(i - L_{qm} - \Delta F_q), \tag{4.14}$$
$$\widetilde{n}_q(j) \equiv \min\{\, \widetilde{n} \mid K_q^* - F + 1 \leq j + \widetilde{n}F,\ \widetilde{n} \text{ is an integer.}\}. \tag{4.15}$$

Applying equation (4.14) to equation (4.13), and introducing new variables $x_{qm}(\cdot)$, we get

$$D_m = \sum_{q \in \mathcal{Q}} x_{qm}(i),\ K^*_{(m)} - F + 1 \leq i \leq K^*_{(m)},\ m \in \mathcal{M}, \tag{4.16}$$
$$x_{qm}(i) \equiv X_{qm}(\widetilde{i}_{qm}(i), \widetilde{i}_{qm}(i) - \ell^*_{qm(\widetilde{i}_{qm}(i))}) f(L_{qm} + \ell^*_{qm(\widetilde{i}_{qm}(i))}),\ m \in \mathcal{M},\ q \in \mathcal{Q}. \tag{4.17}$$

If $\widetilde{i}_{qm}(i) - \ell^*_{qm(\widetilde{i}_{qm}(i))} < K_q^* - F + 1$ in equation (4.17), we have to add $F\widetilde{n}_q(\widetilde{i}_{qm}(i) - \ell^*_{qm(\widetilde{i}_{qm}(i))})$ to both arguments in $X_{qm}(\widetilde{i}_{qm}(i), \widetilde{i}_{qm}(i) - \ell^*_{q(\widetilde{i}_{qm}(i))})$, but we omit this term from this equation for simplicity.

Substituting equations (4.12) and (4.17) into equation (3.36), we get

$$\begin{aligned}
\sum_{j=K_q^*-F+1}^{K_q^*} \frac{X_{qm}(j)}{u_q(j)} &= \sum_{j=K_q^*-F+1}^{K_q^*} \sum_{\ell \in I^2_{qm}(j)} \frac{X_{qm}(j+\ell, j)}{u_q(j)} \\
&= \sum_{i=K^*_{(m)}-F+1}^{K^*_{(m)}} \sum_{\ell \in I^2_{qm}(\widetilde{i}_{qm}(i))} \frac{X_{qm}(\widetilde{i}_{qm}(i) + \ell, \widetilde{i}_{qm}(i))}{u_q(\widetilde{i}_{qm}(i))} \\
&= \sum_{i=K^*_{(m)}-F+1}^{K^*_{(m)}} \frac{X_{qm}(\widetilde{i}_{qm}(i), \widetilde{i}_{qm}(i) - \ell^*_{qm(\widetilde{i}_{qm}(i))})}{u_q(\widetilde{i}_{qm}(i) - \ell^*_{qm(\widetilde{i}_{qm}(i))})}
\end{aligned}$$

$$\equiv \sum_{i=K^*_{(m)}-F+1}^{K^*_{(m)}} \widetilde{\alpha}_{qm}(i)x_{qm}(i),$$

$$\widetilde{\alpha}_{qm}(i) \equiv \frac{1}{u_q(\widetilde{i}_{qm}(i) - \ell^*_{qm(\widetilde{i}_{qm}(i))})f(L_{qm} + \ell^*_{qm(\widetilde{i}_{qm}(i))})},$$

$$K^*_{(m)} - F + 1 \leq i \leq K^*_{(m)},\ m \in \mathcal{M}. \tag{4.18}$$

To avoid any confusion, we describe the relationships between $\ell^*_{qm(\widetilde{i}_{qm}(i))}$ and $I^2_{qm}(\widetilde{i}_{qm}(i))$. The duration $\ell^*_{qm(\widetilde{i}_{qm}(i))}$ denotes how many periods the fresh products harvested in farm q are carried as on-hand inventory in market m before consumed at period i, that is, the corresponding fresh products are harvested at the adjusted period $\widetilde{i}_{qm}(i) - \ell^*_{qm(\widetilde{i}_{qm}(i))}$ in farm q, transported to each market with lead-time L_{qm} and carried for $\ell^*_{qm(\widetilde{i}_{qm}(i))}$ periods in each market. In other words, the fresh products to be consumed at period i in the market should be harvested at the adjusted period $\widetilde{i}_q(i)$ if we do not use any inventory carrying in the market, but $\ell^*_{qm(\widetilde{i}_{qm}(i))} > 0$ means that it is better to shift the harvesting period earlier by $\ell^*_{qm(\widetilde{i}_{qm}(i))}$ periods and to carry them for $\ell^*_{qm(\widetilde{i}_{qm}(i))}$ periods in market m. On the other hand, $I^2_{qm}(\widetilde{i}_{qm}(i))$ stands for the set of durations of inventory holding in market m with respect to the fresh products harvested together at the adjusted period $\widetilde{i}_{qm}(i)$ in farm q, that is, if $I^2_{qm}(\widetilde{i}_{qm}(i)) = \{\ell_1, \ell_2\}$(it is possible that $\ell_1 = 0$), then the fresh products obtained from multiple-harvest are transported together to market m with lead-time L_{qm} and some of them are carried for ℓ_1 periods and the others are carried for ℓ_2 periods before consumption.

Using these equations, we can finally derive the following problem equivalent to the original problem defined by equations (4.1) to (4.11):

$$\text{maximize } D, \tag{4.19}$$

$$\text{subject to } \sum_{m \in \mathcal{M}} P_{qm} = P_q,\ q \in \mathcal{Q}, \tag{4.20}$$

$$\sum_{i=K^*_{(m)}-F+1}^{K^*_{(m)}} \widetilde{\alpha}_{qm}(i)x_{qm}(i) \leq P_{qm},\ q \in \mathcal{Q},\ m \in \mathcal{M}, \tag{4.21}$$

$$\sum_{q \in \mathcal{Q}} x_{qm}(i) \geq \gamma_m D,\ K^*_{(m)} - F + 1 \leq i \leq K^*_{(m)},\ m \in \mathcal{M}, \tag{4.22}$$

$$D \geq 0,\ P_{qm} \geq 0,\ q \in \mathcal{Q},\ m \in \mathcal{M}, \tag{4.23}$$

$$x_{qm}(i) \geq 0,\ K^*_{(m)} - F + 1 \leq i \leq K^*_{(m)},\ q \in \mathcal{Q},\ m \in \mathcal{M}. \tag{4.24}$$

The decision variables are $x_{qm}(\cdot)$, P_{qm} and D. The above problem is very simple compared to the original problem which includes integer decision variables $K_{(m)}$, $m \in \mathcal{M}$, and a lot of periodic variables $X_{qm}(\cdot, \cdot)$. Letting $x^*_{qm}(i)$, $K^*_{(m)} - F + 1 \leq i \leq K^*_{(m)}$, be the optimal solution to the above problem, we finally get the optimal harvesting pattern as

$$X^*_q(j) = \sum_{m \in \mathcal{M}} X^*_{qm}(j),\ K^*_q - F + 1 \leq j \leq K^*_q,\ q \in \mathcal{Q}, \tag{4.25}$$

$$X^*_{qm}(j) = \sum_{\ell \in I^2_{qm}(j)} X^*_{qm}(j + \ell, j),\ K^*_q - F + 1 \leq j \leq K^*_q,\ m \in \mathcal{M},\ q \in \mathcal{Q}, \tag{4.26}$$

$$X_{qm}(j, j - \ell^*_{qm(j)}) = \frac{x^*_{qm}(j + L_{qm} + \Delta F_q)}{f(L_{qm} + \ell^*_{qm(j)})},\ K^*_q - F + 1 \leq j \leq K^*_q,\ m \in \mathcal{M},\ q \in \mathcal{Q}. \tag{4.27}$$

5. Numerical Analyses

We show some results of numerical analyses to evaluate the cooperation effects in terms of the relative increment of the optimal consumption level D^* by solving the above cooperative farm model with two farms and two markets and by comparing the results with those in the individual un-cooperative farm model.

Maturing curve and deterioration curve are given in Table 1 and Figures 1 and 2, where $P_1 = P_2 = 1000$. Flowering cycle is set as $F = 10$, and the largest ten values of $u_q(i)$ in Table 1 are obtained at periods 11 through 19, resulting in $K_1^* = K_2^* = 19$. In a symmetric case, the values of lead time L_{12} and L_{21} are both fixed as 3 and L_{11} and L_{22} are simultaneously varied from 0 to 6 by a step size of one. In an asymmetric case, the values of lead time from farm 2 to two markets are fixed as $L_{21} = 5$ and $L_{22} = 1$, and the values of lead time from Farm 1 to two markets (L_{11}, L_{12}) are varied from (0,6) to (6,0) by a step size of one on condition that $L_{11} = 6 - L_{12}$.

Optimal values of harvesting pattern $X^*_{qm}(j)$, consumption pattern $x^*_{qm}(j)$ and consumption level D^* in the symmetric case for $\Delta F_2 = 0$, 1, 3 and 5 are given in Tables 2 and 3, where the values in parentheses correspond to either the next or the previous flowering cycle. For example, in case of $\Delta F_2 = 0$, the optimal harvesting pattern is the same for both farms such as (0, 0, 96.7, 96.7, 96.7, 96.7, 96.7, 497.3, 0, 0) at periods 10 through 19, and this pattern is repeated every 10 periods for the succeeding flowering cycles. $X^*_{11}(12) = 96.7$ implies that the fresh products are harvested at period 12 in Farm 1 and immediately transferred to market 1 with lead time $L_{11} = 3$ to satisfy the daily consumption level of 94.7 at period 15 in market 1. The difference between 96.7 and 94.7 implies the loss caused by deterioration during transportation and carrying inventory. $X^*_{11}(19) = 497.3$ means the multiple-harvest to satisfy the daily consumption level for 5 periods, $i = 20$ through 24 in market 1. The deterioration loss for 5 periods is given by the difference between 497.3 and 94.7×5. It should be noted that $X^*_{21}(i)$ and $X^*_{22}(i)$ in case of $\Delta F_2 = 0$ are replaced with $X^*_{21}(i - \Delta F_2)$ and $X^*_{22}(i - \Delta F_2)$ if $\Delta F_2 > 0$ in Tables 2 and 3, since period i expresses the duration from flowering for each farm and since the value of i in Tables 2 and 3 stands for the period value in Farm 1. On the other hand, $x^*_{qm}(i + L_{1m})$ in Tables 2 and 3 means that fresh products harvested at some period earlier than or equal to period $i - \Delta F_q$ in Farm q are immediately transferred to Market m with lead time L_{qm}, carried for some periods(possibly zero period), and consumed at period $i + L_{1m}$(the origin of period is set at the flowering period in Farm 1), and the consumption volume of fresh products at period j in Market m is given by $x^*_{1m}(j) + x^*_{2m}(j)$ from Tables 2 and 3.

The values of optimal consumption level D^* are obtained in Tables 4 and 5 for the symmetric case and the asymmetric case, respectively. In these tables, we can find that the value of D^* in the un-cooperative model is the same as that in the cooperative model with $\Delta F_2 = 0$ for $L_{11} = L_{12} = L_{21} = L_{22} = 3$ in Table 4 and for $L_{11} = L_{21} = 5$ and $L_{12} = L_{22} = 1$ in Table 5. This is because both farms can not be distinguished in these cases from a view point of each market, as shown in Table 2 with $\Delta F_2 = 0$ where the optimal harvesting patterns are the same for both farms. The values of relative increment ratio compared with this un-cooperative solution are given in parentheses in Tables 4 and 5 and illustrated in Figure 4. In both cases, the cooperation effect increases as the shift period between flowering cycles in two farms increases, and becomes largest when $\Delta F_2 = 5$, just half a flowering cycle. Excluding the special case where both farms can not be distinguished from a view point of markets, the cooperation effect can be expected up to 1 to 4 %, which varies in a convex style as the lead time L_{11} increases in both symmetric and asymmetric

Table 1: Normalized maturing curve $u_q(i)$ and deterioration curve $f(i)$

i	0	1	2	3	4	5	6	7	8	9	10	11
$u_q(i)$	0.00	0.005	0.05	0.13	0.22	0.32	0.44	0.55	0.65	0.74	0.82	0.87
$f(i)$	1.00	0.995	0.99	0.98	0.97	0.955	0.94	0.92	0.90	0.87	0.83	0.79
i	12	13	14	15	16	17	18	19	20	21	22	23
$u_q(i)$	0.93	0.96	0.98	0.99	1.00	0.99	0.94	0.84	0.71	0.53	0.27	0.00
$f(i)$	0.74	0.69	0.63	0.56	0.47	0.385	0.28	0.15	0.00	0.00	0.00	0.00

Table 2: Values of $X^*_{qm}(i-\Delta F_q)$, $x^*_{qm}(i+L_{1m})$ and D^* ($L_{11}=L_{12}=L_{21}=L_{22}=3$, $\Delta F_1=0$)

Period i at Farm 1		10	11	12	13	14	15	16	17	18
	$X^*_{11}(i)$	0	0	96.7	96.7	96.7	96.7	96.7	497.3	0
ΔF_2	$X^*_{12}(i)$	0	0	0	0	0	0	0	0	0
$=0$	$X^*_{21}(i)$	0	0	0	0	0	0	0	0	0
	$X^*_{22}(i)$	0	0	96.7	96.7	96.7	96.7	96.7	497.3	0
	$x^*_{11}(i+3)$	(94.7)	(94.7)	94.7	94.7	94.7	94.7	94.7	94.7	94.7
D^*	$x^*_{12}(i+3)$	(0)	(0)	0	0	0	0	0	0	0
$=189.5$	$x^*_{21}(i+3)$	(0)	(0)	0	0	0	0	0	0	0
	$x^*_{22}(i+3)$	(94.7)	(94.7)	94.7	94.7	94.7	94.7	94.7	94.7	94.7
		19	20	21	22	23	24	25	26	27
	$X^*_{11}(i)$	0	(0)	(0)	(96.7)	(96.7)	(96.7)	(96.7)	(96.7)	(497.3)
	$X^*_{12}(i)$	0	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)
	$X^*_{21}(i)$	0	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)
	$X^*_{22}(i)$	0	(0)	(0)	(96.7)	(96.7)	(96.7)	(96.7)	(96.7)	(497.3)
	$x^*_{11}(i+3)$	94.7	94.7	94.7	(94.7)	(94.7)	(94.7)	(94.7)	(94.7)	(94.7)
	$x^*_{12}(i+3)$	0	0	0	(0)	(0)	(0)	(0)	(0)	(0)
	$x^*_{21}(i+3)$	0	0	0	(0)	(0)	(0)	(0)	(0)	(0)
	$x^*_{22}(i+3)$	94.7	94.7	94.7	(94.7)	(94.7)	(94.7)	(94.7)	(94.7)	(94.7)
Period i at Farm 1		10	11	12	13	14	15	16	17	18
	$X^*_{11}(i)$	0	0	97.4	97.4	97.4	97.4	97.4	0	0
ΔF_2	$X^*_{12}(i)$	0	0	94.9	97.4	97.4	97.4	97.4	0	0
$=1$	$X^*_{21}(i-1)$	(0)	0	0	0	0	0	0	97.4	397.3
	$X^*_{22}(i-1)$	(0)	0	0	0	0	0	0	97.4	399.9
	$x^*_{11}(i+3)$	0	0	95.4	95.4	95.4	95.4	95.4	0	0
D^*	$x^*_{12}(i+3)$	0	0	93.0	95.4	95.4	95.4	95.4	0	0
$=190.9$	$x^*_{21}(i+3)$	(95.4)	(95.4)	(0)	0	0	0	0	95.4	95.4
	$x^*_{22}(i+3)$	(95.4)	(95.4)	(2.4)	0	0	0	0	95.4	95.4
		19	20	21	22	23	24	25	26	27
	$X^*_{11}(i)$	0	(0)	(0)	(97.4)	(97.4)	(97.4)	(97.4)	(97.4)	(0)
	$X^*_{12}(i)$	0	(0)	(0)	(94.9)	(97.4)	(97.4)	(97.4)	(97.4)	(0)
	$X^*_{21}(i-1)$	0	0	(0)	(0)	(0)	(0)	(0)	(0)	(97.4)
	$X^*_{22}(i-1)$	0	0	(0)	(0)	(0)	(0)	(0)	(0)	(97.4)
	$x^*_{11}(i+3)$	0	0	0	(95.4)	(95.4)	(95.4)	(95.4)	(95.4)	(0)
	$x^*_{12}(i+3)$	0	0	0	(93.0)	(95.4)	(95.4)	(95.4)	(95.4)	(0)
	$x^*_{21}(i+3)$	95.4	95.4	95.4	0	(0)	(0)	(0)	(0)	(95.4)
	$x^*_{22}(i+3)$	95.4	95.4	95.4	2.4	(0)	(0)	(0)	(0)	(95.4)

Table 3: Values of $X^*_{qm}(i-\Delta F_q)$, $x^*_{qm}(i+L_{1m})$ and D^* ($L_{11}=L_{12}=L_{21}=L_{22}=3$, $\Delta F_1=0$)

Period i at Farm 1		10	11	12	13	14	15	16	17	18
	$X^*_{11}(i)$	0	0	0	98.4	98.4	98.4	98.4	98.4	0
ΔF_2	$X^*_{12}(i)$	0	0	0	98.4	98.4	98.4	98.4	97.9	0
$=3$	$X^*_{21}(i-3)$	(298.9)	(0)	(0)	0	0	0	0	0	98.4
	$X^*_{22}(i-3)$	(298.9)	(0)	(0)	0	0	0	0	0.5	98.4
	$x^*_{11}(i+3)$	0	0	0	96.5	96.5	96.5	96.5	96.5	0
D^*	$x^*_{12}(i+3)$	0	0	0	96.5	96.5	96.5	96.5	96.0	0
$=192.9$	$x^*_{21}(i+3)$	(96.5)	(96.5)	(96.5)	0	0	0	0	0	96.5
	$x^*_{22}(i+3)$	(96.5)	(96.5)	(96.5)	0	0	0	0	0.5	96.5
		19	20	21	22	23	24	25	26	27
	$X^*_{11}(i)$	0	(0)	(0)	(0)	(98.4)	(98.4)	(98.4)	(98.4)	(98.4)
	$X^*_{12}(i)$	0	(0)	(0)	(0)	(98.4)	(98.4)	(98.4)	(98.4)	(97.9)
	$X^*_{21}(i-3)$	98.4	298.9	0	0	(0)	(0)	(0)	(0)	(0)
	$X^*_{22}(i-3)$	98.4	298.9	0	0	(0)	(0)	(0)	(0)	(0.5)
	$x^*_{11}(i+3)$	0	(0)	(0)	(0)	(96.5)	(96.5)	(96.5)	(96.5)	(96.5)
	$x^*_{12}(i+3)$	0	(0)	(0)	(0)	(96.5)	(96.5)	(96.5)	(96.5)	(96.0)
	$x^*_{21}(i+3)$	96.5	96.5	96.5	96.5	(0)	(0)	(0)	(0)	(0)
	$x^*_{22}(i+3)$	96.5	96.5	96.5	96.5	(0)	(0)	(0)	(0)	(0.5)
Period i at Farm 1		10	11	12	13	14	15	16	17	18
	$X^*_{11}(i)$	0	0	0	0	98.8	98.8	98.8	198.6	0
ΔF_2	$X^*_{12}(i)$	0	0	0	0	98.8	98.8	98.8	198.6	0
$=5$	$X^*_{21}(i-5)$	(98.8)	(98.8)	(198.6)	(0)	(0)	0	0	0	0
	$X^*_{22}(i-5)$	(98.8)	(98.8)	(198.6)	(0)	(0)	0	0	0	0
	$x^*_{11}(i+3)$	0	0	0	0	96.8	96.8	96.8	96.8	96.8
D^*	$x^*_{12}(i+3)$	0	0	0	0	96.8	96.8	96.8	96.8	96.8
$=193.6$	$x^*_{21}(i+3)$	(96.8)	(96.8)	(96.8)	(96.8)	(0)	0	0	0	0
	$x^*_{22}(i+3)$	(96.8)	(96.8)	(96.8)	(96.8)	(0)	0	0	0	0
		19	20	21	22	23	24	25	26	27
	$X^*_{11}(i)$	0	(0)	(0)	(0)	(0)	(98.8)	(98.8)	(98.8)	(198.6)
	$X^*_{12}(i)$	0	(0)	(0)	(0)	(0)	(98.8)	(98.8)	(98.8)	(198.6)
	$X^*_{21}(i-5)$	98.8	98.8	98.8	198.6	0	0	(0)	(0)	(0)
	$X^*_{22}(i-5)$	98.8	98.8	98.8	198.6	0	0	(0)	(0)	(0)
	$x^*_{11}(i+3)$	0	(0)	(0)	(0)	(0)	(96.8)	(96.8)	(96.8)	(96.8)
	$x^*_{12}(i+3)$	0	(0)	(0)	(0)	(0)	(96.8)	(96.8)	(96.8)	(96.8)
	$x^*_{21}(i+3)$	96.8	96.8	96.8	96.8	96.8	0	(0)	(0)	(0)
	$x^*_{22}(i+3)$	96.8	96.8	96.8	96.8	96.8	0	(0)	(0)	(0)

cases.

From the above results, we can provide some conjectures for multiple-farm, multiple-market models with more than two farms and more than two markets, that is, (1) the cooperation effect increases as the difference of peak periods among maturing curves in multiple farms increases, and (2) the maximum effect can be obtained when the difference becomes largest. It should be noted that to prove these conjectures exactly in multiple-farm, multiple-market models is not easy and requires a vast simulation run because the cooperative effect is affected by the combination of delivery lead times from farms to markets.

Table 4: Value of D^* (the relative value % in parenthises) with respect to ΔF_2 and $L_{11}(= L_{22})$ $(L_{12} = L_{21} = 3)$

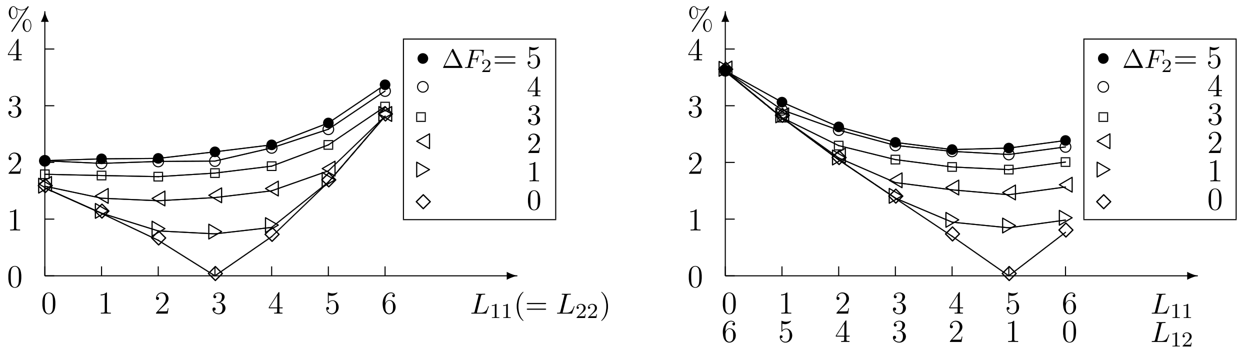
L_{11}, L_{22}	Un-Coop	$\Delta F_2 = 0$	$\Delta F_2 = 1$	$\Delta F_2 = 2$
0, 0	192.4	195.4 (1.550)	195.4 (1.550)	195.4 (1.581)
1, 1	191.6	193.7 (1.101)	193.7 (1.105)	194.2 (1.369)
2, 2	190.7	191.9 (0.627)	192.2 (0.792)	193.2 (1.324)
3, 3	189.5	189.5 (0.000)	190.9 (0.740)	192.1 (1.380)
4, 4	188.2	189.5 (0.691)	189.8 (0.854)	191.0 (1.499)
5, 5	186.4	189.5 (1.652)	189.5 (1.652)	189.9 (1.851)
6, 6	184.3	189.5 (2.814)	189.5 (2.814)	189.5 (2.814)
L_{11}, L_{22}	Un-Coop	$\Delta F_2 = 3$	$\Delta F_2 = 4$	$\Delta F_2 = 5$
0, 0	192.4	195.8 (1.791)	196.3 (2.029)	196.3 (2.031)
1, 1	191.6	194.9 (1.769)	195.3 (1.983)	195.5 (2.063)
2, 2	190.7	194.0 (1.749)	194.5 (2.017)	194.6 (2.071)
3, 3	189.5	192.9 (1.812)	193. 3(2.024)	193.6 (2.187)
4, 4	188.2	191.8 (1.934)	192.4 (2.256)	192.5 (2.310)
5, 5	186.4	190.7 (2.309)	191.2 (2.585)	191.4 (2.696)
6, 6	184.3	189.8 (2.985)	190.3 (3.259)	190.5 (3.370)

Table 5: Value of D^* (the relative value in parentheses) with respect to ΔF_2, L_{11} and L_{12} $(L_{21} = 5,\ L_{22} = 1)$

L_{11}, L_{12}	Un-Coop	$\Delta F_2 = 0$	$\Delta F_2 = 1$	$\Delta F_2 = 2$
0, 6	187.7	194.5 (3.603)	194.5 (3.609)	194.5 (3.613)
1, 5	188.4	193.7 (2.790)	193.7 (2.790)	193.7 (2.790)
2, 4	188.9	192.7 (2.045)	192.7 (2.048)	192.8 (2.080)
3, 3	188.9	191.5 (1.365)	191.5 (1.370)	192.1 (1.645)
4, 2	188.9	190.2 (0.698)	190.7 (0.947)	191.7 (1.516)
5, 1	188.4	188.4 (0.000)	190.0 (0.846)	191.1 (1.429)
6, 0	187.7	189.2 (0.767)	189.6 (0.983)	190.7 (1.565)
L_{11}, L_{12}	Un-Coop	$\Delta F_2 = 3$	$\Delta F_2 = 4$	$\Delta F_2 = 5$
0, 6	187.7	194.5 (3.616)	194.5 (3.618)	194.5 (3.616)
1, 5	188.4	193.7 (2.791)	193.9 (2.926)	194.2 (3.064)
2, 4	188.9	193.2 (2.296)	193.7 (2.570)	193.8 (2.627)
3, 3	188.9	192.8 (2.051)	193.3(2.299)	193.4 (2.353)
4, 2	188.9	192.5 (1.920)	193.0 (2.195)	193.1 (2.226)
5, 1	188.4	191.9 (1.873)	192.4 (2.146)	192.6 (2.252)
6, 0	187.7	191.5 (2.006)	192.0 (2.274)	192.2 (2.386)

6. Conclusions

Basic supply chain management models of agricultural fresh products under periodical flowering were constructed to express un-cooperation and cooperation between farmers in mathematical forms. Optimal harvesting pattern to maximize the daily consumption level in each market in proportion to its market size was derived analytically for the individual un-cooperative farm model. Since the cooperative farm model was too complicated to solve directly, it was converted into a simple linear programming problem by exploiting properties



(a) Symmetric case ($L_{12} = L_{21} = 3$) (b) Asymmetric case ($L_{21} = 5,\ L_{22} = 1$)

Figure 4: Relative increment ratio of D^* compared with un-cooperative case

of the optimal harvesting pattern in the un-cooperative model. Some numerical analyses for a two-farm, two-market model made it clear that the effect of cooperation between farmers became largest when the shift period between flowering cycles in two farms was just half a flowering cycle, and that the cooperation effect also depended on the combination of delivery lead times from farms to markets.

Some conjectures were provided by extending the results obtained from a two-farm, two-market model to multiple-farm, multiple-market models, as long as a plant has a usual property for its maturing curve, that is, quasi-concavity in period after periodical flowering, which is necessary for deriving the optimal harvesting pattern. Since the effect of cooperation among farmers depends on the combination of delivery lead times from farms to markets, it is difficult to show these conjectures hold precisely for any cases. Once the farms and markets are given, we can, however, find optimal harvesting patterns for all farms through the reduced simple linear problem derived in this paper. The maturing curve used in the numerical example is not a real curve but only a sample curve reflecting that a real maturing curve for any plant has the similar pattern such as flowering phase, growing phase, maturing phase and deteriorating phase. If flowering pattern of the plant is not periodical but seasonal, the criterion function should be changed into maximization of daily consumption level during a given interval of the related season. The optimal harvesting pattern derived by supposing the flowering pattern be periodical can be used as a near optimal pattern for generating an optimal harvesting pattern in seasonal flowering.

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