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EFFECT OF FACILITY CLOSING INFORMATION ON TRAVEL DISTANCE

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Abstract This paper examines how providing closing information affects travel distances when facilities are closed. Two cases are considered to assess the effect of closing information. In the first case, customers have perfect information and travel directly to the nearest open facility. In the second case, customers have no information and search for an open facility. The average distances from customers to an open facility with and without information are derived for regular and random patterns of facilities. The effect of information is measured as the reduction in the average distances.

Keywords: Facility planning, kth nearest distance, average distance, regular and random patterns

1. Introduction

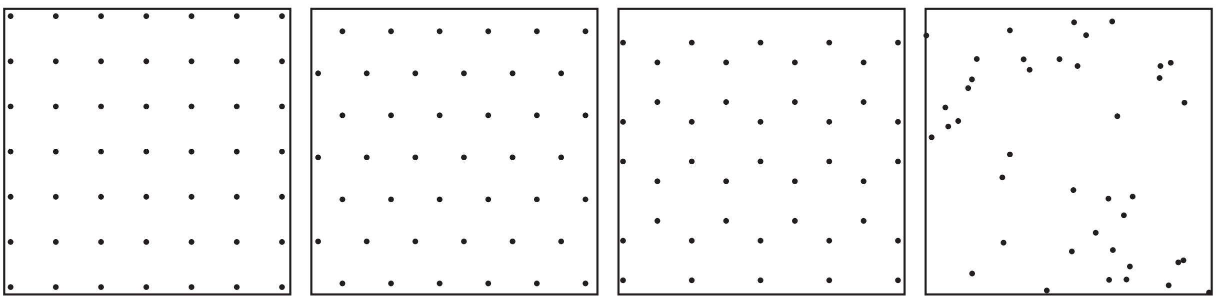
Classical facility location models assume that customers always use their nearest facility. Facilities may, however, be closed or disrupted due to accidents and natural disasters. Customers then have to use more distant facilities. Since such disruptive events are unpredictable, the possibility of closing should be taken into account when locating facilities.

Introducing a reliability aspect into facility location problems has been considered in various ways. Daskin [5] generalized the maximum covering problem to include the possibility that some of the facilities could fail. ReVelle and Hogan [16] developed the α-reliable p-center problem and the maximum reliability location problem. The objective of the former problem is to minimize the maximum time within which service is available with probability α, and the objective of the latter problem is to maximize the probability of service being available. Weaver and Church [19] addressed the vector assignment p-median problem, where a certain percentage of customers could be serviced by the kth nearest facility. Pirkul [15] studied a similar problem in which customers are served by two facilities designated as primary and secondary facilities. Drezner [6] formulated the unreliable p-median and p-center problem. Efficient solution methods for the unreliable p-median problem were developed in Lee [12]. Berman et al. [1] extended Drezner's work and demonstrated that the probability of facility failure has a strong effect on the optimal facility location. Snyder and Daskin [18] presented two reliability models based on the p-median problem and the uncapacitated fixed-charge location problem. They considered the trade-off between the operating cost and the expected failure cost. A facility location problem with congestion was analysed as a spatially distributed queueing system in Larson [9, 10]. A survey of a wide variety of models for facility location problems under uncertainty was provided in Snyder [17].

In a large number of works concerning facility location problems with closing of facilities, few studies explicitly discuss the effect of information. Information on the state of facilities would affect the behaviour of customers when some facilities are closed. Informed customers can travel directly to the nearest open facility, whereas uninformed customers have to search for an open facility. Information is important, particularly for emergency facilities. For example, providing ambulances with hospital information can reduce time to find an operating hospital. Evaluating the effect of information helps planners in prioritizing investments for urban risk management. This paper therefore examines how closing information reduces travel distances from customers to an open facility.

Another characteristic of this paper is to employ an analytical approach based on the kth nearest distance on the continuous plane. The kth nearest distance, which is the distance from customers to the kth nearest facility, provides analytical expressions of travel distances to the nearest open facility when some facilities are closed. The analytical expressions are useful to find fundamental relationships between variables, leading to a better understanding of empirical results. The probability density functions of the kth nearest distance have been obtained for several patterns. The nearest distance was derived in Clark and Evans [2] for the random pattern, Persson [14] for the square lattice, and Holgate [7] for the triangular lattice. The kth nearest distance was derived in Dacey [4] for the random pattern, Koshizuka [8] for the square lattice for $k = 1, 2, 3$, and Miyagawa [13] for the square, triangular, and hexagonal lattices for $k = 1, 2, \ldots, 7$.

This paper focuses on regular and random patterns of facilities, as shown in Figure 1. The regular patterns that we consider are the square, triangular, and hexagonal lattices. Although actual patterns of facilities are neither regular nor random, they can be regarded as the intermediate between regular and random. The theoretical results of these extremes will give an insight into empirical studies on actual patterns. We assume that these patterns continue infinitely. This assumption allows us to examine the travel distance without taking into account the boundary effect. We also assume that customers are uniformly distributed. Although this uniformity assumption is unrealistic, the method is applicable to realistic situations if the study region can be divided into subregions with nearly uniform distributions.



(a) square lattice (b) triangular lattice (c) hexagonal lattice (d) random

Figure 1: Regular and random patterns

The remainder of this paper is organized as follows. The next section discusses the situation where perfect information is available. Section 3 considers the situation where information is unavailable. The effect of closing information is then evaluated. The final section presents the conclusion of this paper.

2. Perfect Information

Consider first the situation where all customers know which facilities are open in advance. In this situation, customers can travel directly to the nearest open facility. For example, if the nearest facility is closed, customers use the second nearest facility. If both first and second nearest facilities are closed, they use the third nearest facility. Let R_k be the Euclidean distance from customers to the kth nearest facility. We call R_k the kth nearest distance. In this section, using the kth nearest distance, we obtain the average distance to the nearest open facility.

For the three regular patterns, the average kth nearest distances $E(R_k)$ for $k = 1, 2, \ldots, 7$ were derived in Miyagawa [13]. For example, $E(R_1)$ of the square, triangular, and hexagonal lattices are

$$E(R_1^S) = \frac{1}{6\sqrt{\rho}} \left\{ \sqrt{2} + \ln(1+\sqrt{2}) \right\} \quad (1)$$

$$E(R_1^T) = \frac{\sqrt{2}}{3^{3/4}\sqrt{\rho}} \left(\frac{1}{3} + \frac{1}{2} \ln \sqrt{3} \right) \quad (2)$$

$$E(R_1^H) = \frac{2}{3^{3/4}\sqrt{\rho}} \left\{ \frac{1}{3} + \frac{1}{6\sqrt{3}} \ln(2+\sqrt{3}) \right\} \quad (3)$$

respectively, where ρ is the density of facilities. For higher order ($k \geq 8$) distances, the upper and lower bounds of R_k were obtained in Miyagawa [13]. The upper and lower bounds of the square, triangular, and hexagonal lattices are

$$\frac{\left\lfloor \sqrt{k} \right\rfloor - 1}{2\sqrt{\rho}} < R_k^S < \frac{\left\lceil \sqrt{k} \right\rceil}{\sqrt{2\rho}}, \quad (4)$$

$$\left\lfloor \frac{1}{6} \left(\sqrt{12k-3} - 3 \right) \right\rfloor \frac{3^{1/4}}{\sqrt{2\rho}} < R_k^T < \left\{ \left\lceil \frac{1}{6} \left(\sqrt{12k-3} - 3 \right) \right\rceil + \frac{2}{3} \right\} \frac{\sqrt{2}}{3^{1/4}\sqrt{\rho}}, \quad (5)$$

$$\left(3 \left\lfloor \sqrt{\frac{k}{6}} \right\rfloor - 1 \right) \frac{1}{3^{3/4}\sqrt{\rho}} < R_k^H < \left(3 \left\lceil \sqrt{\frac{k}{6}} \right\rceil + 1 \right) \frac{2}{3^{5/4}\sqrt{\rho}}, \quad (6)$$

respectively.

For the random pattern, $E(R_k)$ was obtained in Dacey [3] as

$$E(R_k) = \frac{(2k-1)!!}{(2k-2)!!} \frac{1}{2\sqrt{\rho}}. \quad (7)$$

$E(R_k)$ of the regular and random patterns for $k = 1, 2, \ldots, 7$ are given in Table 1, where the density of facilities is $\rho = 1$. Note that $E(R_1)$ of the triangular lattice is the smallest,

Table 1: Average kth nearest distance

	$E(R_1)$	$E(R_2)$	$E(R_3)$	$E(R_4)$	$E(R_5)$	$E(R_6)$	$E(R_7)$
square	0.383	0.700	0.908	**1.023**	1.243	1.309	1.413
triangle	**0.377**	0.729	**0.854**	1.058	1.225	1.326	**1.408**
hexagon	0.404	**0.663**	0.909	1.066	**1.220**	**1.282**	1.453
random	0.500	0.750	0.938	1.094	1.230	1.354	1.466

indicating that if all facilities are open, the triangular lattice is the best. This optimality of the triangular lattice was also shown in Leamer [11]. Note, however, that $E(R_2)$, $E(R_5)$, $E(R_6)$ of the hexagonal lattice and $E(R_4)$ of the square lattice are smaller than those of the triangular lattice.

Suppose that facilities are closed independently and at random. Facilities are open with probability p, and closed with probability $1-p$. Then the probability that customers use the second nearest facility, that is, the nearest facility is closed and the second nearest is open is $(1-p)p$. In general, the probability that customers use the kth nearest facility is $(1-p)^{k-1}p$. Using this probability and the average kth nearest distance $E(R_k)$, we can express the average distance to the nearest open facility $E(R)$ as

$$E(R) = p\sum_{k=1}^{\infty}(1-p)^{k-1}E(R_k). \tag{8}$$

Substituting $E(R_k)$ for $k = 1, 2, \ldots, 7$ and the upper and lower bounds of R_k for $k \geq 8$ of the regular patterns into Equation (8) yields the upper and lower bounds of the average distance to the nearest open facility.

$E(R)$ of the random pattern is obtained from Equation (7) as

$$E(R) = \frac{1}{2\sqrt{p\rho}}. \tag{9}$$

Figure 2 shows the average distance to the nearest open facility $E(R)$ as a function of probability p. For each regular pattern, the upper and lower bounds of $E(R)$ are equivalent at $p = 1$ where all facilities are open, and the difference increases with decreasing p. By

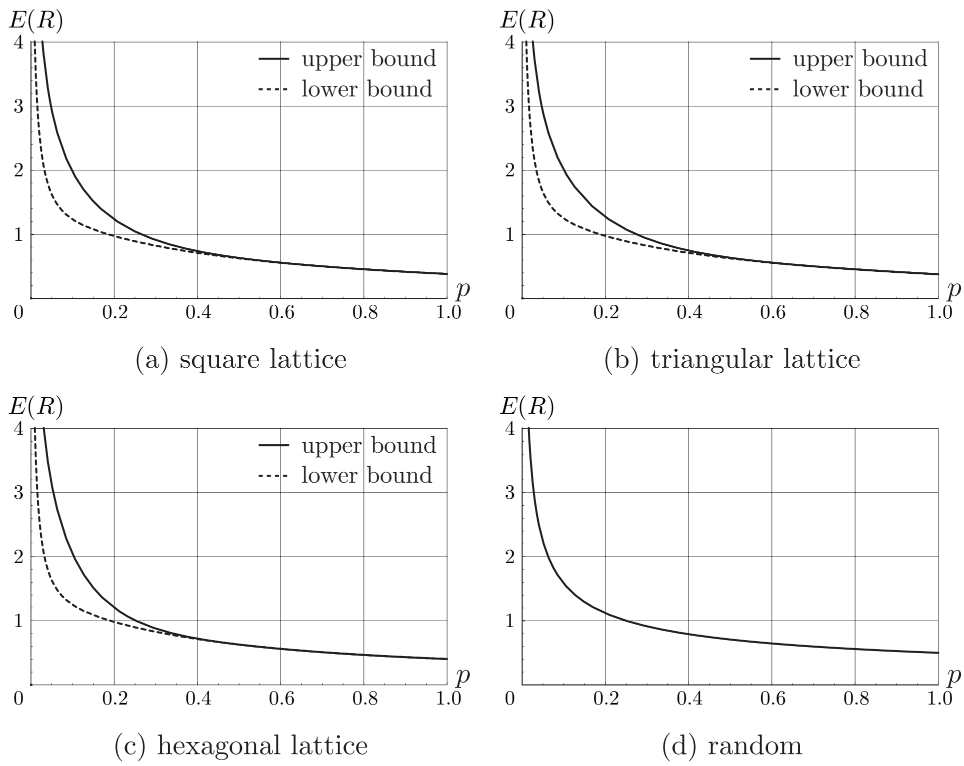


Figure 2: Average distance to the nearest open facility with information

comparing the upper and lower bounds of $E(R)$, we can show the range of p that one pattern outperforms the other. The triangular lattice outperforms the square and hexagonal lattices, that is, the upper bound of the triangular lattice is smaller than the lower bounds of the square and hexagonal lattices when $0.6800 < p \leq 1$. This means that the triangular lattice is the best if at least 68% of facilities are open.

3. No Information

Consider next the situation where no customers know which facilities are open. In this situation, customers have to travel from facility to facility in searching for an open one. Customers first travel to the nearest facility and if the facility is closed, they travel to the nearest facility form this location, and repeat until they find an open facility, as depicted in Figure 3. The travel distance is thus likely to be larger than that with information. Let R_k be the total travel distance from customers to the kth visited facility. We call R_k the kth visited distance. In this section, using the kth visited distance, we obtain the average distance to an open facility. By comparing the average distances with and without information, we evaluate the effect of information.

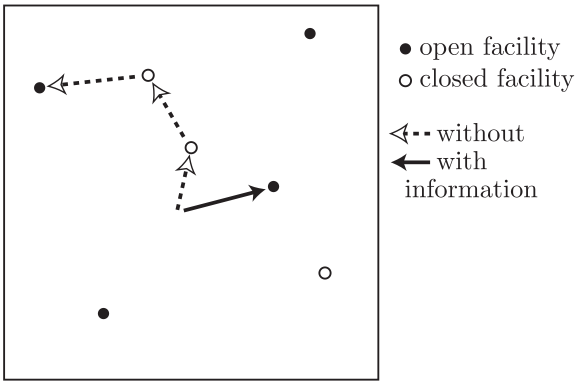


Figure 3: Travel to an open facility

For the regular patterns, the second visited distance is the sum of the distance to the nearest facility and the distance between two adjacent facilities. In general, the average kth visited distance $E(R_k)$ is

$$E(R_k) = E(R_1) + (k-1)a \tag{10}$$

where a is the distance between two adjacent facilities. The distances between adjacent facilities of the square, triangular, and hexagonal lattices are expressed in terms of the density of facilities ρ as

$$a^S = \frac{1}{\sqrt{\rho}}, \quad a^T = \frac{\sqrt{2}}{3^{1/4}\sqrt{\rho}}, \quad a^H = \frac{2}{3^{3/4}\sqrt{\rho}}, \tag{11}$$

respectively. Substituting $E(R_1)$ in the previous section (Equations (1)–(3)) and a into Equation (10) gives the average kth visited distances $E(R_k)$ of the square, triangular, and hexagonal lattices as

$$E(R_k^S) = \frac{1}{6\sqrt{\rho}} \left\{ \sqrt{2} + \ln\left(1+\sqrt{2}\right) + 6(k-1) \right\}, \tag{12}$$

$$E(R_k^T) = \frac{\sqrt{2}}{3^{3/4}\sqrt{\rho}} \left\{ \frac{1}{3} + \frac{1}{2}\ln\sqrt{3} + \sqrt{3}(k-1) \right\}, \tag{13}$$

$$E(R_k^H) = \frac{2}{3^{3/4}\sqrt{\rho}} \left\{ \frac{1}{3} + \frac{1}{6\sqrt{3}}\ln\left(2+\sqrt{3}\right) + (k-1) \right\}, \tag{14}$$

respectively.

For the random pattern, $E(R_k)$ is numerically calculated by using a random pattern in a square shown in Figure 4. To avoid the boundary effect, we assume that customers are distributed only in the inner square, but can use all facilities in the outer square.

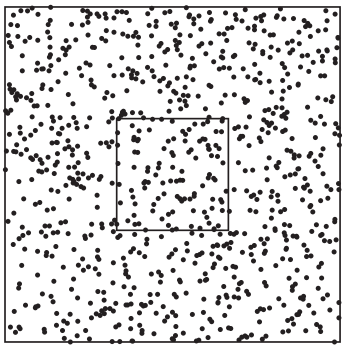


Figure 4: Random pattern in a square

$E(R_k)$ of the regular and random patterns are given in Table 2, where the density of facilities is $\rho = 1$. $E(R_1)$ are the same as those in Table 1. Note that $E(R_k)$ of the hexagonal lattice for $k \geq 2$ are the smallest among the regular patterns. This is because the distance between two adjacent facilities of the hexagonal lattice is the smallest. $E(R_k)$ of the random pattern for $k \geq 2$ are smaller than those of the regular patterns. Since regular patterns are typical dispersed patterns, the distance between adjacent facilities tends to be large, thus increasing the kth visited distance.

Table 2: Average kth visited distance

	$E(R_1)$	$E(R_2)$	$E(R_3)$	$E(R_4)$	$E(R_5)$	$E(R_6)$	$E(R_7)$
square	0.383	1.383	2.383	3.383	4.383	5.383	6.383
triangle	**0.377**	1.452	2.526	3.601	4.675	5.750	6.825
hexagon	0.404	**1.281**	**2.158**	**3.036**	**3.913**	**4.791**	**5.668**
random	0.500	1.051	1.642	2.324	3.060	3.757	4.419

Substituting $E(R_k)$ of the regular patterns into Equation (8) yields the average distance to an open facility $E(R)$. $E(R)$ of the square, triangular, and hexagonal lattices are

$$E(R) = \frac{1}{6\sqrt{\rho}} \left\{ \sqrt{2} + \ln\left(1+\sqrt{2}\right) + 6\left(\frac{1}{p} - 1\right) \right\}, \tag{15}$$

$$E(R) = \frac{\sqrt{2}}{3^{3/4}\sqrt{\rho}} \left\{ \frac{1}{3} + \frac{1}{2}\ln\sqrt{3} + \sqrt{3}\left(\frac{1}{p} - 1\right) \right\}, \tag{16}$$

$$E(R) = \frac{2}{3^{3/4}\sqrt{\rho}} \left\{ \frac{1}{3} + \frac{1}{6\sqrt{3}}\ln\left(2+\sqrt{3}\right) + \left(\frac{1}{p} - 1\right) \right\}, \tag{17}$$

respectively.

$E(R)$ of the random pattern is numerically calculated by using a random pattern in a square shown in Figure 4. The probability of closing is varied from 0 to 1 in increments of 0.01. For each probability of closing, we randomly select closed facilities, and calculate the travel distance from customers in the inner square to an open facility. $E(R)$ is obtained as the average of the distances for 10,000 closing patterns. The standard deviation of the distances is 0.039 at $p = 0.8$, 0.137 at $p = 0.5$, and 0.710 at $p = 0.2$. The dispersion of the distances increases as the number of closed facility increases. Obviously, $E(R)$ varies according to the initial pattern of facilities. The standard deviation of $E(R)$ for 100 different patterns is 0.008 at $p = 0.8$, 0.029 at $p = 0.5$, and 0.105 at $p = 0.2$.

Figure 5 shows the average distance to an open facility $E(R)$ as a function of probability p. By comparing $E(R)$, we can show that the best regular pattern is the triangular lattice when $p \geq 0.932$, the square lattice when $0.853 \leq p < 0.932$, and the hexagonal lattice when $p < 0.853$. That is, not only the triangular lattice but also the square and hexagonal lattices have the possibility of being the best depending on the probability of closing. This makes a clear contrast with the perfect information case where the triangular lattice is the best.

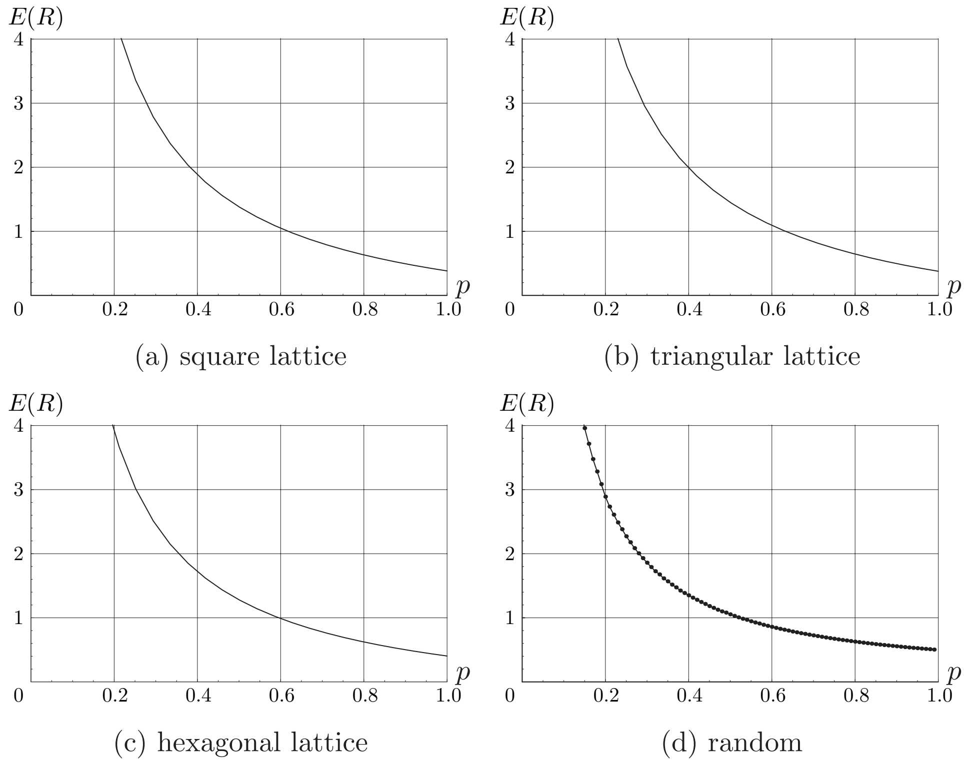


Figure 5: Average distance to an open facility without information

Now, let us examine how providing closing information reduces travel distances. By comparing Figures 2 and 5, we can see that the average distance with information is smaller than that without information, and that the difference increases with decreasing p. For the regular patterns, the average distance with information is 70-75% of that without information at $p = 0.8$, 44-49% at $p = 0.5$, and 27-31% at $p = 0.2$. For the random pattern, the ratio is 89% at $p = 0.8$, 67% at $p = 0.5$, and 39% at $p = 0.2$. It follows that the more facilities are closed, the more effective is providing information. In addition, information is particularly effective for the regular patterns.

4. Conclusion

This paper has examined how closing information affects travel distances when facilities are closed. As expected, information can greatly reduce travel distances from customers to an open facility. Although the analysis has focused on simple regular and random patterns, this reduction in travel distances gives an estimate for the effect of information for actual facility locations. Less expected is that if information is unavailable, the square and hexagonal lattices have the possibility of being the best among the regular patterns. That is, the triangular lattice is not always the best if the availability of information is considered.

The main limitation of our approach is that facilities are assumed to be closed independently. This assumption might be invalid in disastrous situations where facilities are disrupted simultaneously in an area. Incorporating the nonindependent probability of closing is a promising direction for future research.

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