ABSTRACT

ALGORITHMS USING A BRANCH AND BOUND METHOD FOR FINDING ALL REAL SOLUTIONS TO AN EQUATION OF ONE VARIABLE

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In this paper, we propose algorithms using a branch and bound method for finding all real solutions to an equation $f(x)=0$ of one variable x on an interval $[a,b]$. We denote by $\mathrm{P}([c,d])$ the subproblem in which we find all approximate solutions to $f(x)=0$ on $[c,d]\subset[a,b]$. The algorithms repeat the following procedure for each subproblem $\mathrm{P}([c,d])$ (the first subproblem is $\mathrm{P}([a,b])$) until we terminate all the subproblems: If we solve the subproblem $\mathrm{P}([c,d])$ then we terminate it, otherwise we split it into two new subproblems $\mathrm{P}([c,e])$ and $\mathrm{P}([e,d])$ for $e=(c+d)/2$. Here we can solve the subproblem $\mathrm{P}([c,d])$ when there is no solution of $f(x)=0$ on $[c,d]$ or when there exists a solution of $f(x)=0$ on $[c,d]$ and the width $d-c$ is sufficiently small.

We assume that there are two functions $g(x)$ and $h(x)$ such that $f(x) = g(x)-h(x)$ and one of the following conditions holds:

(1) The functions $g(x)$ and $h(x)$ are continuous and monotone increasing.

(2) The first derivatives $g'(x)$ and $h'(x)$ are continuous and monotone increasing.

(3) The second derivatives $g''(x)$ and $h''(x)$ are continuous and monotone increasing.

Then we present new sufficient conditions that the equation $f(x)=0$ has no solution on $[c,d]$. Those conditions are used in the algorithms for solving subproblems. We also show that if the function $f(x)$ is a sum of elementary functions then there exist the functions $g(x)$ and $h(x)$ which satisfy the above assumption.

Furthermore we present sufficient conditions that the equation $f(x)=0$ has exactly one solution on $[c,d]$. We propose two algorithms which solve the subproblem $\mathrm{P}([c,d])$ efficiently when the conditions hold.