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DETERMINATION OF BOUNDS IN DEA ASSURANCE REGION METHOD — ITS APPLICATION TO EVALUATION OF BASEBALL PLAYERS AND CHEMICAL COMPANIES —

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Abstract In Data Envelopment Analysis (DEA) many optimal weights (multipliers) for inputs and outputs may become zeros. This means that corresponding inputs or outputs are neglected. To improve this shortcoming the assurance region methods which have bounds on the ratios of weights have been proposed. Deciding bounds depends on the data, and in some cases it requires judgments from experts. However, it is generally a difficult task to put their judgments into the quantitative bounds. We propose new methods by which the bounds are derived easily from limited information, i.e., partial ranking data. The methods are applied to the evaluation of baseball players and chemical companies.

Keywords: DEA, assurance region, baseball

1. Introduction

In Data Envelopment Analysis (DEA) many optimal weights (multipliers) for inputs and outputs may become zeros because the evaluated Decision Making Units (DMU) can obtain the efficiency score of 1 by neglecting inputs or outputs that are inferior to the inputs or outputs of other DMUs (See Cooper [3]). This means that if inputs or outputs showing the performances of DMUs are neglected, valuable information may consequently be lost.

To improve this shortcoming, the assurance region methods, which have bounds relating to weights were proposed (For example, Allen [1], Beasley [2], Dyson and Thanassoukis [4], Kornbluth [5], Roll et al.(1991) [6], Roll and Golany [7], Takamura and Tone [9], Ueda(2000) [10], Ueda(2007) [11]). However, Allen [1] states, “No method is all-purpose and different approaches may be appropriate in different contexts” and Dyson and Thanassoukis [4] states, “There is no single correct process for determining numerical values of bounds”.

We agree with these opinions and several researchers have proposed various methods of determining bounds. To determine bounds on weights, Dyson and Thanassoukis [4] discusses the use of regression analysis, Ueda(2000) [10] and Ueda(2007) [11]discuss the use of canonical correlation analysis, and to set upper and lower bounds on weights in the “bounded” formulation, Roll et al.(1991) [6] and Roll and Golany [7] use weights which were obtained from unbounded runs of DEA. Beasley [2] and Kornbluth [5] suggest the setting of bounds based on expert judgments, and Takamura and Tone [9] is a concrete realization of them. Takamura and Tone [9] proposed a method that decides bounds by utilizing the judgments of people who know well the characteristics of the evaluated objects. Quantification of bounds is accomplished by Saaty’s Analytic Hierarchy Process (AHP) (Saaty [8]) based on paired comparison results, but when the number of objects is M, $M(M-1)/2$ comparisons are needed and comparisons between unimportant objects are difficult in general. We can take fewer comparisons for rank order data than for paired comparison data and if ranking among unimportant objects can be avoided, ranking becomes easier.

In this paper we discuss cases where more important m $(< M)$ objects than others are ranked and propose a method which does not use the ranking of all objects and transforms the ranking data into positive real numbers. The proposed method is applied to the evaluation of batters in Nippon Professional Baseball and chemical companies.

2. Derivation of Importance Scores and Bounds on Ratios of Weights

We would like to know importance of M objects and we ask N persons to rank them. However, ordering all objects, especially ranking among unimportant objects, may be difficult. More important m $(< M)$ objects than others are ranked. We give score t_1 for the most favorite object, t_2 for the second favorite object,..., t_m for the m-th favorite object, and t_{m+1} for non-selected objects. Variant rankings are usually obtained from person to person. Let a score of person i and object j be e_{ij}. If person$i$ answers object 2 as the most favorite object and object 5 as the second favorite object, object 2 is given score t_1, that is, $e_{i2}= t_1$, and object 5 is given score t_2, that is, $e_{i5} = t_2$ (See Table 1). Because we would like to know ratios among t_i, let t_{m+1}=1 and log t_i is discussed as it becomes familiar with mean and variance. Differentiation among objects is realized through Maximization of the Variance Between Objects (MVBO), that is, the following formulation, MVBO1.

Table 1: Ranking data (a)

	rank1	rank2	rank3	rank4	rank5
person1	**2**	**5**	1	10	7
person2	2	1	5	10	3
Scoring image (b)					
person1	object 1	object 2	object 3	object 4	object 5
	$e_{11} = t_3$	$\boldsymbol{e_{12}=t_1}$	$e_{13} = t_6$	$e_{14} = t_6$	$\boldsymbol{e_{15}=t_2}$
	object 6	object 7	object 8	object 9	object 10
	$e_{16} = t_6$	$e_{17} = t_5$	$e_{18} = t_6$	$e_{19} = t_6$	$e_{1,10} = t_4$
person2	object 1	object 2	object 3	object 4	object 5
	$e_{21} = t_2$	$e_{22} = t_1$	$e_{23} = t_5$	$e_{24} = t_6$	$e_{25} = t_3$
	object 6	object 7	object 8	object 9	object 10
	$e_{26} = t_6$	$e_{27} = t_6$	$e_{28} = t_6$	$e_{29} = t_6$	$e_{2,10} = t_4$

(MVBO1)

$$maximize \sum_{j=1}^{M} (\mu_j - \mu)^2 \tag{1}$$

subject to

$$\sum_{j=1}^{M} \sum_{i=1}^{N} \{(\log e_{ij}) - \mu\}^2/(MN) = V : \text{constant} \tag{2}$$

$$t_1 \geq t_2 \geq \cdots \geq t_{m+1} = 1 \tag{3}$$

where

$$\mu = \sum_{j=1}^{M} \sum_{i=1}^{N} (\log e_{ij})/(MN) \ ; \ \mu_j = \sum_{i=1}^{N} (\log e_{ij})/N. \tag{4}$$

In MVBO1 some t_hs may become a same value, but it means that we cannot make the best use of ranking among objects. For example, if $t_h = t_{h+1}$, discrimination of the h-th faviorite and (h+1)-th favorite objects comes to nothing. Therefore the following constraint is added.

$$\log(t_h) - \log(t_{h+1}) \geq C \geq 0. \tag{5}$$

Also the objective function is changed into

$$\max \sum_{j=1}^{M} (\mu_j - \mu)^2 \ + C^2. \tag{6}$$

If t_hs are decided, we obtain values of e_{ij}. Thus, we use these values of e_{ij} in order to decide bounds on ratios of multipliers (weights) for inputs or outputs (objects) in DEA as follows.

[Derivation of bounds] The following is the same procedure as Takamura and Tone [9].
(1) Obtain the following quantities.

$$L_{jk} = \min_i \ e_{ik}/e_{ij}; \ U_{jk} = \max_i \ e_{ik}/e_{ij} \tag{7}$$

(2) Let constraints on a ratio u_k/u_j of weights for k-th object and j-th object be

$$L_{jk} \leq u_k/u_j \leq U_{jk}. \tag{8}$$

3. Evaluation of Baseball Players

In order to concrete our discussion our method is applied to evaluation of baseball players, especially 115 batters over 220 times at bat in the 2005 season. We use the following items (M=10) as objects.

1: batting average, 2: on-base percentage, 3: rate of runs batted in,
4: slugging percentage, 5: rate of stolen bases, 6: rate of home runs,
7: batting average in scoring position, 8: rate of sacrifice batting,
9: rate of double play, 10: rate of strikeout.

These items are used as outputs of DEA, where the best value and the worst value of each item are transformed into 1 and 0, respectively. We must note that for items 9 and 10 the largest values are transformed into 0 and the smallest values are transformed into 1. Each DMU has single input and is set to 1.

Which items are important is different in each batting order. We asked 10 persons (N=10) to select more important five items (m=5) than others corresponding to each batting order. Orders of selected items are also asked. In the following Sec.3.1 the case of $\{m$=5$\}$ is discussed. For the purpose of comparison the case of $\{m <5\}$ is also discussed in Sec.3.2.

3.1. The case of $\{m=5\}$

More important items than others are shown in Table 2 for the first batter and Table 3 for the second batter. Important items for other batting order were also selected. From Table 2 we can see that item 2 (on-base percentage) may be the most important for the first batter.

Table 4 shows values of objective functions. Especially the second and third columns show values for MVBO1, but except for the second batter the same values were obtained. This means that Equation (6) which has a parameter C was not effective. Therefore we gave some fixed values for C. As a result we propose the following formulation.

Table 2: More important items than others for the first batter

person$i$	important item j most	2nd	3rd	4th	5th
1	2	5	1	10	7
2	2	1	5	10	3
3	2	10	5	1	7
4	2	1	5	10	8
5	2	1	5	10	4
6	2	10	9	1	5
7	1	2	5	10	3
8	2	1	5	10	7
9	2	10	1	5	4
10	2	1	4	3	7

Table 3: More important items than others for the second batter

person$i$	important item j most	2nd	3rd	4th	5th
1	1	2	3	7	4
2	2	1	8	9	10
3	8	9	10	2	1
4	2	8	1	7	10
5	8	2	9	1	7
6	2	9	1	8	5
7	2	1	8	7	9
8	1	8	9	10	7
9	2	1	9	10	8
10	2	8	1	9	10

Table 4: Values of objective functions

batting order	[1]	[2]	[3]	[4]	[5]
1	**8.717**	**8.717**	8.694	8.660	8.610
2	6.674	6.877	6.674	6.674	6.674
3	**7.078**	**7.078**	7.054	7.016	6.965
4	**7.522**	**7.522**	7.504	7.434	7.266
5	**7.907**	**7.907**	7.892	7.846	7.729
6	**4.816**	**4.816**	4.805	4.780	4.740
7,8,9	**6.761**	**6.761**	6.761	6.761	6.756

[1] Equation (1), [2] Equation (6),
[3] Equation (9), C=0.1,
[4] Equation (9), C=0.2,
[5] Equation (9), C=0.3

(MVBO2)

$$\max\text{imize} \sum_{j=1}^{M} (\mu_j - \mu)^2 \tag{9}$$

subject to

$$\sum_{j=1}^{M} \sum_{i=1}^{N} \{(\log e_{ij}) - \mu\}^2/(MN) = V : \text{constant} \tag{10}$$

$$\log(t_h) - \log(t_{h+1}) \geq C \ \ (h = 1, 2, ..., m-1) \tag{11}$$

$$t_{m+1} = 1 \tag{12}$$

When V=1 and C=0.3, the number of the cases where the left hand side of Equation (11) is equal to C increased. This means that C=0.3 became a decisive factor for values of t_h. Therefore let C=0.2.

Table 5 shows values of t_h, where C=0.2. Table 6 shows values of μ_j in the first batter when using values of t_h in Table 5. Suppose that items which are detached from 0 be important. In Table 6 items 1, 2, 5 and 10 are important. Then bounds L_{jk} and U_{jk}(k=1,2,5,10;$j \neq k$) are calculated. For example $u_2 = t_1$=25.56, $u_4 = t_6$=1 and u_2/u_4=25.56 as person 1 answered item 2 as rank 1 and item 4 as rank 6 for the first batter. Table 7 shows ratios between item 2 and item 4 for the first batter, and $L_{42} = 4.60 \leq u_2/u_4 \leq U_{42} = 25.56$. These bounds L_{jk} and U_{jk} (k=1,2,5,10;$j \neq k$) were used as bounds of the following assurance region method for the batter o (See Sec.6.1 in Cooper et al.[3] and Takamura and Tone [9]). The efficiency score of the batter o is calculated by Equation (13).

Table 5: Values of t_h(C=0.2)

batting order	t_1	t_2	t_3	t_4	t_5
1	25.56	4.60	2.84	2.33	1.22
2	13.45	10.11	5.65	3.84	2.72
3	13.48	11.04	5.63	3.38	2.77
4	10.59	8.67	7.10	5.81	4.76
5	10.63	8.70	7.13	5.84	4.60
6	13.67	8.07	6.61	5.41	2.44
7,8,9	17.68	9.33	3.21	1.80	1.22

(assurance region method 1)

$$\max\text{imize} \sum_{j=1}^{M=10} u_j y_{jo} \tag{13}$$

subject to

$$\sum_{j=1}^{M=10} u_j y_{jg} \leq 1 \ (g = 1, ..., 115) \tag{14}$$

$$L_{jk} \leq u_k/u_j \leq U_{jk} \ (k = 1, 2, 5, 10; \ j \neq k) \tag{15}$$

$$u_j \geq 0 \ (j = 1, 2, ..., [M = 10]) \tag{16}$$

Table 8 shows efficient batters for each batting order. Table 9 shows efficiency scores of 22 batters efficient in the ordinary CCR model by each batting order (Normalized statistics

Table 6: Values of $\mu_j$for the first batter

j	μ_j	j	μ_j
1	**1.465**	6	0.000
2	**3.070**	7	0.080
3	0.124	8	0.000
4	0.144	9	0.104
5	**0.883**	10	**0.964**

Table 7: Ratios between item 2 and item 4 for the first batter

person	item 2/item4	person	item 2/item4
1	25.56	6	25.56
2	25.56	7	4.60
3	25.56	8	25.56
4	25.56	9	20.93
5	20.93	10	9.00
		max	25.56
		min	4.60

of these batters are shown in Table A1). Aoki is efficient as the first batter, but inefficient as the fifth batter. Araki is inefficient for every batting order. Kanemoto is efficient for every batting order. Yamazaki is only efficient as the second batter (Efficiency scores of these batters are shown in Table 9, and normalized statistics of Aoki, Araki, Kanemoto and Yamazaki are shown in Table 10). These facts show effectiveness of the assurance region model. Maeda is an excellent batter, but he is efficient in the second batter only, because his statistics are inferior to Kanemoto except for item 10 and he refers to Kanemoto in batting orders except for the second batter. Since for the second batter item 10 was judged as more important than item 8 by person 9, Maeda became efficient in the second batter.

Table 8: Efficient batters by each batting order

1	Aoki	3	Ibata
	Akaboshi		Kanemoto
	Kanemoto		Matsunaka
2	Aoki	4	Kanemoto
	Akaboshi		Matsunaka
	Ibata	5	Kanemoto
	Kanemoto		Matsunaka
	Maeda	6	Kanemoto
	K.Yamazaki		Matsunaka
		7,8,9	Akaboshi
			Kanemoto

Matsunaka was efficient for batting order 3, 4, 5 and 6. If Kanemoto and Matsunaka are selected for the third, fourth or fifth batter, there is only one candidate, Akaboshi, for the sixth $\sim$ ninth batter. Table 11 shows efficient batters when Kanemoto and Matsunaka are

Table 9: Efficiency scores of CCR efficient batters by each batting order

	1	2	3	4	5	6	7,8,9
Aoki	1	1	0.917	0.795	0.733	0.950	0.997
Akaboshi	1	1	0.868	0.760	0.723	0.932	1
Araki	0.786	0.837	0.708	0.619	0.598	0.738	0.733
Ibata	0.946	1	1	0.893	0.853	0.946	0.951
Imaoka	0.741	0.811	0.959	0.990	0.931	0.973	0.814
Iwamura	0.886	0.979	0.890	0.857	0.852	0.958	0.987
Ogata	0.828	0.968	0.846	0.804	0.780	0.916	0.956
Kanemoto	1	1	1	1	1	1	1
Kawasaki	0.671	0.971	0.721	0.626	0.590	0.768	0.802
Kinjoh	0.904	0.990	0.950	0.884	0.860	0.944	0.927
Koike	0.473	0.855	0.475	0.551	0.473	0.586	0.614
Shimizu	0.765	0.947	0.747	0.701	0.693	0.849	0.846
Johjima	0.854	0.897	0.803	0.812	0.819	0.847	0.844
Zuleta	0.903	0.914	0.952	0.946	0.941	0.958	0.917
Tsuboi	0.788	0.924	0.693	0.604	0.568	0.862	0.939
Nakamura	0.614	0.664	0.732	0.860	0.793	0.852	0.683
Nishioka	0.811	0.918	0.873	0.785	0.725	0.817	0.827
Fukudome	0.989	0.977	0.945	0.926	0.934	0.966	0.985
Maeda	0.909	1	0.898	0.893	0.896	0.936	0.909
Matsunaka	0.987	0.997	1	1	1	1	0.989
K.Yamazaki	0.433	1	0.522	0.482	0.392	0.632	0.651
LaRocca	0.826	0.903	0.824	0.852	0.855	0.885	0.855

Table 10: Normalized statistics of particular batters

item	Aoki	Araki	Kanemoto	K.Yamazaki
1	1	0.647	0.887	0.407
2	0.706	0.465	0.992	0
3	0.034	0.124	0.774	0
4	0.364	0.179	0.878	0.032
5	0.522	0.781	0.125	0.121
6	0.054	0.034	0.751	0.050
7	0.643	0.571	0.824	0.391
8	0.221	0.071	0	1
9	0.857	0.649	0.819	0.757
10	0.573	0.847	0.716	0.573
mean	0.497	0.437	0.677	0.333

neglected. Table 12 shows an example of the batting order (line-up) constructed by batters with higher efficiency scores than others.

MVBO2 maximizes the variance between items under constant total variance. This corresponds to maximization of the correlation ratio. Therefore the following formulation can be taken.

Table 11: Efficient batters when two batters are neglected

6	Aoki	6	Fukudome
	Akaboshi		Maeda
	Ibata		LaRocca
	Imaoka	7,8,9	Aoki
	Iwamura		Akaboshi
	Garcia		Ibata
	Kinjoh		Iwamura
	Zuleta		Fukudome

Table 12: An ideal line-up

1	Aoki	6	Imaoka
2	Maeda	7,8,9	Akaboshi
3	Ibata		Iwamura
4	Kanemoto		Fukudome
5	Matunaka		

(MVBO3)

$$\max\text{imize} \sum_{j=1}^{M} (\mu_j - \mu)^2/V \tag{17}$$

subject to

$$\log(t_i) - \log(t_{i+1}) \geq C \tag{18}$$

$$t_{m+1} = 1 \tag{19}$$

$$t_1 : \text{ a fixed value} \tag{20}$$

where $\sum_{j=1}^{M} \sum_{i=1}^{N} \{(\log e_{ij}) - \mu\}^2/(MN) = V$.

t_1 must be given a fixed value, otherwise even if t_1 becomes infinitive, the same value of correlation ratio can be achieved. Table 13 shows ratios t_i/t_{i+1} (i=1, 2,.., 5), where "MVBO2"s are results of MVBO2 shown by Equation (9)$\sim$Equation (12), "t_1=MVBO2"s are results of MVBO3 when using values of t_1 obtained by MVBO2 and "t_1=15"s are results of MVBO3 when t_1 is given a fixed value, 15. A little difference is brought about by $t_{m+1} = 1$ as shown in Table 13. Both MVBO2 and MVBO3 have the same set of efficient batters. Also when Kanemoto and Matsunaka are neglected, both methods have the same set of efficient batters except for LaRocca in the sixth batter. From these facts we can use either MVBO2 or MVBO3.

3.2. The cases of $\{m < 5\}$

In the above discussion, persons selected more important five items (m=5) than others, corresponding to each batting order. We suppose that the same items as Table 2 are selected for (m<5), for example, person 1 selected item 2 as the most important item for (m =2,..,5). Table 14 shows efficient batters at each batting order corresponding to each m. Table 15 shows efficiency scores corresponding to each m of batters who are efficient at some batting orders, but are not efficient at other batting orders. There is no large difference between m and $m-1$. For example, in the case of m=2, μ_j of seven items were evaluated as zero,

Table 13: Ratios t_i/t_{i+1} (i=1,2,..,5)

batting order	C=0.2	t_1/t_2	t_2/t_3	t_3/t_4	t_4/t_5	t_5/t_6
1	MVBO2	5.555	1.620	1.221	1.904	1.221
	t_1=MVBO2	5.460	1.627	1.221	1.928	1.221
	t_1=15	4.113	1.473	1.221	1.659	1.221
2	MVBO2	1.331	1.790	1.470	1.414	2.717
	t_1=MVBO2	1.331	1.790	1.470	1.412	2.717
	t_1=15	1.347	1.835	1.494	1.433	2.834
3	MVBO2	1.221	1.961	1.665	1.221	2.768
	t_1=MVBO2	1.221	1.942	1.674	1.221	2.780
	t_1=15	1.221	2.006	1.719	1.221	2.917
4	MVBO2	1.221	1.221	1.221	1.221	4.757
	t_1=MVBO2	1.221	1.221	1.221	1.221	4.757
	t_1=15	1.221	1.221	1.221	1.221	6.740
5	MVBO2	1.221	1.221	1.221	1.268	4.602
	t_1=MVBO2	1.221	1.221	1.221	1.259	4.633
	t_1=15	1.221	1.221	1.221	1.364	6.036
6	MVBO2	1.694	1.221	1.221	2.217	2.440
	t_1=MVBO2	1.636	1.221	1.221	2.280	2.456
	t_1=15	1.676	1.221	1.221	2.367	2.535
7,8,9	MVBO2	1.895	2.909	1.786	1.470	1.221
	t_1=MVBO2	1.892	2.912	1.787	1.471	1.221
	t_1=15	1.821	2.739	1.726	1.426	1.221

but the close efficiency scores between m=2 and m=3 were obtained. Efficiency scores of Aoki have large difference between m=5 and m=4, because two persons selected item 3, for which Aoki recorded a very low value, as the fifth favorite item and at m=4, item 3 was evaluated as t_5=1 by the above-mentioned two persons. The similar results as Aoki may occur, though they were rare cases in this study.

4. Evaluation of Chemical Companies

We evaluate a total of 25 chemical companies (See Table 16) in the year 2004 as a case in which each DMU has two inputs besides outputs, the inputs are total assets and the numbers of employees and the outputs are the volume of sales and incomes (These data were derived from Electronic Disclosure for Investors' Network (EDINET) in Japan). Since DEA cannot handle zero in any input, each value of four items (two inputs and two outputs) was divided by the maximum of each item; that is, this normalization is different from Sec.3 (See Table A2). We asked 20 students at Seikei University (N=20), who are not economists, to select two items (m=2) more important than others (See Table 17). For example, student 1 selected "incomes" as the most important item and "sales" as the second important item. When V=0.2257 (which is the total variance obtained at $\{t_1$=3, t_2=2 and t_3=1$\}$) and C=0.2, the values of t_i (t_1=3.0915, t_2=1.2214 and t_3=1) are obtained according to MVBO2. Let w_k be as follows: $w_k = v_k(k$=1,2) and $w_k = u_{k-2}(k$=3,4). Since μ_1=0.266, μ_2=0.153, μ_3=0.213 and μ_4=0.744, bounds L_{jk} and U_{jk}(j=1,2,3; k=4) were calculated by Equation (7). These bounds, L_{jk} and U_{jk}, were used as bounds of the following assurance region method. The efficiency score of company o is calculated by Equation (21).

Table 14: Efficient batters by each batting order and each m

	m=5	m=4	m=3	m=2
1	Aoki	Aoki	Aoki	Aoki
	Akaboshi		Matsunaka	Matsunaka
	Kanemoto	Kanemoto	Kanemoto	Kanemoto
2	Aoki	Aoki	Aoki	Aoki
	Akaboshi	Akaboshi	Akaboshi	Akaboshi
	Ibata	Ibata	Ibata	Ibata
	Kanemoto	Kanemoto	Kanemoto	Kanemoto
		Kinjoh	Kinjoh	
	Maeda	Maeda		
		Matsunaka	Matsunaka	Matsunaka
	K.Yamazaki	K.Yamazaki	K.Yamazaki	K.Yamazaki
3	Ibata	Ibata	Ibata	Ibata
				Imaoka
	Kanemoto	Kanemoto	Kanemoto	Kanemoto
	Matsunaka	Matsunaka	Matsunaka	Matsunaka
4				Aoki
	Kanemoto	Kanemoto	Kanemoto	Kanemoto
	Matsunaka	Matsunaka	Matsunaka	Matsunaka
5			Ibata	Ibata
		Imaoka	Imaoka	
	Kanemoto	Kanemoto	Kanemoto	Kanemoto
	Matsunaka	Matsunaka	Matsunaka	Matsunaka
6			Aoki	Aoki
			Ibata	Ibata
	Kanemoto	Kanemoto	Kanemoto	Kanemoto
	Matsunaka	Matsunaka	Matsunaka	Matsunaka
7,8,9				Aoki
	Akaboshi	Akaboshi	Akaboshi	Akaboshi
				Iwamura
	Kanemoto	Kanemoto	Kanemoto	Kanemoto
			Matsunaka	Matsunaka

(assurance region method 2)

$$\max\text{imize} \sum_{j=1}^{2} w_{j+2} y_{jo} \tag{21}$$

subject to

$$\sum_{j=1}^{2} w_j x_{jo} = 1 \tag{22}$$

$$-\sum_{j=1}^{2} w_j x_{jg} + \sum_{j=1}^{2} w_{j+2} y_{jg} \leq 0 \ (g = 1, 2, ..., 25) \tag{23}$$

$$L_{j4} \leq w_4/w_j \leq U_{j4} \ (j = 1, 2, 3) \tag{24}$$

$$w_j \geq 0 \ (j = 1, 2, 3, 4) \tag{25}$$

Table 15: Efficiency scores by each batting order and each m

		m=5	m=4	m=3	m=2
1	Akaboshi	1	0.993	0.943	0.924
	Matsunaka	0.987	0.996	1	1
2	Kinjoh	0.990	1	1	0.953
	Maeda	1	1	0.998	0.933
	Matsunaka	0.997	1	1	1
3	Imaoka	0.959	0.964	0.993	1
4	Aoki	0.795	0.943	0.987	1
5	Ibata	0.853	0.893	1	1
	Imaoka	0.931	1	1	0.990
6	Aoki	0.950	0.946	1	1
	Ibata	0.946	0.930	1	1
7,8,9	Aoki	0.997	0.996	0.995	1
	Iwamura	0.987	0.988	0.988	1
	Matsunaka	0.989	0.995	1	1

Table 16: DMU numbers and names of chemical companies

DMU number	name of company	DMU number	name of company
1	Asahi Chemical	14	Dainippon Ink
2	UBE Industries	15	Denka
3	Kao Corporation	16	Tosoh
4	Kaneka	17	Toyo Ink
5	Kyowa Hakko	18	Tokuyama
6	JSR	19	Zeon
7	Shiseido	20	Hitachi Chemical
8	Showa Denko	21	Fujifilm
9	Shinetsu	22	Mitsui Chemical
10	Sumitomo Chemi	23	Mitubishi Chemi
11	Sumitomo Bake	24	Mitubishi Gas
12	Sekisui-Chemi	25	Lion Corporation
13	Daisel Chemical		

Table 18 shows the efficiency scores of chemical companies, where values in AR-I-C were obtained by the above assurance region method 2. All efficiency scores except for efficient DMU 3 are smaller than the scores in the CCR model. This shows that more severe evaluation was conducted.

5. Conclusion

The assurance region model needs to decide bounds on the ratios of multipliers (weights) for inputs or outputs (objects). We proposed three derivation methods through the maximum variance between objects.

We showed the effectiveness of the assurance region model by an evaluation of batters in baseball, which have the same input and ten outputs and an evaluation of chemical companies, which have two inputs and two outputs. Especially, the batters were studied in

Table 17: Importance ranking of each item for companies

Student i	Assets	Employee	Sales	Incomes
1	3	3	2	1
2	1	3	3	2
3	3	3	2	1
4	3	3	2	1
5	2	3	3	1
6	1	2	3	3
7	3	3	2	1
8	3	2	3	1
9	2	3	1	3
10	1	2	3	3
11	3	3	1	1
12	3	3	2	1
13	3	3	2	1
14	3	3	2	1
15	2	1	3	3
16	3	3	2	1
17	3	3	2	1
18	3	3	2	1
19	1	2	3	3
20	2	1	3	3

1: the most important, 2:secondly important,
3: others

detail. In the case of batters, similar results were obtained by MVBO2 and MVBO3 and the cases of m <5 were also studied, but we did not find large differences among the different values of m.

References

[1] R. Allen, A. Athanassopoulos, R.G. Dyson and E. Thanassoulis: Weights restrictions and value judgments in Data Envelopment Analysis: Evolution, development and future directions. *Annals of Operations Research*, **73** (1997), 13–34.

[2] J. Beasley: Comparing university departments. *Omega*, **18-2** (1990), 171–183.

[3] W.W. Cooper, L.M. Seiford and K. Tone: *Data Envelopment Analysis*, 2nd Ed. (Springer, 2007).

[4] R.G. Dyson and E. Thanassoulis: Reducing weight flexibility in data envelopment analysis. *Journal of the Operational Research Society*, **39-6** (1988), 563–576.

[5] J.S.H. Kornbluth: Analyzing policy effectiveness using cone restricted Data Envelopment Analysis. *Journal of the Operational Research Society*, **42-12** (1991), 1097–1104.

[6] Y. Roll, W.D. Cook and B. Golany: Controlling factor weights in Data Envbelopment Analysis. *IIE Transactions*, **23-1** (1991), 2–9.

[7] Y. Roll and B. Golany: Alternate methods of treating factor weights in DEA. *Omega*, **21-1** (1993), 99–109.

[8] TL. Saaty: *Analytic Hierarchy Process* (McGraw-Hill, 1980).

Table 18: Efficiency scores of companies

DMU#	CCR	AR-I-C
1	0.912	0.802
2	0.702	0.584
3	1	1
4	0.962	0.824
5	0.848	0.717
6	1	0.750
7	0.671	0.645
8	0.745	0.585
9	0.826	0.549
10	0.778	0.596
11	0.647	0.635
12	0.899	0.828
13	0.677	0.555
14	0.738	0.712
15	0.773	0.637
16	0.886	0.732
17	0.662	0.610
18	0.690	0.571
19	0.949	0.737
20	0.993	0.967
21	0.638	0.610
22	1	0.767
23	0.962	0.821
24	1	0.601
25	1	0.934

[9] Y. Takamura and K. Tone: A comparative site evaluation study for relocating Japanese government agencies out of Tokyo. *Socio-Economic Planning Sciences*, **37** (2003), 85–102.

[10] T. Ueda: Objective restriction on multipliers in Data Envelopment Analysis (DEA). *Technical Reports of Seikei University*, **37-1** (2000), 27–31 (in Japanese).

[11] T. Ueda: Application of multivariate analysis for DEA. *Proceedings of DEA Symposium 2007* (2007), 96–101.

Table A 1: Normalized statistics of efficient batters in the ordinal CCR model

item	1	2	3	4	5	6	7	8	9	10
Aoki	1	0.706	0.034	0.364	0.522	0.054	0.643	0.221	0.857	0.573
Akaboshi	0.813	0.773	0.095	0.259	1	0.017	0.685	0.093	0.916	0.732
Araki	0.647	0.465	0.124	0.179	0.781	0.034	0.571	0.071	0.649	0.847
Ibata	0.860	0.800	0.300	0.353	0.353	0.113	1	0.230	0.669	0.777
Imaoka	0.567	0.570	1	0.549	0.091	0.545	0.845	0.000	0.368	0.703
Iwamura	0.833	0.785	0.653	0.721	0.148	0.575	0.634	0.000	0.939	0.296
Ogata	0.747	0.734	0.406	0.613	0.053	0.512	0.630	0.033	0.922	0.728
Kanemoto	0.887	0.992	0.774	0.878	0.125	0.751	0.824	0.000	0.819	0.716
Kawasaki	0.513	0.363	0.214	0.181	0.351	0.105	0.651	0.383	0.831	0.804
Kinjoh	0.867	0.632	0.503	0.422	0.071	0.214	0.845	0.062	0.829	0.892
Koike	0.327	0.276	0.362	0.400	0.066	0.520	0.235	0.599	0.625	0.413
Shimizu	0.707	0.516	0.293	0.458	0.113	0.321	0.454	0.075	0.828	0.955
Johjima	0.767	0.738	0.440	0.727	0.072	0.613	0.529	0.000	0.222	1
Zuleta	0.833	0.810	0.771	0.958	0.073	0.979	0.643	0.000	0.489	0.327
Tsuboi	0.767	0.659	0.091	0.233	0.131	0.067	0.277	0.068	1	0.649
Nakamura	0.433	0.414	0.933	0.847	0.045	0.975	0.244	0.000	0.503	0.314
Nishioka	0.493	0.400	0.309	0.305	0.830	0.094	0.870	0.097	0.774	0.865
Fukudome	0.893	1	0.670	0.813	0.228	0.571	0.668	0.000	0.739	0.363
Maeda	0.833	0.704	0.564	0.699	0.072	0.622	0.664	0.000	0.720	0.993
Matsunaka	0.807	0.922	0.884	1	0.109	1	0.718	0.000	0.791	0.634
Yamazaki	0.407	0.000	0.000	0.032	0.121	0.050	0.391	1	0.757	0.573
LaRocca	0.727	0.668	0.776	0.730	0.091	0.708	0.500	0.000	0.496	0.871

Table A 2: Normalized statistics of chemical companues

DMU	(I)Asset	(I)Employee	(O)Sales	(O)Incomes
Asahi Chemical	0.4257	0.3149	0.5451	0.6952
UBE Industries	0.2369	0.1464	0.2226	0.1456
Kao Corporation	0.2309	0.2531	0.3707	0.7721
Kaneka	0.1331	0.0879	0.1733	0.2547
Kyowa Hakko	0.1255	0.0788	0.1420	0.1994
JSR	0.1089	0.0577	0.1208	0.2715
Shiseido	0.2350	0.3197	0.2532	0.1883
Showa Denko	0.3164	0.1476	0.2931	0.2397
Shinetsu	0.4948	0.2400	0.3828	0.9332
Sumitomo Chemi	0.5526	0.2670	0.5129	0.7606
Sumitomo Bake	0.0851	0.1037	0.0884	0.1265
Sekisui-Chemi	0.2493	0.2248	0.3391	0.2340
Daisel Chemical	0.1386	0.0769	0.1212	0.1549
Dainippon Ink	0.3348	0.3672	0.3968	0.2787
Denka	0.1100	0.0628	0.1108	0.1349
Tosoh	0.2022	0.1209	0.2328	0.3434
Toyo Ink	0.0910	0.0815	0.0907	0.0770
Tokuyama	0.1035	0.0606	0.0940	0.0945
Zeon	0.0794	0.0368	0.0915	0.1158
Hitachi Chemical	0.1379	0.2188	0.2198	0.2847
Fujifilm	1	1	1	1
Mitsui Chemical	0.4040	0.1617	0.4857	0.4912
Mitubishi Chemi	0.6605	0.4397	0.8663	0.9121
Mitubishi Gas	0.1658	0.0585	0.1538	0.2324
Lion Corporation	0.0789	0.0756	0.1225	0.0509

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