ABSTRACT

SOLUTION ALGORITHMS OF A SYSTEM OF EQUATIONS AND MINIMIZATION OF A FUNCTION BY A BRANCH AND BOUND METHOD

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This paper proposes new algorithms for solving a system of equations and minimizing a function in one or two variables. The algorithms use the Branch and Bound method. We show the algorithm for solving a system of equations in two variables. Let I be the rectangular set $\{X=(x_1,x_2): a_1 \leqq x_1 \leqq b_1,\ a_2 \leqq x_2 \leqq b_2\}$ and F be a mapping from I into the m-dimensional Euclidean space. The j-th component of F is denoted by f_j. We assume that the gradient vector Df_j of f_j is Lipschitz continuous on I, i.e., we have

$$\| Df_j(X) - Df_j(Y) \| \leqq L_j \| X-Y \| \text{ for each } X,Y \in I,\ j=1,2,\ldots,m,$$

where $\| \cdot \|$ denotes the ℓ_2-norm.

At first we divide the set I into two triangles. In branching operation, each triangle is divided into 4 small triangles. Then the function $f_j(x)$ is bounded on each triangle σ, i.e., we calculate v_j and u_j such that

$$v_j \leqq f_j(X) \leqq u_j \quad \text{for each} \quad X \in \sigma, \quad j=1,2,\ldots,m.$$

We easily see that there are no solutions of the equation $F(X)=0$ on σ if $v_j>0$ or $u_j<0$ for some j. Hence we can obtain an approximate set U of the solution set $S = \{X:F(X)=0\}$ by the next algorithm.

Step 1: let J be a set of two triangles into which the initial rectangular set is divided and $U=\phi$.

Step 2: if $J=\phi$ then end.

Step 3: pick out a triangle σ from J.

Step 4: calculate the lower bound v_j and the upper bound u_j of $f_j(x)$ on σ for each j.

Step 5: if $v_j>0$ or $u_j<0$ for some j then go to Step 2.

Step 6: if the size of σ is small enough then add the representative point of σ to U and go to Step 2.

Step 7: add 4 small triangles into which σ is divided to J and go to Step 2.

In the same way, we also propose an algorithm for minimizing a function.