

ABSTRACT

OPTIMAL WEIGHING PROBLEMS

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The false coin problem is stated as : You are given a balance and N coins ($N \geq 3$). $N-1$ coins are equal in weight, but the N -th is defective, and weighs somewhat more or less than each of $N-1$ coins. Devise a procedure to determine, in the minimum number of weighings, which is the false coin, and whether it is lighter or heavier than the others. This problem has been studied in various approaches for the last four decades, and several methods by which the problem can be solved have been given.

Almost no progress has been made, however, towards the solution of the problem of minimizing the expectation of the number of weighings in handling the false coin problem, and it has been still unsolved.

In this paper we call the problem of minimizing the expectation of the number of weighings 'Optimal Weighing Problem', and solve it by a new method named 'Stochastic Skelton Method'. At first, we classify the ways of weighing coins into three types in terms of probability. Then, we present a graphical representation of a procedure of weighing and call it 'Skelton Diagram'. Next, we present an algorithm for finding an optimal procedure of weighings by means of the Skelton Diagram.