

ABSTRACT

On the Concepts of Nondominated Solutions for Multiobjective Linear Programming Problems with Interval Coefficients

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The coefficients of linear programming problems have been assumed to be determined as crisp values by the experts. However, the knowledge of the experts is not always so accurate to determine them as crisp values. The experts often have vague or imprecise knowledge of the coefficients. In such cases, it is not sufficient to fix the coefficients definitely. It is more sufficient to determine them only as regions in which the coefficients possibly take. From this point of view, the inexact linear programming, the interval linear programming and possibilistic linear programming have been proposed. In multiobjective linear programming, the ones with interval coefficients or with fuzzy coefficients have been proposed and the concept of efficiency is extended.

Bitran directly extended the efficiency to multiobjective linear programming problem with interval objective functions:

$$\text{Max}\{\Phi\mathbf{x}/\mathbf{x} \in \mathbf{R}^n, A\mathbf{x} = \mathbf{b}, \mathbf{x} \geq 0\},$$

where A is an m by n matrix, \mathbf{b} and \mathbf{x} are respectively m and n vectors. Φ is a set of p by n matrices with components c_{ij} in the interval $[\ell_{ij}, u_{ij}]$, $i = 1, 2, \dots, p$ and $j = 1, 2, \dots, n$. By Bitran, it was pointed out that two kinds of efficient solutions can be considered.

In this paper, the solution concepts for the interval multiobjective linear program are proposed in a different manner from Bitran's approach. First, given a reflective domination relation in the objective space, a strong (nonreflective) domination relation is defined. The domination relations in the decision space are composed from the domination relations in the objective space, and using them the nonreflective domination relations are defined. The properties of nonreflective domination relations are investigated. Next, four concepts of nondominated solutions for the interval multiobjective linear programming problem, i.e. possibly nondominated solutions, necessarily nondominated solutions, strong possibly nondominated solutions and weak necessarily nondominated solutions, are defined using four nonreflective domination relations. The properties of these nondominated solutions are discussed and it is shown that an arbitrary element of each nondominated solution set can be expressed by a convex combination of the corresponding nondominated extreme points. Moreover, the relationships with Bitran's two efficient solutions are clarified. Two of four nondominated solutions are equivalent to Bitran's two efficient solutions and the others are intermediated ones between them.