ABSTRACT

OPTIMAL ALLOCATION OF THE ADVERTISING EXPENDITURES IN CONSIDERATION OF THE CARRY-OVER EFFECT

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When two competing companies, A and B, plan to advertise their products in a market for the advertising campaign term which can be divided into some periods, they face the problem of allocation of their advertising budgets to each period over the term. In this paper, this problem is treated with the techniques of game theory.

The effect of advertising expenditure allocated for a certain period in the advertising campaign term appears not only for this period but also for the periods after it. This after-effect that may be called "carry-over effect" is taken into account in the determination of the optimal allocation of the advertising budget.

The following assumptions is made.

(1) The advertising campaign term can be divided into n periods.

(2) Companies A and B have fixed advertising budgets A and B respectively.

(3) The potential sales in the ith period is s_i, where $\Sigma_{i=1}^{i=n} s_i = S$, the total potential sales.

(4) The carry-over effect of the advertising expenditure of a allocated for a certain period corresponds to the advertising expenditure of ar^k $(0 \leqq r \leqq 1)$ after k $(k = 1, 2, \ldots)$ periods.

(5) Sales by each company for the ith period can be calculate by multiplying s_i by his share of cumulative advertising capital (the sum total of the advertising expenditure for this period and the carry-over effect of all past advertising expenditures).

On these assumptions, let the pland of the advertising budget allocation of companies A and B for the advertising campaign term be $\mathbf{A}=(a_1, a_2, \ldots, a_n)$ and $\mathbf{B}=(b_1, b_2, \ldots, b_n)$ respectively, and difference $M(\mathbf{A}, \mathbf{B})$ in total sales between companies A and B will be given by the equation

$$M(\mathbf{A}, \mathbf{B}) = \sum_{i=1}^{n} \frac{\sum_{i=1}^{i} (a_j - b_j) r^{i-j}}{\sum_{j=1}^{i} (a_j + b_j) r^{i-j}} \; s_i$$

If this difference is taken as payoff to the company A and negative as payoff to the company B, this problem is treated as a two-person zero-sum game in which the company A tries to maximize $M(\mathbf{A}, \mathbf{B})$ and the company B to minimize it.