ABSTRACT

Mathematical Considerations on the Relationship between the Ordering of Players and Winning Probability in Certain Types of Team Sports

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This paper is concerned with the relationship between the ordering of players and winning probability in two types of methods adopted in team sports. Suppose that there are two teams A and B each of which consists of more than one player. The team game between A and B is composed of a sequence of individual matches each of which is done between one player from A and one player from B, and the win or loss of team A is determined as a result of such individual matches.

There are several known methods to organize such a team game. Among them, this paper deals with the following two methods. In both methods, an initial ordering of players in each team is determined in advance. The first method is called a tie (or elimination series) which is carried out as follows. Initially the first players in the orderings of teams A and B play an individual match against each other. The winning player successively plays a match against the second player of the other team. The player that once lost the match is eliminated from his team. In this manner, a player successively plays matches until he loses. The team that eliminated all players of the other team wins the game. This method is adopted in judo and kendo. The second method is the same as the first one except that when a player wins the match, he is place at the last position of the ordering of remaining players of his team, and that at each round the first two players in the current orderings of both teams play a next individual match. This method is called an exterminatory series and is adopted in soft tennis.

This paper assumes that a nonnegative value is associated with each player that represents his strength, and that when player 1 with strength a plays an individual match against player 2 with strength b($a > 0$ or $b > 0$), the probability such that player 1 wins over player 2 is given by $p(a, b)$. Under this assumption, we first give a necessary and sufficient condition under which the probability P_A such that team A wins the game over team B by both methods is independent of the initial orderings of both teams. We then show that under this condition, the probability P_A by the first method is equal to P_A by the second method. Thirdly, Bradley-Terry model that has been used in the literature (i.e., $p(a, b) = a/(a + b)$) satisfies this condition. Finally, we show that $p(a, b)$ satisfies this condition if and only if $p(a, b) = f(a)/(f(a) + f(b))$, where $f(a)$ is an arbitrary monotone increasing function with $f(0) = 0$.