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REDUCTION OF ULTRAMETRIC MINIMUM COST SPANNING TREE GAMES TO COST ALLOCATION GAMES ON ROOTED TREES

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Abstract A minimum cost spanning tree game is called ultrametric if the cost function on the edges of the underlying network is an ultrametric. We show that every ultrametric minimum cost spanning tree game is reduced to a cost allocation game on a rooted tree. It follows that there exist $O(n^2)$ time algorithms for computing the Shapley value, the nucleolus and the egalitarian allocation of the ultrametric minimum cost spanning tree games, where n is the number of players.

Keywords: Game theory, algorithm, combinatorial optimization.

1. Introduction

Let $N = \{1, \cdots, n\}$, where $n \geq 1$ is an integer. Suppose that $K_{N'}$ is the complete graph whose vertex set is $N' = N \cup \{0\}$ and a function w which assigns a nonnegative cost $w(e)$ to each edge e of $K_{N'}$ is given. A minimum cost spanning tree game (MCST game for short) is a cooperative (cost) game (N, c_w) defined as follows: for $S \subseteq N$ define $c_w(S)$ as the cost of a minimum cost spanning tree of the subgraph of $K_{N'}$ induced by $S \cup \{0\}$. Bird [2] showed that the core of an MCST game is always nonempty by explicitly constructing a core allocation, which is often called a Bird allocation (also see [8]). An ultrametric MCST game is an MCST game where the cost function w on the edges of the underlying graph is an ultrametric, i.e., for each distinct $i, j, k \in N'$ we have

$$w(i, k) \leq \max\{w(i, j), w(j, k)\}. \tag{1.1}$$

In [2], Bird also showed that the core of any MCST game (N, c_w) contains the core of another MCST game $(N, c_{\bar{w}})$ associated with the cost function $\bar{w}$, where for each $i, j \in N'$ $\bar{w}(i, j)$ is defined as the maximum of $w(k, l)$ over all the edges (k, l) in the path from i to j in some minimum cost spanning tree of $K_{N'}$. Bird called the latter core the *irreducible core*. The cost function $\bar{w}$ is known to be an ultrametric (see [19]), and conversely, each ultrametric function is derived in this way (see [17]). The irreducible core of an MCST game (N, c_w) and the associated game $(N, c_{\bar{w}})$ have been studied by many authors (e.g. [2], [1], [14] and [19]).

Cost allocation games on rooted trees are another class of cooperative (cost) games. Let $T = (V, A)$ be a rooted tree whose set of leaves is $N = \{1, \ldots, n\}$ and let l be a function which assigns a nonnegative length $l(a)$ to each edge a of T. For $S \subseteq N$ define $t_l(S)$ as the total length of edges that belongs to some path from a leaf $i \in S$ to the root. We call the resulting game (N, t_l) a cost allocation game on a rooted tree. This class of games is equivalent to the games studied by Megiddo [15] and the standard tree games [9] (see [12]). Any cost allocation game on a rooted tree is submodular and there exist efficient algorithms for computing solutions like the nucleolus and the egalitarian allocation for them ([15], [7], [12]).

In this paper, we show that any ultrametric MCST game can be represented as a cost allocation game on a rooted tree. It follows that for an ultrametric MCST game we can compute the Shapley value, the nucleolus and the egalitarian allocation in $\mathrm{O}(n^2)$ time. It should be noted here, in contrast, that computing solutions of a general MCST game are intractable: computing the nucleolus of the MCST games is NP-hard [5] and testing membership in the core of MCST games is co-NP-complete [4]. The computational complexities of the Shapley value and the egalitarian allocation of the MCST games are still open problems.

2. Basic Definitions

In this section, we review definitions from cooperative game theory, and give definitions of ultrametric MCST games and cost allocation games on rooted trees.

We denote by $\mathbb{R}$ the set of real numbers and by $\mathbb{R}_+$ the set of nonnegative real numbers.

2.1. Cooperative games

A *cooperative (cost) game* (N, c) is a pair of a finite set $N = \{1, \cdots, n\}$ and a function $c\colon 2^N \to \mathbb{R}$ with $c(\emptyset) = 0$. We call $N = \{1, \cdots, n\}$ the set of the *players* and the function c is called the *characteristic function*. In the context of this paper, the value $c(S)$ for $S \subseteq N$ is interpreted as the total cost of some activity when only the members in S cooperate.

A cooperative game (N, c) is *subadditive* if for all $S, T \subseteq N$ with $S \cap T = \emptyset$ we have $c(S) + c(T) \geq c(S \cup T)$. Also, a game (N, c) is *submodular* (or *concave*) if for all $S, T \subseteq N$ we have $c(S) + c(T) \geq c(S \cup T) + c(S \cap T)$. The *core* of the cooperative game (N, c) is defined as follows

$$\mathrm{core}(c) = \{x \mid x \in \mathbb{R}^N, \forall S \subseteq N\colon x(S) \leq c(S), x(N) = c(N)\}, \tag{2.1}$$

where $x(S) = \sum_{i \in S} x(i)$ for $S \subseteq N$. Note that the directions of the inequalities in the usual definition of the core are reversed. The core of a submodular game is nonempty [18].

The *Shapley value* $\Phi\colon N \to \mathbb{R}$ of game (N, c) is defined as

$$\Phi(i) = \sum_{i \notin S \subseteq N} \frac{|S|!(n - |S| - 1)!}{n!}(c(S \cup \{i\}) - c(S)) \quad (i \in N). \tag{2.2}$$

For a vector $x \in \mathbb{R}^N$ let us denote by $\tilde{x}$ the vector in $\mathbb{R}^N$ obtained by rearranging the components of x in nondecreasing order. For vectors $\tilde{x}$ and $\tilde{y}$ in $\mathbb{R}^n$ we say $\tilde{x}$ is *lexicographically greater than* $\tilde{y}$ if there exists $k = 1, \cdots, n$ such that $\tilde{x}_i = \tilde{y}_i$ $(i = 1, \cdots, k-1)$ and $\tilde{x}_k > \tilde{y}_k$. For a submodular game (N, c) the *egalitarian allocation* is the unique vector x in the core which lexicographically maximizes $\tilde{x}$ over the core. The concept of egalitarian allocation for general cooperative games was introduced in [3] and that for concave games was studied in [6].

For a cooperative game (N, c) and a vector x such that $x(N) = c(N)$, the *excess* $e(S, x)$ of x for subset $S \subseteq N$ is defined as

$$e(S, x) = c(S) - x(S). \tag{2.3}$$

Given a vector x with $x(N) = c(N)$ let us denote by $\theta(x)$ the sequence of components $e(S, x)$ $(\emptyset \subset S \subset N)$ arranged in order of nondecreasing magnitude. The *nucleolus* [16] of game (N, c) is defined to be the unique vector x which lexicographically maximizes $\theta(x)$ over all the vectors x with $x(N) = c(N)$.

2.2. (Ultrametric) MCST games

All graphs we consider in this paper are simple undirected graphs (without self-loop and parallel edges). Therefore, an edge a of a graph $G = (V, A)$ is an unordered pair of distinct vertices $u, v \in V$ but we write $a = (u, v)$ instead of $a = \{u, v\}$. A graph $G = (V, A)$ is *complete* if $A = \{(u, v) \mid u, v \in V, u \neq v\}$ and we denote such a complete graph by K_V.

A graph $G = (V, A)$ is called a *tree* if it is connected and contains no cycle. For a tree $T = (V, A)$, a vertex $v \in V$ is called a *leaf* if there exists exactly one edge incident to v. For a graph $G = (V, A)$ a subgraph $H = (W, B)$ is called a *spanning tree* if $V = W$ and H is a tree. We also say that B is a spanning tree of $G = (V, A)$ if $H = (W, B)$ is a spanning tree of G.

Let $\mathrm{K}_{N'}$ be the complete graph with vertex set $N' = \{0, 1, \cdots, n\}$ and let $w \colon N' \times N' \to \mathbb{R}_+$ be a function such that $w(i, i) = 0$ for all $i \in N'$ and $w(i, j) = w(j, i)$ for all $i, j \in N'$. We call such a pair $(\mathrm{K}_{N'}, w)$ a *network*. For each subset Γ of edges of $\mathrm{K}_{N'}$, we define the *cost* $w(\Gamma)$ of Γ by

$$w(\Gamma) = \sum_{(i,j) \in \Gamma} w(i, j). \tag{2.4}$$

For each $S \subseteq N$ we write $S' = S \cup \{0\}$. The *minimum cost spanning tree game* (or *MCST game* for short) associated with network $(\mathrm{K}_{N'}, w)$ is a cooperative game (N, c_w) defined by

$$c_w(S) = \min\{w(\Gamma) \mid \Gamma \text{ is a spanning tree of } \mathrm{K}_{S'}\} \quad (S \subseteq N), \tag{2.5}$$

where $\mathrm{K}_{S'}$ is the complete subgraph of $\mathrm{K}_{N'}$ with vertex set S'. The core of an MCST game is always nonempty. Indeed, a vector called a Bird allocation [2] is in the core (see [8]). It is easy to see that an MCST game is subadditive. However, an MCST game is not submodular in general even if w is a metric.

A function $w \colon N' \times N' \to \mathbb{R}_+$ is called an *ultrametric* if for each distinct $i, j, k \in N'$ we have

$$w(i, k) \leq \max\{w(i, j), w(j, k)\}. \tag{2.6}$$

Equivalently, w is an ultrametric if and only if for each distinct $i, j, k \in N'$ the maximum of $w(i, j), w(j, k), w(i, k)$ is attained by at least two pairs. An MCST game (N, c_w) is called *ultrametric* if w is an ultrametric. It can be shown that every ultrametric MCST game is submodular [14].

2.3. Cost allocation game on rooted trees

Let $T = (V, A)$ be a tree with a distinguished vertex r and the set of leaves being $N = \{1, \ldots, n\}$. We call the vertex r the *root* of T and do not consider r to be a leaf. Let $l \colon A \to \mathbb{R}_+$ be a function on A. We call such a pair (T, l) a *rooted tree*.

Denote by A_i the set of edges on the unique path from i to r and for each $S \subseteq N$ define A_S by $A_S = \bigcup_{i \in S} A_i$. Then, the *cost allocation game* (N, t_l) on a rooted tree (T, l) is defined by

$$t_l(S) = \sum_{a \in A_S} l(a) \quad (S \subseteq N). \tag{2.7}$$

It is easy to see that any cost allocation game (N, t_l) on a rooted tree is submodular. Megiddo [15] showed that the Shapley value and the nucleolus of any cost allocation game on a rooted tree can be found in $\mathrm{O}(n)$ and $\mathrm{O}(n^3)$, respectively. Galil [7] improved the latter time bound to $\mathrm{O}(n \log n)$. Iwata and Zuiki [12] gave $\mathrm{O}(n \log n)$ algorithms for computing the nucleolus and the egalitarian allocation of cost allocation games on rooted trees. Summarizing, we have the following lemma.

Lemma 2.1 (Megiddo [15], Galil [7], Iwata and Zuiki [12]) *For each cost allocation game* (N, t_l) *on a rooted tree the Shapley value, the nucleolus and the egalitarian allocation can be computed in* $\mathrm{O}(n)$, $\mathrm{O}(n \log n)$ *and* $\mathrm{O}(n \log n)$ *time, respectively.*

3. The Reduction to Cost Allocation Games on Rooted Trees

Let $(T = (V, A), l)$ be a rooted tree with root r and the set of leaves being M. For each pair (u, v) of vertices of T, let us denote by $d_l(u, v)$ the length of the path from u to v with respect to the function $l\colon A \to \mathbb{R}_+$. We call a rooted tree (T, l) *equidistant* if for all $i, j \in M$ we have $d_l(i, r) = d_l(j, r)$. A rooted tree (T, l) with the set of leaves being M is said to *represent* a function $w\colon M \times M \to \mathbb{R}_+$ if

$$w(i, j) = d_l(i, j) \quad (i, j \in M). \tag{3.1}$$

Lemma 3.1 (cf. Gusfield [10]) *For a function* $w\colon N' \times N' \to \mathbb{R}_+$*,* w *is an ultrametric if and only if there exists an equidistant rooted tree which represents* w*.* □

The statement of the following lemma can be found in [2]. Recall that we defined $S' = S \cup \{0\}$ for each $S \subseteq N$.

Lemma 3.2 *Suppose that* (N, c_w) *is an ultrametric MCST game associated with network* $(\mathrm{K}_{N'}, w)$*. For* $S \subseteq N$ *and* $i \notin S$ *we have*

$$c_w(S \cup \{i\}) = c_w(S) + w(i, j^*), \tag{3.2}$$

where $j^* \in S'$ *is such that* $w(i, j^*) = \min\{w(i, j) \mid j \in S'\}$*.*

(Proof) Let Γ be a minimum cost spanning tree of $\mathrm{K}_{S'}$. It suffices to show that $\Gamma \cup \{(i, j^*)\}$ is a minimum cost spanning tree of $\mathrm{K}_{S' \cup \{i\}}$. For $j \in S'$ with $j \neq j^*$ let us consider the path

$$j^* = j_0, j_1, \cdots, j_k = j \tag{3.3}$$

from j^* to j in Γ. By the definition of j^*, we have $w(i, j^*) \leq w(i, j)$. Then, since w is an ultrametric, we must have $w(j, j^*) \leq w(i, j)$. Since Γ is a minimum cost spanning tree of $\mathrm{K}_{S'}$ we must have

$$w(j_{p-1}, j_p) \leq w(j, j^*) \quad (p = 1, \cdots, k). \tag{3.4}$$

Therefore, we have

$$w(j_{p-1}, j_p) \leq w(i, j) \quad (p = 1, \cdots, k). \tag{3.5}$$

Hence, it follows from the optimality condition of the minimum cost spanning tree [13, Theorem 6.2] that $\Gamma \cup \{(i, j^*)\}$ is a minimum cost spanning tree of $\mathrm{K}_{S' \cup \{i\}}$ as required. □

Let $(T = (V, A), l)$ be a rooted tree and let r be the root of T. For $u, v \in V$, if u is on the unique path from v to r, we say that u is an *ancestor* of v and that v is a *descendant* of u. For $u, u' \in V$, v is called the *least common ancestor* if v is a common ancestor (i.e, v is an ancestor of both u and u') and every common ancestor of u and u' is an ancestor of v.

Theorem 3.3 *For each ultrametric MCST game* (N, c_w) *there exists a cost allocation game* (N, t_l) *on a rooted tree* (T, l) *such that*

$$c_w(S) = t_l(S) \quad (S \subseteq N). \tag{3.6}$$

(Proof) Let (N, c_w) be an ultrametric MCST game, where $w\colon N' \times N' \to \mathbb{R}_+$ is an ultrametric. By Lemma 3.1, there exists an equidistant rooted tree $(T' = (V', A'), l')$ which represents w where the set of leaves of T' is N'. Define $l\colon A' \to \mathbb{R}_+$ by

$$l(u, v) = \begin{cases} 0 & \text{if } (u, v) \text{ is on the path from 0 to the root,} \\ 2l'(u, v) & \text{otherwise} \end{cases} \quad ((u, v) \in A') \tag{3.7}$$

and let us consider the rooted tree (T', l).

It suffices to show that

$$c_w(S) = t_l(S') \quad (S \subseteq N) \tag{3.8}$$

since the desired rooted tree (T, l) can be derived by contracting all the edges on the path from 0 to the root of T', where we let the newly created vertex be the root of T, provided that we have (3.8).

We prove (3.8) by induction on $|S|$. For $S = \emptyset$ this is trivial. If $S = \{i\}$ for some $i \in N$, then we have

$$t_l(S') = d_{l'}(i, 0) = w(i, 0) = c_w(S) \tag{3.9}$$

since (T', l') represents w and (T', l') is equidistant.

Let $1 \leq |S| < n$ and $i \in N - S$. We will show $c_w(S \cup \{i\}) = t_l((S \cup \{i\})')$. Let $j^* \in S'$ be such that

$$w(i, j^*) = \min\{w(i, j) \mid j \in S'\} \tag{3.10}$$

and let $v^* \in V$ be the least common ancestor of i and j^* in T'. Let

$$P : i = v_0, a_1, v_1, a_2, \cdots, v_{k-1}, a_k, v_k = v^* \tag{3.11}$$

be the path from i to v^* in T'. Then, we have

$$w(i, j^*) = d_{l'}(i, j^*) = d_l(i, v^*) = \sum_{p=1}^{k} l(a_p) \tag{3.12}$$

since (T', l') represents w and (T', l') is equidistant.

Claim. For all $p = 1, \cdots, k$, if $a_p \in A_{S'}$, then we have $l(a_p) = 0$.

(Proof) Suppose that $a_p \in A_{S'}$ and $l(a_p) > 0$ for some $p = 1, \cdots, k$. Since $a_p \in A_{S'}$, vertex v_{p-1} is a common ancestor of i and some $\hat{\jmath} \in S'$. Then, since $l(a_p) > 0$ we must have $w(i, \hat{\jmath}) < w(i, j^*)$, which contradicts the choice (3.10) of j^*. (End of the proof of the Claim)

It follows from the Claim, the induction hypothesis and Lemma 3.2 that

$$\begin{aligned} t_l((S \cup \{i\})') &= \sum_{a \in A_{S'}} l(a) + \sum_{p=1}^{k} l(a_p) && (3.13)\\ &= t_l(S') + d_l(i, v^*) && (3.14)\\ &= c_w(S) + w(i, j^*) && (3.15)\\ &= c_w(S \cup \{i\}), && (3.16) \end{aligned}$$

which completes the proof of the present theorem. □

We have the following corollary from Theorem 3.3.

Corollary 3.4 *For any ultrametric MCST game the Shapley value, the nucleolus and the egalitarian allocation can be computed in* $\mathrm{O}(n^2)$ *time.*

(Proof) Given an ultrametric function w, we can construct the equidistant tree (T', l') which represents w in $\mathrm{O}(n^2)$ time (see [10], [11]). Then, by Lemma 2.1, the Shapley value, the nucleolus and the egalitarian allocation of the game (N, t_l) can be found in time dominated by $\mathrm{O}(n^2)$. Therefore, we have $\mathrm{O}(n^2)$ time bound for computations of all these solutions. □

We have seen that any ultrametric MCST game can be represented as a cost allocation game on a rooted tree (T, l). The rooted tree (T, l) can be derived from an equidistant rooted tree (T', l') by compressing the path from 0 to the root. We call such a rooted tree nearly equidistant. More precisely, a rooted tree (T, l) is called *nearly equidistant* if for each immediate descendant v of the root of T, the subtree rooted at v is equidistant. Note that an equidistant rooted tree is nearly equidistant.

Theorem 3.5 *For each ultrametric MCST game (N, c_w) there exists a cost allocation game (N, t_l) on a nearly equidistant rooted tree (T, l) such that $c_w = t_l$. Conversely, for each cost allocation game (N, t_l) on a nearly equidistant rooted tree (T, l), there exists an ultrametric MCST game (N, c_w) such that $c_w = t_l$.*

(Proof) The first statement follows from Theorem 3.3.

We prove the second statement. Let $(T = (V, A), l)$ be a nearly equidistant rooted tree whose set of leaves is N. Let v_p $(p = 0, 1, \cdots, k)$ be the immediate descendants of the root r and let T_p be the equidistant subtree rooted at v_p $(p = 0, 1, \cdots, k)$. For each $p = 0, 1, \cdots, k$ let us denote by δ_p the distance $d_l(i, r)$ from a leaf i of T_p to the root r. We can assume without loss of generality that $\delta_0 \geq \delta_1 \geq \cdots \geq \delta_k$.

Suppose that $\{r_1, \cdots, r_k, 0\}$ is a set of new vertices such that $\{r_1, \cdots, r_k, 0\} \cap V = \emptyset$. Define a rooted tree $(T' = (V', A'), l')$ as follows.

$$V' = V \cup \{r_1, \cdots, r_k, 0\}, \tag{3.17}$$

$$\begin{aligned} A' = {} & (A - \{(v_p, r) \mid p = 1, \cdots, k\}) \cup \{(v_p, r_p) \mid p = 1, \cdots, k\} \\ & \cup \{(r_p, r_{p-1}) \mid p = 2, \cdots, k\} \cup \{(r_1, r), (0, r_k)\}, \end{aligned} \tag{3.18}$$

$$l'(a) = \begin{cases} l(v_p, r) & \text{if } a = (v_p, r_p) \text{ for some } p = 1, \cdots, k, \\ \delta_0 - \delta_1 & \text{if } a = (r_1, r), \\ \delta_{p-1} - \delta_p & \text{if } a = (r_p, r_{p-1}) \text{ for some } p = 2, \cdots, k, \\ \delta_k & \text{if } a = (0, r_k), \\ l(a) & \text{otherwise} \end{cases} \quad (a \in A'). \tag{3.19}$$

It is easy to see that rooted tree (T', l') is equidistant, and hence, it follows from Lemma 3.1 that there exists an ultrametric $w \colon N' \times N' \to \mathbb{R}_+$ which is represented by (T', l'). The construction of (T, l) in the proof of Theorem 3.3 shows that we have $c_w = t_l$. □

4. Conclusion

We showed that any ultrametric MCST game can be represented as a cost allocation game on a rooted tree. The reduction is done in time $\mathrm{O}(n^2)$ and it follows that the Shapley value, the egalitarian allocation and the nucleolus of an ultrametric MCST game can be computed in time $\mathrm{O}(n^2)$.

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