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BOUNDS FOR CALL COMPLETION PROBABILITIES IN LARGE-SCALE MOBILE COMMUNICATION NETWORKS

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Abstract In this paper, we propose a method to obtain upper and lower bounds of call completion probabilities in a mobile communication network having a large number of base stations. Each base station has its own zone where its radio wave reaches. These zones are overlapping with neighboring zones, and handovers become possible. However, the existence of overlapping zones makes the analysis of a network difficult. To overcome this difficulty, we introduce two smaller size models and show that the upper and lower bounds can be represented in terms of certain performance measures obtained from the smaller size models. Using our method, we can get the values of the bounds by standard methods such as simulation with less computational burden, where the tightness of the bounds can be controlled by choosing a set of focused base stations with appropriate size.

Keywords: Telecommunication, queue, Markov process, sample path argument, stochastic comparison

1. Introduction

In this paper, we deal with a mobile communication network having a large number of base stations. Each base station has its own zone where its radio wave reaches. Users can make calls within zones and moreover may try to move from a zone to another with keeping their talks on the calls. From the viewpoint of the system, this mechanism is referred as *handover* between base stations. The handovers become possible because the zones of base stations are overlapping. However, the existence of the overlapping zones makes the analysis of the network much difficult, because the behavior of calls in a base station affects the others in the network.

In Ref. [11], we have proposed a method to evaluate performance measures in such a mobile communication network. There, first we modeled the whole system as a large-scale Markov chain. Then we applied the aggregation method to derive an aggregated process that described the stochastic behavior of a smaller model in which only a certain number of base stations were focused and others were unfocused or ignored. Tight upper and lower bounds of various performance measures were derived from the smaller model. One remarkable result of that paper (Theorem 2 of Ref. [11]) is that upper and lower bounds of some performance measures can be obtained by using two particular models; one is the model corresponding to the original one under the situation that all the unfocused base stations are always full (this model is referred as *Model 1*), and the other is the model corresponding to the original one under the situation that all the unfocused base stations are idle at all times (this model is referred as *Model 2*). This result enables us to obtain the bounds numerically or by simulation with less computational burden. However, performance measures that we can evaluate in that manner are restricted to a certain class of performance measures. For example, call loss probabilities are included in the class but call completion probabilities, which we want to evaluate in this paper, are not.

The aim of this paper is to show that upper and lower bounds of call completion probabilities are given in terms of certain performance measures obtained from Models 1 and 2 mentioned above. The call completion probability is one of the most important measures since, in mobile communication networks, calls having started their talks may forcibly be terminated because of unsuccessful handovers and such call terminations are very unpleasant for users. For the purpose, we take an approach different from that taken in the previous paper; while in the previous paper we used the aggregation method, the weak D-Markov chain (Markov set-chain) theory [2, 12] and the Markov decision theory [4, 9] as mathematical tools, in this paper we use sample path arguments and comparison methods for stochastic models [7].

Steps of our analysis are as follows. First we construct Markov chains representing the behaviors of the original model, Model 1 and Model 2 on a common probability space, where the state of each model is represented as a random vector whose kth element is the number of calls served at base station k. We show that the states of the Markov chains satisfy certain monotone properties. From the monotone properties and the Markov chain convergence theorem, it is derived that the ordinary stochastic order holds between the states of the chains in steady state. This result corresponds to Theorem 2 of Ref. [11]. Furthermore, studying the Markov chains, we derive certain monotone properties for the conditional call-completion probabilities when the initial states of the chains are given. Upper and lower bounds of call completion probabilities are obtained by using these monotone properties, and they can easily be estimated by simulation.

Here we would shortly mention related works. Hong and Rappaport [3] proposed models for a single base station capturing handover mechanisms, and derived algorithms to evaluate performance measures. Ohmikawa and Takagi [8] and Takagi, Sakamaki and Miyashiro [10] dealt with models similar to [3], but they further considered a local network to make the evaluations more accurate. Mcmillan [6] proposed models capturing handover mechanisms and assignment mechanisms. Lagrange and Jabbari [5] gave a model with call generations in a zone with some overlapping areas to neighboring zones, and described assignment mechanisms for them. However, all of these papers cited above considered only a single base station or a local network. Hence their results are somewhat approximations and accuracy of them is not known. There are some other papers that considered the whole network as Everitt [1], but their models are too simple for practical applications.

The rest of the paper is constructed as follows. In Section 2 a mobile communication network that we concern is described. Section 3 is the central part of the paper. In Subsection 3.1, we define two models (Model 1 and Model 2) and state the main theorem that gives upper and lower bounds of call completion probabilities. Lemmas needed to prove the theorem are prepared in Subsections 3.2 and 3.3, and the proof of the theorem is presented in Subsection 3.4. In Section 4 we show some numerical results and in Section 5 we conclude the paper with a brief summary.

2. Model Description

2.1. Zones and areas

Here we present our model of a mobile communication network [11]. There are N base stations in the network and they are labeled from 1 to N. Each base station has its own zone. The zone of base station k is referred as *zone* k. The area that is covered with zone k

only is called *area* k, and the area where zone k and zone l overlap is called *area* (k,l) (see Figure 1). Area (l,k) is the same area as area (k,l). For simplicity, we assume that there are no areas where three or more zones overlap. Let $\mathcal{A} = \{k : 1 \le k \le N\}$ be the index set of base stations and $\mathcal{B} = \{(k,l) : k,l \in \mathcal{A},\ k \ne l,\ \text{zone } k \text{ overlaps with zone } l\}$ the index set of overlapping areas. For $k \in \mathcal{A}$, let $\mathcal{C}^{(k)} = \{l : (k,l) \in \mathcal{B}\}$ be the index set of zones overlapping with zone k.

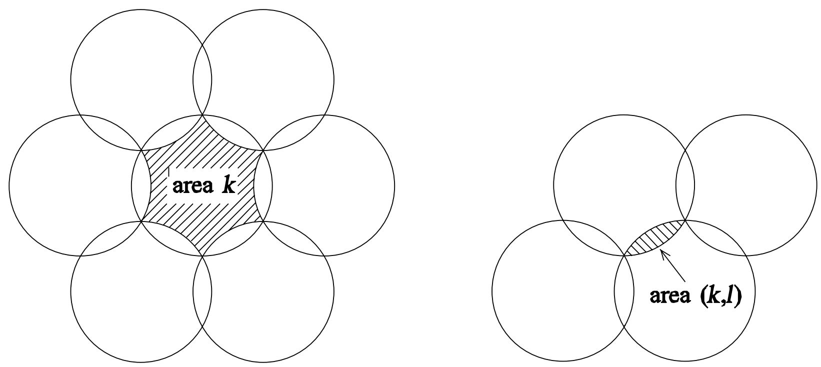

Figure 1: Area k and area (k,l)

Remark 1 *We have assumed that there exist no areas where three or more zones overlap. We make this assumption from the following two reasons. First, in many mobile communication systems, users (mobile stations) receive at most two strongest radio waves among various waves from various base stations. Our model imagines such a situation. The second reason is a technical one. If we allow areas where three or more zones overlap, the model and analysis become very complicated. We have to introduce a new assignment scheme for handover calls and newly generated calls in such an area, and the proof of monotone properties we use in this paper would become tremendously difficult, even if it is not impossible.*

On the other hand, we have not put restrictions on the configuration of the network. A typical configuration is a hexagon model shown in Figure 2 in Section 4. But our model is not restricted to it. The number of neighboring base stations may be different among base stations, and parameters (except for call holding times) may be different among areas and base stations.

2.2. Channels and calls

Base station $k \in \mathcal{A}$ has c_k channels among which g_k channels are reserved for handover calls as *guard channels* $(0 \le g_k \le c_k)$. Calls behave in the following manner.

(1) *Call generation.* New calls are generated in area $k \in \mathcal{A}$ according to a Poisson process with parameter λ_k, and in area $(k,l) \in \mathcal{B}$ according to a Poisson process with parameter $\lambda_{(k,l)}$. (Of course we assume that $\lambda_{(k,l)} = \lambda_{(l,k)}$.)

(2) *Channel assignment for new calls.* When a new call is generated in area $k \in \mathcal{A}$, it is served at base station k if more than g_k channels are idle. If not, it is rejected. When a new call is generated in area $(k,l) \in \mathcal{B}$, base station k or base station l is selected with equal probability. Suppose that base station k is selected. Then the call is served at base station k if more than g_k channels are idle. If not, base station l is reselected. The call is served at base station l if more than g_l channels are idle. If not, it is rejected. If a call is rejected from the first selected base station, it is referred as *an overflow call.*

(3) *Call holding times.* Call holding times (lengths of talks) are exponentially distributed random variables with parameter μ irrespective of areas in which they are generated and base stations at which they are served.

(4) *Call residence times.* Call residence times in base station k are exponentially distributed random variables with parameter γ_k. If the holding time of a call served at base station k ends before the residence time is over, then the call leaves the system. We refer to such a call as *a completed call.* If the residence time is over before the holding time ends, then the call tries to move to a neighboring base station in $\mathcal{C}^{(k)}$. At that time the call selects base station l in $\mathcal{C}^{(k)}$ with probability $p_{k,l}$. We denote by $\gamma_{k,l} = p_{k,l}\gamma_k$, the rate at which a call in base station k is handed over to base station l.

(5) *Channel assignment for handover calls.* When a call in base station k is handed over to base station l, it is served in base station l if there exists an idle channel in base station l. In this case the handover is said to be successful. If not, the call is forcibly terminated. In this case the handover is said to be unsuccessful.

Call generation processes, call holding times, call residence times, selections of first assigned base stations, and selections of handover destinations are assumed to be mutually independent.

2.3. Markov chain model

We denote by $X_k(t)$ the number of calls (active channels) in base station k at time t. Since our model is Markovian, $X_k(t)$ represents the state of base station k. The state of the whole network is represented as a vector $\boldsymbol{X}(t) = (X_1(t), X_2(t), \cdots, X_N(t))$. It is easily seen that $\{\boldsymbol{X}(t)\}$ forms a Markov chain on the state space $\mathcal{S} = \{(x_1, x_2, \cdots, x_N) : 0 \le x_k \le c_k,\ 1 \le k \le N\}$. We denote by $\boldsymbol{Q}$ the transition rate matrix of the chain but here we do not give its precise description [11] because it is very complicate and we do not use it in this paper.

3. Upper and Lower Bounds for a Call Completion Probability

3.1. Main theorem

Let θ_k denote the call completion probability of an arbitrary call whose talk starts at base station k. Remind that such a call is generated in zone k. Since the model is Markovian, we may calculate the value of θ_k by using some standard method. However, the number of states in $\mathcal{S}$ is equal to $M = \prod_{k=1}^{N}(c_k + 1)$. If N is large, then M becomes tremendously large, and it is practically impossible to get θ_k numerically. Even it is not easy to estimate θ_k by simulation.

To overcome this difficulty, we will introduce two smaller size models called *Model 1* and *Model 2*. Upper and lower bounds for θ_k will be given in terms of measures that can be obtained from Model 1 and Model 2. Let us focus on N_0 base stations here, and without loss of generality we assume that they are labeled from 1 to N_0. Other unfocused base stations are labeled from $N_0 + 1$ to N. Let $\mathcal{A}_0 = \{k : 1 \le k \le N_0\}$ be the index set of focused base stations and $\overline{\mathcal{A}}_0 = \{k : N_0 + 1 \le k \le N\}$ the index set of unfocused base stations. We denote by $\mathcal{S}_0 = \{(s_1, s_2, \cdots, s_{N_0}) : 0 \le s_k \le c_k,\ 1 \le k \le N_0\}$ the set of possible states for base stations in $\mathcal{A}_0$. Model 1 corresponds to the original model under the situation that all the unfocused base stations are always full, and Model 2 corresponds to the original model under the situation that all the unfocused base stations are always idle. Since the states of the unfocused base stations are fixed in both Model 1 and Model 2, the state space of each model becomes $\mathcal{S}_0$. The number of states in $\mathcal{S}_0$ is equal to $M_0 = \prod_{k=1}^{N_0}(c_k + 1)$, and usually it is far smaller than M.

Let $\mathcal{B}_0 = \{(k,l) : k,l \in \mathcal{A}_0,\ k \neq l$, zone k overlaps with zone $l\}$ be the index set of overlapping areas between the zones of the focused base stations. For $k \in \mathcal{A}_0$, let $\mathcal{C}_0^{(k)} = \mathcal{A}_0 \cap \mathcal{C}^{(k)}$ be the index set of focused base stations whose zones overlap with that of base station k and $\overline{\mathcal{C}}_0^{(k)} = \overline{\mathcal{A}}_0 \cap \mathcal{C}^{(k)}$ the index set of unfocused base stations whose zones overlap with that of base station k. Precise descriptions of Model 1 and Model 2 are as follows.

- The set of base stations in each model is $\mathcal{A}_0$ and that of overlapping areas is $\mathcal{B}_0$. Calls in areas indexed in $\mathcal{A}_0$ and $\mathcal{B}_0$ behave as in the original model. In Model 1, new calls generated in area (k,l), $k \in \mathcal{A}_0$, $l \in \overline{\mathcal{A}}_0$, are all regarded as new calls generated in area k. On the other hand, in Model 2, a half of such new calls are regarded as new calls generated in area k since the other half will be immediately served in base station $l \in \overline{\mathcal{A}}_0$. This requires a change in the call generation rate in area $k \in \mathcal{A}_0$. Handover calls from base stations in $\mathcal{A}_0$ to those in $\overline{\mathcal{A}}_0$ are regarded as forcibly terminated calls in Model 1, and as completed calls in Model 2. For $k \in \mathcal{A}_0$, we denote by $\gamma'_{k,\infty}$ the rate at which calls served at base station k move to other base stations in $\overline{\mathcal{A}}_0$. In Model 1, there are handover calls from base stations in $\overline{\mathcal{A}}_0$ to those in $\mathcal{A}_0$. For $k \in \mathcal{A}_0$, the total rate at which such handovers to base station k occur is denoted by $\Gamma'_{\infty,k}$.
- Parameters of Model 1 and Model 2 are set as follows. They are represented with primes (') to distinguish from the parameters of the original model. For $k \in \mathcal{A}_0$,

$$\lambda'_k = \begin{cases} \lambda_k + \sum_{j \in \overline{\mathcal{C}}_0^{(k)}} \lambda_{(j,k)} & \text{for Model 1,} \\ \lambda_k + \frac{1}{2}\sum_{j \in \overline{\mathcal{C}}_0^{(k)}} \lambda_{(j,k)} & \text{for Model 2,} \end{cases} \qquad \Gamma'_{\infty,k} = \sum_{l \in \overline{\mathcal{C}}_0^{(k)}} c_l \gamma_{l,k},$$

$$\lambda'_{(k,l)} = \lambda_{(k,l)},\ l \in \mathcal{C}_0^{(k)}, \quad \mu' = \mu, \quad \gamma'_{k,l} = \gamma_{k,l},\ l \in \mathcal{C}_0^{(k)}, \quad \gamma'_{k,\infty} = \sum_{l \in \overline{\mathcal{C}}_0^{(k)}} \gamma_{k,l}\,.$$

For Model m ($m \in \{1,2\}$), we denote by $X_k^{[m]}(t)$ the number of calls in base station k at time t. Since our model is Markovian, $X_k^{[m]}(t)$ represents the state of base station k. The state of the whole network is represented as a vector $\boldsymbol{X}^{[m]}(t) = (X_1^{[m]}(t), \cdots, X_{N_0}^{[m]}(t))$. It is easily seen that $\{\boldsymbol{X}^{[m]}(t)\}$ forms a Markov chain on the state space $\mathcal{S}_0$. In the following sections, we refer to the original model as *Model 0* and $\boldsymbol{X}(t)$ as $\boldsymbol{X}^{[0]}(t)$.

In steady state, for Model m ($m \in \{0,1,2\}$), let $X_k^{[m]}$ denote the number of calls in base station k and $\boldsymbol{X}^{[m]}$ denote a vector $(X_k^{[m]},\ k \in \mathcal{A}^{[m]})$, where $\mathcal{A}^{[0]} = \mathcal{A}$ and $\mathcal{A}^{[1]} = \mathcal{A}^{[2]} = \mathcal{A}_0$. We define $P^{[m]}(\boldsymbol{x})$, $\bar{P}_k^{[m]}(x_k)$ and $\bar{P}_{k,l}^{[m]}(x_k, x_l)$ as $P^{[m]}(\boldsymbol{x}) = \Pr(\boldsymbol{X}^{[m]} = \boldsymbol{x})$, $\bar{P}_k^{[m]}(x_k) = \Pr(X_k^{[m]} \geq x_k)$ and $\bar{P}_{k,l}^{[m]}(x_k, x_l) = \Pr(X_k^{[m]} \geq x_k,\ X_l^{[m]} \geq x_l)$, respectively. We also define the following measures for $k \in \mathcal{A}_0$.

- $\zeta_k^{[m]}$, $m \in \{0,1,2\}$: the probability in Model m that an arbitrary call generated in area k starts its talk (holding time) at base station k and successfully completes the talk at some base station.
- $\xi_{(k,l)}^{[m]}$, $m \in \{0,1,2\}$: the probability in Model m that an arbitrary call generated in area (k,l) finds less than $c_l - g_l$ calls in base station l, first selects base station k, starts its talk (holding time) at the base station, and successfully completed the talk at some base station. Note that, for $m \in \{1,2\}$, $\xi_{(k,l)}^{[m]}$ is undefined when $l \in \overline{\mathcal{C}}_0^{(k)}$.

Upper and lower bounds for θ_k are given in the next theorem.

Theorem 1 *For $k \in \mathcal{A}_0$, let θ_k^{lower} and θ_k^{upper} be defined as*

$$\theta_k^{lower} = \frac{\Lambda_k \zeta_k^{[1]} - \left(\sum_{l \in \mathcal{C}_0^{(k)}} \lambda_{(k,l)} \xi_{(k,l)}^{[2]} + \frac{1}{2} \sum_{l \in \overline{\mathcal{C}}_0^{(k)}} \lambda_{(k,l)} \zeta_k^{[2]}\right)}{\left(\lambda_k + \frac{1}{2} \sum_{l \in \mathcal{C}_0^{(k)}} \lambda_{(k,l)} + \sum_{l \in \overline{\mathcal{C}}_0^{(k)}} \lambda_{(k,l)}\right)\left(1 - \phi_k^{[2]}\right) + \frac{1}{2} \sum_{l \in \mathcal{C}_0^{(k)}} \lambda_{(k,l)} \left(\phi_l^{[1]} - \phi_{(k,l)}^{[2]}\right)}, \tag{1}$$

$$\theta_k^{upper} = \frac{\Lambda_k \zeta_k^{[2]} - \sum_{l \in \mathcal{C}_0^{(k)}} \lambda_{(k,l)} \xi_{(k,l)}^{[1]}}{\left(\lambda_k + \frac{1}{2} \sum_{l \in \mathcal{C}^{(k)}} \lambda_{(k,l)}\right)\left(1 - \phi_k^{[1]}\right) + \frac{1}{2} \sum_{l \in \mathcal{C}_0^{(k)}} \lambda_{(k,l)} \left(\phi_l^{[2]} - \phi_{(k,l)}^{[1]}\right)}, \tag{2}$$

where $\Lambda_k = \lambda_k + \sum_{l \in \mathcal{C}^{(k)}} \lambda_{(k,l)}$, $\phi_k^{[m]} = \bar{P}_k^{[m]}(c_k - g_k)$ and $\phi_{(k,l)}^{[m]} = \bar{P}_{k,l}^{[m]}(c_k - g_k, c_l - g_l)$. Then, θ_k satisfies the following inequality.

$$\theta_k^{lower} \le \theta_k \le \theta_k^{upper} \tag{3}$$

Remark 2 *To apply formulas (1) and (2), we need to know the values of $\phi_k^{[m]}$, $\phi_{(k,l)}^{[m]}$, $\zeta_k^{[m]}$ and $\xi_{(k,l)}^{[m]}$, $m = 1, 2$. Call loss probabilities $\phi_k^{[m]}$ and $\phi_{(k,l)}^{[m]}$, $m = 1, 2$, are calculated from the steady state probabilities of Model m, and their values are easily obtained from numerical analysis or simulation. For call completion probabilities $\zeta_k^{[m]}$ and $\xi_{(k,l)}^{[m]}$, $m = 1, 2$, it seems difficult to get their values from numerical analysis, and hence we have to use simulation. Fortunately, the size of Model m is not so large, we can easily accomplish a simulation to estimate them. In the numerical examples shown in Section 4, we use simulation.*

In order to prove Theorem 1, we prepare some lemmas in the next two subsections. The proof of Theorem 1 will be given in Subsection 3.4.

3.2. Monotone properties for the models

Here we will present some monotone properties for the Markov chains $\{\boldsymbol{X}^{[m]}(t)\}$, $m = 0, 1, 2$, defined in the previous subsection. For the purpose, first we introduce sequences of independent random variables, and using them we construct Markov chains $\{\hat{\boldsymbol{X}}^{[m]}(t) = (\hat{X}_k^{[m]}(t),\ k \in \mathcal{A}^{[m]})\}$, $m = 0, 1, 2$, on a common probability space $(\Omega, \mathcal{F}, P)$ such that, for $m \in \{0, 1, 2\}$, the state transitions of the Markov chain $\{\hat{\boldsymbol{X}}^{[m]}(t)\}$ and those of $\{\boldsymbol{X}^{[m]}(t)\}$ obey the same probability law, i.e., they are governed by the same transition rate matrix.

We introduce sequences of interarrival times, call holding times, call residence times, selections of base stations at generations of calls in overlapping areas and selections of destination base stations at handovers in the original model. Since these random variables are mutually independent and random variables representing times are exponentially distributed, we may consider from the strong Markov property that the processes restart at every epoch of state transition in stochastic sense. This allows us to regenerate these random variables at every transition epoch without changing the probability law of the processes.

A precise construction of the sequences of random variables is as follows. Indices k, l, i_k and n used below are for $k \in \mathcal{A}$, $l \in \mathcal{C}^{(k)}$, $i_k \in [1, c_k]$ and $n \ge 1$. Let $\tau_k(n)$ be an exponentially distributed random variable with parameter λ_k. $\tau_k(n)$ is used for determining a call generation epoch in area k. Let $\tau_{(k,l)}(n)$ denote an exponentially distributed random variable with parameter $\lambda_{(k,l)}$ and $j_{(k,l)}(n)$ denote a randomly selected base station from $\{k, l\}$ with equal probability. The pair $(\tau_{(k,l)}(n), j_{(k,l)}(n))$ is used for determining a call generation epoch in area (k, l) and the first selected base station by the call. Let $s_{k,i_k}(n)$ denote an exponentially distributed random variable with parameter μ. $s_{k,i_k}(n)$ is used for determining the remaining service time of a call at channel i_k of base station k. We use a vector representation $\boldsymbol{s}_k(n) = (s_{k,i_k}(n))$ for them. Let $\eta_{k,i_k}(n)$ denote an exponentially distributed random variable with parameter γ_k and $d_{k,i_k}(n)$ denote a randomly selected base station from $\mathcal{C}^{(k)}$ with probabilities $(p_{k,l}, l \in \mathcal{C}^{(k)})$. The pair $(\eta_{k,i_k}(n), d_{k,i_k}(n))$ is used for determining the remaining residence time of a call at channel i_k of base station k and the base station to which the call is handed over. We use a vector representation $\boldsymbol{\eta}_k(n) = ((\eta_{k,i_k}(n), d_{k,i_k}(n)))$ for them. We assume that all these random variables and random selections of base stations are mutually independent. To the basic probability space $(\Omega, \mathcal{F}, P)$, we define Ω as the set of possible realizations of the sequences of random variables $\{\tau_k(n), (\tau_{(k,l)}(n), j_{(k,l)}(n)), \boldsymbol{s}_k(n), \boldsymbol{\eta}_k(n), k \in \mathcal{A}, l \in \mathcal{C}^{(k)}\}_{n=1}^{\infty}$ and assume that $\mathcal{F}$ and P are properly defined such that the sequences of random variables have the properties described above.

We define a sequence of inter-transition times $\{T(n)\}$ as

$$T(n) = \min\{\tau_k(n), \tau_{(k,l)}(n), s_{k,i_k}(n), \eta_{k,i_k}(n) \,:\, k \in \mathcal{A},\, l \in \mathcal{C}^{(k)},\, i_k \in [1, c_k]\}, \tag{4}$$

and a sequence of transition times as $t_0 = 0$ and

$$t_n = \sum_{m=1}^{n} T(m) \tag{5}$$

for $n \geq 1$. We consider t_n is the nth transition epoch of the chains and, in each chain, the random variable attaining the minimum in (4) indicates the kind of transition at that time. Namely, if $T(n) = \tau_k(n)$, a call is generated in area k at time t_n; if $T(n) = \tau_{(k,l)}(n)$, a call is generated in area (k, l) at time t_n and base station $j_{(k,l)}(n)$ is selected first; if $T(n) = s_{k,i_k}(n)$, a call served by channel i_k in base station k ends its talk at time t_n; if $T(n) = \eta_{k,i_k}(n)$, a call served by channel i_k in base station k moves to base station $d_{k,i_k}(n)$ at time t_n. Note that, in each chain, if it is impossible for such a transition to occur at time t_n, no real state transitions occur at that time and we regard such t_n as a shadow epoch of state transition.

For each $m \in \{0, 1, 2\}$, we give the initial state of $\{\hat{\boldsymbol{X}}^{[m]}(t)\}$ by $\boldsymbol{x}^{[m]} \in \mathcal{S}^{[m]}$, i.e., $\hat{\boldsymbol{X}}^{[m]}(0) = \boldsymbol{x}^{[m]}$, where $\mathcal{S}^{[0]} = \mathcal{S}$ and $\mathcal{S}^{[1]} = \mathcal{S}^{[2]} = \mathcal{S}_0$, and construct sample paths of $\{\hat{\boldsymbol{X}}^{[m]}(t)\}$ from realizations of the sequences of random variables $\{\tau_k(n), (\tau_{(k,l)}(n), j_{(k,l)}(n)), \boldsymbol{s}_k(n), \boldsymbol{\eta}_k(n), k \in \mathcal{A}, l \in \mathcal{C}^{(k)}\}_{n=1}^{\infty}$. That is, for each $\omega \in \Omega$, a sample path $\{\hat{\boldsymbol{X}}^{[m]}(t;\omega)\}$ is given as a function of the sequence $\{\tau_k(n;\omega), (\tau_{(k,l)}(n;\omega), j_{(k,l)}(n;\omega)), \boldsymbol{s}_k(n;\omega), \boldsymbol{\eta}_k(n;\omega), k \in \mathcal{A}, l \in \mathcal{C}^{(k)}\}_{n=1}^{\infty}$ in the following manner. Below we assume that, when there exist $\hat{X}_k^{[m]}(t;\omega)$ calls in base station k in Model m, channels labeled from 1 to $\hat{X}_k^{[m]}(t;\omega)$ are occupied and channels labeled from $\hat{X}_k^{[m]}(t;\omega) + 1$ to c_k are idle.

(i) In the case of $T(n;\omega) = \tau_k(n;\omega)$:
   (a) When $k \in \mathcal{A}_0$, for each $m \in \{0, 1, 2\}$, if $\hat{X}_k^{[m]}(t_{n-1};\omega) < c_k - g_k$, then $\hat{X}_k^{[m]}(t_n;\omega) = \hat{X}_k^{[m]}(t_{n-1};\omega) + 1$; if not, $\hat{X}_k^{[m]}(t_n;\omega) = \hat{X}_k^{[m]}(t_{n-1};\omega)$. Other elements of $\hat{\boldsymbol{X}}^{[m]}(t;\omega)$ remain unchanged at time $t_n(\omega)$.
   (b) When $k \in \overline{\mathcal{A}}_0$, $\hat{\boldsymbol{X}}^{[1]}(t;\omega)$ and $\hat{\boldsymbol{X}}^{[2]}(t;\omega)$ remain unchanged at time $t_n(\omega)$ and $\hat{\boldsymbol{X}}^{[0]}(t;\omega)$ changes in the same manner as (i)-(a) at that time.

(ii) In the case of $T(n;\omega) = \tau_{(k,l)}(n;\omega)$ and $j_{(k,l)}(n;\omega) = k$:

(a) When $k, l \in \mathcal{A}_0$, for each $m \in \{0,1,2\}$, if $\hat{X}_k^{[m]}(t_{n-1};\omega) < c_k - g_k$, then $\hat{X}_k^{[m]}(t_n;\omega) = \hat{X}_k^{[m]}(t_{n-1};\omega) + 1$ and $\hat{X}_l^{[m]}(t_n;\omega) = \hat{X}_l^{[m]}(t_{n-1};\omega)$; if $\hat{X}_k^{[m]}(t_{n-1};\omega) \geq c_k - g_k$ and $\hat{X}_l^{[m]}(t_{n-1};\omega) < c_l - g_l$, then we obtain $\hat{X}_k^{[m]}(t_n;\omega) = \hat{X}_k^{[m]}(t_{n-1};\omega)$ and $\hat{X}_l^{[m]}(t_n;\omega) = \hat{X}_l^{[m]}(t_{n-1};\omega)+1$; otherwise, we obtain $\hat{X}_k^{[m]}(t_n;\omega) = \hat{X}_k^{[m]}(t_{n-1};\omega)$ and $\hat{X}_l^{[m]}(t_n;\omega) = \hat{X}_l^{[m]}(t_{n-1};\omega)$. Other elements of $\hat{\boldsymbol{X}}^{[m]}(t;\omega)$ remain unchanged at time $t_n(\omega)$.

(b) When $k \in \mathcal{A}_0$ and $l \in \overline{\mathcal{A}}_0$, for each $m \in \{1,2\}$, if $\hat{X}_k^{[m]}(t_{n-1};\omega) < c_k - g_k$, then $\hat{X}_k^{[m]}(t_n;\omega) = \hat{X}_k^{[m]}(t_{n-1};\omega)+1$; if not, $\hat{X}_k^{[m]}(t_n;\omega) = \hat{X}_k^{[m]}(t_{n-1};\omega)$; other elements of $\hat{\boldsymbol{X}}^{[m]}(t;\omega)$ remain unchanged at time $t_n(\omega)$. On the other hand, $\hat{\boldsymbol{X}}^{[0]}(t;\omega)$ changes in the same manner as (ii)-(a) at time $t_n(\omega)$.

(c) When $k \in \overline{\mathcal{A}}_0$ and $l \in \mathcal{A}_0$, $\hat{\boldsymbol{X}}^{[1]}(t;\omega)$ changes in the same manner as (ii)-(b) but replacing k with l at time $t_n(\omega)$ and $\hat{\boldsymbol{X}}^{[2]}(t;\omega)$ remains unchanged at that time. On the other hand, $\hat{\boldsymbol{X}}^{[0]}(t;\omega)$ changes in the same manner as (ii)-(a) at time $t_n(\omega)$.

(d) When $k, l \in \overline{\mathcal{A}}_0$, $\hat{\boldsymbol{X}}^{[1]}(t;\omega)$ and $\hat{\boldsymbol{X}}^{[2]}(t;\omega)$ remain unchanged at time $t_n(\omega)$ and $\hat{\boldsymbol{X}}^{[0]}(t;\omega)$ changes in the same manner as (ii)-(a) at that time.

(iii) In the case of $T(n;\omega) = s_{k,i_k}(n;\omega)$:

(a) When $k \in \mathcal{A}_0$, for each $m \in \{0,1,2\}$, if $i_k \leq \hat{X}_k^{[m]}(t_{n-1};\omega)$, then $\hat{X}_k^{[m]}(t_n;\omega) = \hat{X}_k^{[m]}(t_{n-1};\omega) - 1$; if not, $\hat{X}_k^{[m]}(t_n;\omega) = \hat{X}_k^{[m]}(t_{n-1};\omega)$. Other elements of $\hat{\boldsymbol{X}}^{[m]}(t;\omega)$ remain unchanged at time $t_n(\omega)$.

(b) When $k \in \overline{\mathcal{A}}_0$, $\hat{\boldsymbol{X}}^{[1]}(t;\omega)$ and $\hat{\boldsymbol{X}}^{[2]}(t;\omega)$ remain unchanged at time $t_n(\omega)$ and $\hat{\boldsymbol{X}}^{[0]}(t;\omega)$ changes in the same manner as (iii)-(a) at that time.

(iv) In the case of $T(n;\omega) = \eta_{k,i_k}(n;\omega)$ and $d_{k,i_k}(n;\omega) = l$:

(a) When $k, l \in \mathcal{A}_0$, for each $m \in \{0,1,2\}$, if $i_k \leq \hat{X}_k^{[m]}(t_{n-1};\omega)$ and $\hat{X}_l^{[m]}(t_{n-1};\omega) < c_l$, then $\hat{X}_k^{[m]}(t_n;\omega) = \hat{X}_k^{[m]}(t_{n-1};\omega) - 1$ and $\hat{X}_l^{[m]}(t_n;\omega) = \hat{X}_l^{[m]}(t_{n-1};\omega) + 1$; if $i_k \leq \hat{X}_k^{[m]}(t_{n-1};\omega)$ and $\hat{X}_l^{[m]}(t_{n-1};\omega) = c_l$, then $\hat{X}_k^{[m]}(t_n;\omega) = \hat{X}_k^{[m]}(t_{n-1};\omega) - 1$ and $\hat{X}_l^{[m]}(t_n;\omega) = \hat{X}_l^{[m]}(t_{n-1};\omega)$; otherwise, $\hat{X}_k^{[m]}(t_n;\omega) = \hat{X}_k^{[m]}(t_{n-1};\omega)$ and $\hat{X}_l^{[m]}(t_n;\omega) = \hat{X}_l^{[m]}(t_{n-1};\omega)$. Other elements of $\hat{\boldsymbol{X}}^{[m]}(t;\omega)$ remain unchanged at time $t_n(\omega)$.

(b) When $k \in \mathcal{A}_0$ and $l \in \overline{\mathcal{A}}_0$, for model $m \in \{1,2\}$, if $i_k \leq \hat{X}_k^{[m]}(t_{n-1};\omega)$, then $\hat{X}_k^{[m]}(t_n;\omega) = \hat{X}_k^{[m]}(t_{n-1};\omega) - 1$; other elements of $\hat{\boldsymbol{X}}^{[m]}(t;\omega)$ remain unchanged at time $t_n(\omega)$. On the other hand, $\hat{\boldsymbol{X}}^{[0]}(t;\omega)$ changes in the same manner as (iv)-(a) at time $t_n(\omega)$.

(c) When $k \in \overline{\mathcal{A}}_0$ and $l \in \mathcal{A}_0$, if $\hat{X}_l^{[1]}(t_{n-1};\omega) < c_l$, then $\hat{X}_l^{[1]}(t_n;\omega) = \hat{X}_l^{[1]}(t_{n-1};\omega) + 1$; if not, $\hat{X}_l^{[1]}(t_n;\omega) = \hat{X}_l^{[1]}(t_{n-1};\omega)$. Other elements of $\hat{\boldsymbol{X}}^{[1]}(t;\omega)$ remain unchanged at time $t_n(\omega)$. On the other hand, $\hat{\boldsymbol{X}}^{[2]}(t;\omega)$ remains unchanged at time $t_n(\omega)$ and $\hat{\boldsymbol{X}}^{[0]}(t;\omega)$ changes in the same manner as (iv)-(a) at that time.

(d) When $k, l \in \overline{\mathcal{A}}_0$, $\hat{\boldsymbol{X}}^{[1]}(t;\omega)$ and $\hat{\boldsymbol{X}}^{[2]}(t;\omega)$ remain unchanged at time $t_n(\omega)$ and $\hat{\boldsymbol{X}}^{[0]}(t;\omega)$ changes in the same manner as (iv)-(a) at that time.

For $t \in (t_{n-1}(\omega), t_n(\omega))$, $\hat{\boldsymbol{X}}^{[m]}(t;\omega)$ is given as $\hat{\boldsymbol{X}}^{[m]}(t;\omega) = \hat{\boldsymbol{X}}^{[m]}(t_{n-1};\omega)$. Note that, since area (k,l) is the same area as area (l,k), case (ii) includes the case where $T(n;\omega) = \tau_{(k,l)}(n;\omega)$ and $j_{(k,l)}(n;\omega) = l$.

Now we compare sample paths of $\{\hat{\boldsymbol{X}}^{[0]}(t)\}$, $\{\hat{\boldsymbol{X}}^{[1]}(t)\}$ and $\{\hat{\boldsymbol{X}}^{[2]}(t)\}$.

Lemma 2 *Let $\hat{\boldsymbol{X}}^{[0]}_{\mathcal{A}_0}(t)$ be the restriction of $\hat{\boldsymbol{X}}^{[0]}(t)$ to the set of focused base stations, $\mathcal{A}_0$, i.e., $\hat{\boldsymbol{X}}^{[0]}_{\mathcal{A}_0}(t) = (\hat{X}^{[0]}_k(t),\, k \in \mathcal{A}_0)$. Let $\boldsymbol{x}^{[0]} \in \mathcal{S}$ and $\boldsymbol{x}^{[1]} \in \mathcal{S}_0$ (resp. $\boldsymbol{x}^{[2]} \in \mathcal{S}_0$ and $\boldsymbol{x}^{[0]} \in \mathcal{S}$) be the initial states of $\{\hat{\boldsymbol{X}}^{[0]}(t)\}$ and $\{\hat{\boldsymbol{X}}^{[1]}(t)\}$ (resp. $\{\hat{\boldsymbol{X}}^{[2]}(t)\}$ and $\{\hat{\boldsymbol{X}}^{[0]}(t)\}$), and assume that $\boldsymbol{x}^{[0]}_{\mathcal{A}_0} \le \boldsymbol{x}^{[1]}$ (resp. $\boldsymbol{x}^{[2]} \le \boldsymbol{x}^{[0]}_{\mathcal{A}_0}$), where $\boldsymbol{x}^{[0]}_{\mathcal{A}_0}$ is the restriction of $\boldsymbol{x}^{[0]}$ to $\mathcal{A}_0$. Then, $\hat{\boldsymbol{X}}^{[0]}_{\mathcal{A}_0}(t) \le \hat{\boldsymbol{X}}^{[1]}(t)$ (resp. $\hat{\boldsymbol{X}}^{[2]}(t) \le \hat{\boldsymbol{X}}^{[0]}_{\mathcal{A}_0}(t)$) for all $t \ge 0$ w.p. 1.*

Proof. For the purpose, it is sufficient to show that, for each $\omega \in \Omega$, for any $n \ge 1$, the inequality $\hat{\boldsymbol{X}}^{[0]}_{\mathcal{A}_0}(t_{n-1};\omega) \le \hat{\boldsymbol{X}}^{[1]}(t_{n-1};\omega)$ (resp. $\hat{\boldsymbol{X}}^{[2]}(t_{n-1};\omega) \le \hat{\boldsymbol{X}}^{[0]}_{\mathcal{A}_0}(t_{n-1};\omega)$) implies that $\hat{\boldsymbol{X}}^{[0]}_{\mathcal{A}_0}(t_n;\omega) \le \hat{\boldsymbol{X}}^{[1]}(t_n;\omega)$ (resp. $\hat{\boldsymbol{X}}^{[2]}(t_n;\omega) \le \hat{\boldsymbol{X}}^{[0]}_{\mathcal{A}_0}(t_n;\omega)$). We assume $\hat{\boldsymbol{X}}^{[0]}_{\mathcal{A}_0}(t_{n-1};\omega) \le \hat{\boldsymbol{X}}^{[1]}(t_{n-1};\omega)$.

(i) In the case of $T(n;\omega) = \tau_k(n;\omega)$:

(a) When $k \in \mathcal{A}_0$, if $\hat{X}^{[1]}_k(t_{n-1};\omega) < c_k - g_k$, then

$$\hat{X}^{[0]}_k(t_n;\omega) = \hat{X}^{[0]}_k(t_{n-1};\omega) + 1 \le \hat{X}^{[1]}_k(t_{n-1};\omega) + 1 = \hat{X}^{[1]}_k(t_n;\omega);$$

if $\hat{X}^{[0]}_k(t_{n-1};\omega) < c_k - g_k \le \hat{X}^{[1]}_k(t_{n-1};\omega)$, then

$$\hat{X}^{[0]}_k(t_n;\omega) = \hat{X}^{[0]}_k(t_{n-1};\omega) + 1 \le c_k - g_k \le \hat{X}^{[1]}_k(t_{n-1};\omega) = \hat{X}^{[1]}_k(t_n;\omega);$$

otherwise,

$$\hat{X}^{[0]}_k(t_n;\omega) = \hat{X}^{[0]}_k(t_{n-1};\omega) \le \hat{X}^{[1]}_k(t_{n-1};\omega) = \hat{X}^{[1]}_k(t_n;\omega).$$

Other elements of $\hat{\boldsymbol{X}}^{[0]}_{\mathcal{A}_0}(t;\omega)$ and $\hat{\boldsymbol{X}}^{[1]}(t;\omega)$ remain unchanged at time $t_n(\omega)$, and hence we obtain $\hat{\boldsymbol{X}}^{[0]}_{\mathcal{A}_0}(t_n;\omega) \le \hat{\boldsymbol{X}}^{[1]}(t_n;\omega)$.

(b) When $k \in \overline{\mathcal{A}}_0$, both $\hat{\boldsymbol{X}}^{[0]}_{\mathcal{A}_0}(t;\omega)$ and $\hat{\boldsymbol{X}}^{[1]}(t;\omega)$ remain unchanged at time $t_n(\omega)$. Hence we obtain $\hat{\boldsymbol{X}}^{[0]}_{\mathcal{A}_0}(t_n;\omega) \le \hat{\boldsymbol{X}}^{[1]}(t_n;\omega)$.

(ii) In the case of $T(n;\omega) = \tau_{(k,l)}(n;\omega)$ and $j_{(k,l)}(n;\omega) = k$:

(a) When $k, l \in \mathcal{A}_0$, if $\hat{X}^{[1]}_k(t_{n-1};\omega) < c_k - g_k$, then

$$\begin{aligned}
&\hat{X}^{[0]}_k(t_n;\omega) = \hat{X}^{[0]}_k(t_{n-1};\omega) + 1 \le \hat{X}^{[1]}_k(t_{n-1};\omega) + 1 = \hat{X}^{[1]}_k(t_n;\omega) \quad \text{and}\\
&\hat{X}^{[0]}_l(t_n;\omega) = \hat{X}^{[0]}_l(t_{n-1};\omega) \le \hat{X}^{[1]}_l(t_{n-1};\omega) = \hat{X}^{[1]}_l(t_n;\omega);
\end{aligned}$$

if $\hat{X}^{[0]}_k(t_{n-1};\omega) < c_k - g_k \le \hat{X}^{[1]}_k(t_{n-1};\omega)$, then

$$\begin{aligned}
&\hat{X}^{[0]}_k(t_n;\omega) = \hat{X}^{[0]}_k(t_{n-1};\omega) + 1 \le c_k - g_k \le \hat{X}^{[1]}_k(t_{n-1};\omega) = \hat{X}^{[1]}_k(t_n;\omega) \quad \text{and}\\
&\hat{X}^{[0]}_l(t_n;\omega) = \hat{X}^{[0]}_l(t_{n-1};\omega) \le \hat{X}^{[1]}_l(t_{n-1};\omega) \le \hat{X}^{[1]}_l(t_n;\omega);
\end{aligned}$$

if $\hat{X}^{[0]}_k(t_{n-1};\omega) \ge c_k - g_k$ and $\hat{X}^{[1]}_l(t_{n-1};\omega) < c_l - g_l$, then

$$\begin{aligned}
&\hat{X}^{[0]}_k(t_n;\omega) = \hat{X}^{[0]}_k(t_{n-1};\omega) \le \hat{X}^{[1]}_k(t_{n-1};\omega) = \hat{X}^{[1]}_k(t_n;\omega) \quad \text{and}\\
&\hat{X}^{[0]}_l(t_n;\omega) = \hat{X}^{[0]}_l(t_{n-1};\omega) + 1 \le \hat{X}^{[1]}_l(t_{n-1};\omega) + 1 = \hat{X}^{[1]}_l(t_n;\omega);
\end{aligned}$$

if $\hat{X}_k^{[0]}(t_{n-1};\omega) \geq c_k - g_k$ and $\hat{X}_l^{[0]}(t_{n-1};\omega) < c_l - g_l \leq \hat{X}_l^{[1]}(t_{n-1};\omega)$, then

$$\hat{X}_k^{[0]}(t_n;\omega) = \hat{X}_k^{[0]}(t_{n-1};\omega) \leq \hat{X}_k^{[1]}(t_{n-1};\omega) = \hat{X}_k^{[1]}(t_n;\omega) \quad \text{and}$$
$$\hat{X}_l^{[0]}(t_n;\omega) = \hat{X}_l^{[0]}(t_{n-1};\omega) + 1 \leq c_l - g_l \leq \hat{X}_l^{[1]}(t_{n-1};\omega) = \hat{X}_l^{[1]}(t_n;\omega);$$

otherwise,

$$\hat{X}_k^{[0]}(t_n;\omega) = \hat{X}_k^{[0]}(t_{n-1};\omega) \leq \hat{X}_k^{[1]}(t_{n-1};\omega) = \hat{X}_k^{[1]}(t_n;\omega) \quad \text{and}$$
$$\hat{X}_l^{[0]}(t_n;\omega) = \hat{X}_l^{[0]}(t_{n-1};\omega) \leq \hat{X}_l^{[1]}(t_{n-1};\omega) = \hat{X}_l^{[1]}(t_n;\omega).$$

Other elements of $\hat{\boldsymbol{X}}_{\mathcal{A}_0}^{[0]}(t;\omega)$ and $\hat{\boldsymbol{X}}^{[1]}(t;\omega)$ remain unchanged at time $t_n(\omega)$, and hence we obtain $\hat{\boldsymbol{X}}_{\mathcal{A}_0}^{[0]}(t_n;\omega) \leq \hat{\boldsymbol{X}}^{[1]}(t_n;\omega)$.

(b) When $k \in \mathcal{A}_0$ and $l \in \overline{\mathcal{A}}_0$, if $\hat{X}_k^{[1]}(t_{n-1};\omega) < c_k - g_k$, then

$$\hat{X}_k^{[0]}(t_n;\omega) = \hat{X}_k^{[0]}(t_{n-1};\omega) + 1 \leq \hat{X}_k^{[1]}(t_{n-1};\omega) + 1 = \hat{X}_k^{[1]}(t_n;\omega);$$

if $\hat{X}_k^{[0]}(t_{n-1};\omega) < c_k - g_k \leq \hat{X}_k^{[1]}(t_{n-1};\omega)$, then

$$\hat{X}_k^{[0]}(t_n;\omega) = \hat{X}_k^{[0]}(t_{n-1};\omega) + 1 \leq c_k - g_k \leq \hat{X}_k^{[1]}(t_{n-1};\omega) = \hat{X}_k^{[1]}(t_n;\omega);$$

otherwise,

$$\hat{X}_k^{[0]}(t_n;\omega) = \hat{X}_k^{[0]}(t_{n-1};\omega) \leq \hat{X}_k^{[1]}(t_{n-1};\omega) = \hat{X}_k^{[1]}(t_n;\omega).$$

Other elements of $\hat{\boldsymbol{X}}_{\mathcal{A}_0}^{[0]}(t;\omega)$ and $\hat{\boldsymbol{X}}^{[1]}(t;\omega)$ remain unchanged at time $t_n(\omega)$, and hence we obtain $\hat{\boldsymbol{X}}_{\mathcal{A}_0}^{[0]}(t_n;\omega) \leq \hat{\boldsymbol{X}}^{[1]}(t_n;\omega)$.

(c) When $k \in \overline{\mathcal{A}}_0$ and $l \in \mathcal{A}_0$, $\hat{X}_l^{[1]}(t;\omega)$ changes in the same manner as (ii)-(b) at time $t_n(\omega)$ but $\hat{X}_l^{[0]}(t;\omega)$ increases only if $\hat{X}_l^{[0]}(t_{n-1};\omega) < c_l - g_l$ and $\hat{X}_k^{[0]}(t_{n-1};\omega) \geq c_k - g_k$. Since other elements of $\hat{\boldsymbol{X}}_{\mathcal{A}_0}^{[0]}(t;\omega)$ and $\hat{\boldsymbol{X}}^{[1]}(t;\omega)$ remain unchanged at time $t_n(\omega)$, we, therefore, obtain $\hat{\boldsymbol{X}}_{\mathcal{A}_0}^{[0]}(t_n;\omega) \leq \hat{\boldsymbol{X}}^{[1]}(t_n;\omega)$.

(d) When $k, l \in \overline{\mathcal{A}}_0$, both $\hat{\boldsymbol{X}}_{\mathcal{A}_0}^{[0]}(t;\omega)$ and $\hat{\boldsymbol{X}}^{[1]}(t;\omega)$ remain unchanged at time $t_n(\omega)$. Hence we obtain $\hat{\boldsymbol{X}}_{\mathcal{A}_0}^{[0]}(t_n;\omega) \leq \hat{\boldsymbol{X}}^{[1]}(t_n;\omega)$.

(iii) In the case of $T(n;\omega) = s_{k,i_k}(n;\omega)$:

(a) When $k \in \mathcal{A}_0$, if $i_k \leq \hat{X}_k^{[0]}(t_{n-1};\omega)$, then

$$\hat{X}_k^{[0]}(t_n;\omega) = \hat{X}_k^{[0]}(t_{n-1};\omega) - 1 \leq \hat{X}_k^{[1]}(t_{n-1};\omega) - 1 = \hat{X}_k^{[1]}(t_n;\omega);$$

if $\hat{X}_k^{[0]}(t_{n-1};\omega) < i_k \leq \hat{X}_k^{[1]}(t_{n-1};\omega)$, then

$$\hat{X}_k^{[0]}(t_n;\omega) = \hat{X}_k^{[0]}(t_{n-1};\omega) \leq i_k - 1 \leq \hat{X}_k^{[1]}(t_{n-1};\omega) - 1 = \hat{X}_k^{[1]}(t_n;\omega);$$

otherwise,

$$\hat{X}_k^{[0]}(t_n;\omega) = \hat{X}_k^{[0]}(t_{n-1};\omega) \leq \hat{X}_k^{[1]}(t_{n-1};\omega) = \hat{X}_k^{[1]}(t_n;\omega).$$

Other elements of $\hat{\boldsymbol{X}}_{\mathcal{A}_0}^{[0]}(t;\omega)$ and $\hat{\boldsymbol{X}}^{[1]}(t;\omega)$ remain unchanged at time $t_n(\omega)$, and hence we obtain $\hat{\boldsymbol{X}}_{\mathcal{A}_0}^{[0]}(t_n;\omega) \leq \hat{\boldsymbol{X}}^{[1]}(t_n;\omega)$.

(b) When $k \in \overline{\mathcal{A}}_0$, both $\hat{\boldsymbol{X}}^{[0]}_{\mathcal{A}_0}(t;\omega)$ and $\hat{\boldsymbol{X}}^{[1]}(t;\omega)$ remain unchanged at time $t_n(\omega)$. Hence we obtain $\hat{\boldsymbol{X}}^{[0]}_{\mathcal{A}_0}(t_n;\omega) \le \hat{\boldsymbol{X}}^{[1]}(t_n;\omega)$.

(iv) In the case of $T(n;\omega) = \eta_{k,i_k}(n;\omega)$ and $d_{k,i_k}(n;\omega) = l$:

(a) When $k, l \in \mathcal{A}_0$, if $i_k \le \hat{X}^{[0]}_k(t_{n-1};\omega)$, then

$$\hat{X}^{[0]}_k(t_n;\omega) = \hat{X}^{[0]}_k(t_{n-1};\omega) - 1 \le \hat{X}^{[1]}_k(t_{n-1};\omega) - 1 = \hat{X}^{[1]}_k(t_n;\omega) \quad \text{and}$$
$$\hat{X}^{[0]}_l(t_n;\omega) = \min\{c_l, \hat{X}^{[0]}_l(t_{n-1};\omega) + 1\} \le \min\{c_l, \hat{X}^{[1]}_l(t_{n-1};\omega) + 1\} = \hat{X}^{[1]}_l(t_n;\omega);$$

if $\hat{X}^{[0]}_k(t_{n-1};\omega) < i_k \le \hat{X}^{[1]}_k(t_{n-1};\omega)$, then

$$\hat{X}^{[0]}_k(t_n;\omega) = \hat{X}^{[0]}_k(t_{n-1};\omega) \le i_k - 1 \le \hat{X}^{[1]}_k(t_{n-1};\omega) - 1 = \hat{X}^{[1]}_k(t_n;\omega) \quad \text{and}$$
$$\hat{X}^{[0]}_l(t_n;\omega) = \hat{X}^{[0]}_l(t_{n-1};\omega) \le \min\{c_l, \hat{X}^{[1]}_l(t_{n-1};\omega) + 1\} = \hat{X}^{[1]}_l(t_n;\omega);$$

otherwise,

$$\hat{X}^{[0]}_k(t_n;\omega) = \hat{X}^{[0]}_k(t_{n-1};\omega) \le \hat{X}^{[1]}_k(t_{n-1};\omega) = \hat{X}^{[1]}_k(t_n;\omega) \quad \text{and}$$
$$\hat{X}^{[0]}_l(t_n;\omega) = \hat{X}^{[0]}_l(t_{n-1};\omega) \le \hat{X}^{[1]}_l(t_{n-1};\omega) = \hat{X}^{[1]}_l(t_n;\omega).$$

Other elements of $\hat{\boldsymbol{X}}^{[0]}_{\mathcal{A}_0}(t;\omega)$ and $\hat{\boldsymbol{X}}^{[1]}(t;\omega)$ remain unchanged at time $t_n(\omega)$, and hence we obtain $\hat{\boldsymbol{X}}^{[0]}_{\mathcal{A}_0}(t_n;\omega) \le \hat{\boldsymbol{X}}^{[1]}(t_n;\omega)$.

(b) When $k \in \mathcal{A}_0$ and $l \in \overline{\mathcal{A}}_0$, $\hat{\boldsymbol{X}}^{[0]}_{\mathcal{A}_0}(t;\omega)$ and $\hat{\boldsymbol{X}}^{[1]}(t;\omega)$ change in the same manner as (iii)-(a) at time $t_n(\omega)$, and hence we obtain $\hat{\boldsymbol{X}}^{[0]}_{\mathcal{A}_0}(t_n;\omega) \le \hat{\boldsymbol{X}}^{[1]}(t_n;\omega)$.

(c) When $k \in \overline{\mathcal{A}}_0$ and $l \in \mathcal{A}_0$,

$$\hat{X}^{[0]}_l(t_n;\omega) \le \min\{c_l, \hat{X}^{[0]}_l(t_{n-1};\omega) + 1\} \le \min\{c_l, \hat{X}^{[1]}_l(t_{n-1};\omega) + 1\} = \hat{X}^{[1]}_l(t_n;\omega).$$

Other elements of $\hat{\boldsymbol{X}}^{[0]}_{\mathcal{A}_0}(t;\omega)$ and $\hat{\boldsymbol{X}}^{[1]}(t;\omega)$ remain unchanged at time $t_n(\omega)$, and hence we obtain $\hat{\boldsymbol{X}}^{[0]}_{\mathcal{A}_0}(t_n;\omega) \le \hat{\boldsymbol{X}}^{[1]}(t_n;\omega)$.

(d) When $k, l \in \overline{\mathcal{A}}_0$, both $\hat{\boldsymbol{X}}^{[0]}_{\mathcal{A}_0}(t;\omega)$ and $\hat{\boldsymbol{X}}^{[1]}(t;\omega)$ remain unchanged at time $t_n(\omega)$. Hence we obtain $\hat{\boldsymbol{X}}^{[0]}_{\mathcal{A}_0}(t_n;\omega) \le \hat{\boldsymbol{X}}^{[1]}(t_n;\omega)$.

In the same manner, we can prove that if $\hat{\boldsymbol{X}}^{[2]}(t_{n-1};\omega) \le \hat{\boldsymbol{X}}^{[0]}_{\mathcal{A}_0}(t_{n-1};\omega)$, then $\hat{\boldsymbol{X}}^{[2]}(t_n;\omega) \le \hat{\boldsymbol{X}}^{[0]}_{\mathcal{A}_0}(t_n;\omega)$ for all $n \ge 0$. □

For each $m \in \{0,1,2\}$, in the same manner as used for constructing $\{\hat{\boldsymbol{X}}^{[m]}(t)\}$, we can also construct Markov chains starting from different initial states, $\{{}^1\hat{\boldsymbol{X}}^{[m]}(t)\}$ and $\{{}^2\hat{\boldsymbol{X}}^{[m]}(t)\}$, on the same probability space $(\Omega, \mathcal{F}, P)$ such that, for $\nu \in \{1,2\}$, the state transitions of the Markov chain $\{{}^\nu\hat{\boldsymbol{X}}^{[m]}(t)\}$ and those of $\{\boldsymbol{X}^{[m]}(t)\}$ obey the same probability law. Comparing sample paths of $\{{}^1\hat{\boldsymbol{X}}^{[m]}(t)\}$ and $\{{}^2\hat{\boldsymbol{X}}^{[m]}(t)\}$ for each $m \in \{0,1,2\}$, we obtain the next lemma, which can be proved by using the same idea as used for deriving Lemma 2.

Lemma 3 *For $m \in \{0,1,2\}$, let ${}^1\boldsymbol{x}^{[m]} \in \mathcal{S}^{[m]}$ and ${}^2\boldsymbol{x}^{[m]} \in \mathcal{S}^{[m]}$ be the initial states of $\{{}^1\hat{\boldsymbol{X}}^{[m]}(t)\}$ and $\{{}^2\hat{\boldsymbol{X}}^{[m]}(t)\}$, and assume that ${}^1\boldsymbol{x}^{[m]} \le {}^2\boldsymbol{x}^{[m]}$. Then, ${}^1\hat{\boldsymbol{X}}^{[m]}(t) \le {}^2\hat{\boldsymbol{X}}^{[m]}(t)$ for all $t \ge 0$ w.p. 1.*

For random vectors $\boldsymbol{A}$ and $\boldsymbol{B}$ with values in $\mathcal{R}^n$, if $E[f(\boldsymbol{A})] \le E[f(\boldsymbol{B})]$ for all bounded nondecreasing function $f : \mathcal{R}^n \to \mathcal{R}$, then we denote $\boldsymbol{A} \le_{st} \boldsymbol{B}$. This relation $\le_{st}$ is called the usual stochastic order[7]. From Lemma 2, we obtain the next lemma.

Lemma 4

$$\boldsymbol{X}^{[2]} \le_{st} \boldsymbol{X}^{[0]}_{\mathcal{A}_0} \le_{st} \boldsymbol{X}^{[1]} \tag{6}$$

Proof. Consider vectors $\boldsymbol{x}^{[m]}$ in $\mathcal{S}^{[m]}$, $m = 0, 1, 2$, such that $\boldsymbol{x}^{[1]} \le \boldsymbol{x}^{[0]}_{\mathcal{A}_0} \le \boldsymbol{x}^{[2]}$, and for $m \in \{0, 1, 2\}$ let the initial states of the Markov chains $\{\hat{\boldsymbol{X}}^{[m]}(t)\}$ and $\{\boldsymbol{X}^{[m]}(t)\}$ be $\boldsymbol{x}^{[m]}$. The state transitions of $\{\hat{\boldsymbol{X}}^{[m]}(t)\}$ and those of $\{\boldsymbol{X}^{[m]}(t)\}$ obey the same probability law and hence we obtain that $\hat{\boldsymbol{X}}^{[m]}(t) =_d \boldsymbol{X}^{[m]}(t)$ for all $t \ge 0$, where the relation $=_d$ represents equality in distribution. Furthermore, Lemma 2 claims that $\Pr(\hat{\boldsymbol{X}}^{[0]}_{\mathcal{A}_0}(t) \le \hat{\boldsymbol{X}}^{[1]}(t)) = 1$ and $\Pr(\hat{\boldsymbol{X}}^{[2]}(t) \le \hat{\boldsymbol{X}}^{[0]}_{\mathcal{A}_0}(t)) = 1$ for all $t \ge 0$. Hence we obtain that $\boldsymbol{X}^{[0]}_{\mathcal{A}_0}(t) \le_{st} \boldsymbol{X}^{[1]}(t)$ and $\boldsymbol{X}^{[2]}(t) \le_{st} \boldsymbol{X}^{[0]}_{\mathcal{A}_0}(t)$ for all $t \ge 0$. For each $m \in \{0, 1, 2\}$, the Markov chain $\{\boldsymbol{X}^{[m]}(t)\}$ is irreducible and finite, and hence $\boldsymbol{X}^{[m]}(t)$ converges weakly to $\boldsymbol{X}^{[m]}$. Since the usual stochastic order is closed with respect to weak convergence [7], we obtain the assertion of the lemma. □

Let 1_A denote an indicator function of condition A. For a fixed x_k, $1_{\{x'_k \ge x_k\}}$ is nondecreasing and bounded with respect to x'_k and, for fixed x_k and x_l, $1_{\{x'_k \ge x_k, x'_l \ge x_l\}}$ is nondecreasing and bounded with respect to x'_k and x'_l. Hence we immediately obtain the next corollary from Lemma 4.

Corollary 5 *For $k, l \in \mathcal{A}_0$, for $x_k \in [0, c_k]$, $x_l \in [0, c_l]$,*

$$\bar{P}^{[2]}_k(x_k) \le \bar{P}^{[0]}_k(x_k) \le \bar{P}^{[1]}_k(x_k) \quad \text{and} \quad \bar{P}^{[2]}_{k,l}(x_k, x_l) \le \bar{P}^{[0]}_{k,l}(x_k, x_l) \le \bar{P}^{[1]}_{k,l}(x_k, x_l). \tag{7}$$

3.3. Monotone properties for the conditional call-completion probabilities

For Model m ($m \in \{0, 1, 2\}$), let $\alpha^{[m]}_k(\boldsymbol{x}^{[m]})$ be the conditional call-completion probability of a call served at base station k, given that the state of the system at that time is $\boldsymbol{x}^{[m]} \in \mathcal{S}^{[m]}$, where $\mathcal{S}^{[0]} = \mathcal{S}$ and $\mathcal{S}^{[1]} = \mathcal{S}^{[2]} = \mathcal{S}_0$. If $x^{[m]}_k = 0$, then we assume $\alpha^{[m]}_k(\boldsymbol{x}^{[m]})$ to be zero. To get monotone properties for $\alpha^{[m]}_k(\boldsymbol{x}^{[m]})$, we use the Markov chains $\{\hat{\boldsymbol{X}}^{[m]}(t)\}$, $m = 0, 1, 2$, described in the previous subsection. From their definitions, it can be seen that, for each $m \in \{0, 1, 2\}$, the Markov chain $\{\hat{\boldsymbol{X}}^{[m]}(t)\}$ is a stochastic process representing the behavior of Model m. Hence we define a tagged call served at base station k in each model and study the behaviors of the tagged calls.

For each $m \in \{0, 1, 2\}$, consider that $\hat{\boldsymbol{X}}^{[m]}(t)$ represents the state of Model m at time t, and assume $\hat{\boldsymbol{X}}^{[m]}(0) = \boldsymbol{x}^{[m]}$. For a fixed $k \in \mathcal{A}_0$, assume $x^{[m]}_k > 0$ and let $\hat{U}^{[m]}_0$ denote a tagged call served at base station k in Model m at time 0. In order to synchronize the behaviors of $\hat{U}^{[0]}_0$, $\hat{U}^{[1]}_0$ and $\hat{U}^{[2]}_0$, we further assume that, for each m, if $\hat{U}^{[m]}_0$ is served in some base station, then it always occupies channel 1 of the base station. That is, when there exist $\hat{X}^{[m]}_l(t)$ calls in base station l in Model m and one of them is $\hat{U}^{[m]}_0$, $\hat{U}^{[m]}_0$ occupies channel 1 and the other calls occupy channels 2 through $\hat{X}^{[m]}_l(t)$. To achieve this, we assume that, when $\hat{U}^{[m]}_0$ is successfully handed over to base station l at time t, calls in base station l change their positions to channels 2 through $\hat{X}^{[m]}_l(t-0)+1$ and $\hat{U}^{[m]}_0$ is assigned to channel 1. It is clear that these modifications do not change the probability law of the processes.

For $n \geq 0$, let t_n be the nth transition time defined by formula (5), and suppose that each tagged call is served in base station l ($l \in \mathcal{A}_0$) in the corresponding model at time t_n. For $\omega \in \Omega$, we assume that $\hat{\boldsymbol{X}}^{[2]}(t_n;\omega) \leq \hat{\boldsymbol{X}}^{[0]}_{\mathcal{A}_0}(t_n;\omega) \leq \hat{\boldsymbol{X}}^{[1]}(t_n;\omega)$. Then, the states of the tagged calls at the next transition time, $t_{n+1}(\omega)$, are given as follows.

(i) In the case of $T(n+1;\omega) = s_{l,1}(n+1;\omega)$, all the tagged calls end their talks (holding times) simultaneously at time $t_{n+1}(\omega)$ and they become completed calls.

(ii) In the case of $T(n+1;\omega) = \eta_{l,1}(n+1;\omega)$ and $d_{l,1}(n+1;\omega) = l'$, we further consider the following two cases.

- (a) If $l' \in \mathcal{A}_0$, then all the tagged calls try to move from base station l to base station l' at time $t_{n+1}(\omega)$. From the assumption that $\hat{\boldsymbol{X}}^{[0]}_{\mathcal{A}_0}(t_n;\omega) \leq \hat{\boldsymbol{X}}^{[1]}(t_n;\omega)$, if $\hat{U}_0^{[1]}$ is successfully handed over at that time, $\hat{U}_0^{[0]}$ is also successfully handed over. Moreover, from the assumption that $\hat{\boldsymbol{X}}^{[2]}(t_n;\omega) \leq \hat{\boldsymbol{X}}^{[0]}_{\mathcal{A}_0}(t_n;\omega)$, if $\hat{U}_0^{[0]}$ is successfully handed over at that time, $\hat{U}_0^{[2]}$ is also successfully handed over.
- (b) If $l' \in \overline{\mathcal{A}}_0$, then $\hat{U}_0^{[0]}$ tries to move from base station l to base station l' at time $t_{n+1}(\omega)$; $\hat{U}_0^{[1]}$ moves from base station l to the outside of $\mathcal{A}_0$ and it becomes a forcibly terminated call; $\hat{U}_0^{[2]}$ moves from base station l to the outside of $\mathcal{A}_0$ and it becomes a completed call.

(iii) In the other cases, all the tagged calls remain in base station l at time $t_{n+1}(\omega)$.

Note that, once case (ii)-(b) occurs, $\hat{U}_0^{[1]}$ and $\hat{U}_0^{[2]}$ disappear from their systems but $\hat{U}_0^{[0]}$ may not. Hence we have to describe the behavior of $\hat{U}_0^{[0]}$ when it is served at base station l in $\overline{\mathcal{A}}_0$ at time $t_n(\omega)$.

(i') In the case of $T(n+1;\omega) = s_{l,1}(n+1;\omega)$, $\hat{U}_0^{[0]}$ ends its talk (holding time) at time $t_{n+1}(\omega)$ and it becomes a completed call.

(ii') In the case of $T(n+1;\omega) = \eta_{l,1}(n+1;\omega)$ and $d_{l,1}(n+1;\omega) = l'$, $\hat{U}_0^{[0]}$ tries to move from base station l to base station l' at time $t_{n+1}(\omega)$. This handover will be successful when $\hat{X}^{[0]}_{l'}(t_n;\omega) < c_{l'}$.

(iii') In the other cases, $\hat{U}_0^{[0]}$ remains in base station l at time $t_{n+1}(\omega)$.

Next we define, for each $m \in \{0,1,2\}$, a process $\{\hat{W}^{[m]}(t)\}$ representing the state of $\hat{U}_0^{[m]}$ as follows.

$$\hat{W}^{[m]}(t) = \begin{cases} w & \text{if } \hat{U}_0^{[m]} \text{ is served at base station } w \text{ at time } t, \\ 0 & \text{if } \hat{U}_0^{[m]} \text{ has successfully completed its talk at or before time } t, \\ -1 & \text{otherwise.} \end{cases}$$

We obtain the next lemma.

Lemma 6 *Let $\boldsymbol{x}$ be a vector in $\mathcal{S}$, and assume that $\hat{\boldsymbol{X}}^{[0]}(0) = \boldsymbol{x}$ and $\hat{\boldsymbol{X}}^{[1]}(0) = \hat{\boldsymbol{X}}^{[2]}(0) = \boldsymbol{x}_{\mathcal{A}_0}$, where $\boldsymbol{x}_{\mathcal{A}_0}$ is the restriction of $\boldsymbol{x}$ to the set of focused base stations, $\mathcal{A}_0$. Moreover we assume that each tagged call is served at base station k ($k \in \mathcal{A}_0$) in the corresponding model at time 0. (This means that $x_k \geq 1$.) Then the following statements hold for all $t \geq 0$*

w.p. 1.

$$\hat{W}^{[1]}(t) = w \ (w \in \mathcal{A}_0) \Rightarrow \hat{W}^{[0]}(t) = w \tag{8}$$

$$\hat{W}^{[1]}(t) = 0 \Rightarrow \hat{W}^{[0]}(t) = 0 \tag{9}$$

$$\hat{W}^{[0]}(t) = w \ (w \in \mathcal{A}_0) \Rightarrow \hat{W}^{[2]}(t) = w \ or \ 0 \tag{10}$$

$$\hat{W}^{[0]}(t) = w \ (w \in \overline{\mathcal{A}}_0) \Rightarrow \hat{W}^{[2]}(t) = 0 \tag{11}$$

$$\hat{W}^{[0]}(t) = 0 \Rightarrow \hat{W}^{[2]}(t) = 0 \tag{12}$$

Proof. Let $\{t_n\}$ be the transition times defined by formula (5). Since the state of $\hat{\boldsymbol{X}}^{[m]}(t)$ does not change between the times, it is sufficient to show that the statements hold for t_n, $n \geq 0$. From the assumption for the tagged calls, we obtain $\hat{W}^{[0]}(0) = \hat{W}^{[1]}(0) = \hat{W}^{[2]}(0) = k$, and taking account of the assumptions of the initial states and Lemma 2, we obtain, for each $\omega \in \Omega$,

$$\hat{\boldsymbol{X}}^{[2]}(t_n;\omega) \leq \hat{\boldsymbol{X}}^{[0]}_{\mathcal{A}_0}(t_n;\omega) \leq \hat{\boldsymbol{X}}^{[1]}(t_n;\omega) \text{ for all } n \geq 0.$$

Hence the tagged calls change their states according to (i)–(iii) and (i')–(iii') described above. Each statement is obtained from the following reasons, where we assume that $n \geq 1$.

- *Statement (8).* $\hat{W}^{[1]}(t_n;\omega) = w \in \mathcal{A}_0$ means that either (ii)-(a) or (iii) occurred at time $t_n(\omega)$. In both the cases, we obtain $\hat{W}^{[0]}(t_n;\omega) = w$.
- *Statement (9).* $\hat{W}^{[1]}(t_n;\omega) = 0$ means that case (i) has occurred at or before time $t_n(\omega)$. Hence we obtain $\hat{W}^{[0]}(t_n;\omega) = 0$.
- *Statement (10).* When $\hat{W}^{[0]}(t_n;\omega) = w \in \mathcal{A}_0$, two cases can be considered: one is that $\hat{U}_0^{[0]}$ has been in the outside of $\mathcal{A}_0$ at least once and the other that it has never. In the former case, from (ii)-(b), we obtain $\hat{W}^{[2]}(t_n;\omega) = 0$. In the latter case, we obtain $\hat{W}^{[2]}(t_n;\omega) = w$ from the same reason as used for statement (8).
- *Statement (11).* $\hat{W}^{[0]}(t_n;\omega) = w \in \overline{\mathcal{A}}_0$ means that case (ii)-(b) has occurred at or before time t_n. Hence we obtain $\hat{W}^{[2]}(t_n;\omega) = 0$.
- *Statement (12).* When $\hat{W}^{[0]}(t_n;\omega) = 0$, two cases can be considered: one is that $\hat{U}_0^{[0]}$ has been in the outside of $\mathcal{A}_0$ at least once and then successfully completed its talk, and the other that it has successfully completed its talk before leaving $\mathcal{A}_0$. In the former case, from (ii)-(b), we obtain $\hat{W}^{[2]}(t_n;\omega) = 0$, and in the latter case, from (i), we also obtain it.

□

From Lemma 6, we obtain the next lemma.

Lemma 7 *For $k \in \mathcal{A}_0$, for every vector $\boldsymbol{x}$ in $\mathcal{S}$,*

$$\alpha_k^{[1]}(\boldsymbol{x}_{\mathcal{A}_0}) \leq \alpha_k^{[0]}(\boldsymbol{x}) \leq \alpha_k^{[2]}(\boldsymbol{x}_{\mathcal{A}_0}). \tag{13}$$

Proof. For each model, we assume that the tagged call is served at base station k in $\mathcal{A}_0$ at time 0. From statements (9) and (12), we obtain $1_{\{\hat{W}^{[1]}(t)=0\}} \leq 1_{\{\hat{W}^{[0]}(t)=0\}} \leq 1_{\{\hat{W}^{[2]}(t)=0\}}$ for all $t \geq 0$ w.p. 1. Since $1_{\{\hat{W}^{[m]}(t)=0\}}$ is nondecreasing with respect to t, the monotone convergence theorem implies that

$$\alpha_k^{[m]}(\boldsymbol{x}^{[m]}) = E\Big[\lim_{t\to\infty} 1_{\{\hat{W}^{[m]}(t)=0\}}\Big] = \lim_{t\to\infty} E\Big[1_{\{\hat{W}^{[m]}(t)=0\}}\Big],$$

where $\boldsymbol{x}^{[0]} = \boldsymbol{x}$ and $\boldsymbol{x}^{[1]} = \boldsymbol{x}^{[2]} = \boldsymbol{x}_{\mathcal{A}_0}$. Hence we obtain

$$\alpha_k^{[1]}(\boldsymbol{x}_{\mathcal{A}_0}) = \lim_{t\to\infty} E\Big[1_{\{\hat{W}^{[1]}(t)=0\}}\Big] \le \lim_{t\to\infty} E\Big[1_{\{\hat{W}^{[0]}(t)=0\}}\Big] = \alpha_k^{[0]}(\boldsymbol{x})$$

and

$$\alpha_k^{[0]}(\boldsymbol{x}) = \lim_{t\to\infty} E\Big[1_{\{\hat{W}^{[0]}(t)=0\}}\Big] \le \lim_{t\to\infty} E\Big[1_{\{\hat{W}^{[2]}(t)=0\}}\Big] = \alpha_k^{[2]}(\boldsymbol{x}_{\mathcal{A}_0}).$$

□

In the same idea as used for obtaining Lemma 7, we can derive another monotone property for $\alpha_k^{[m]}(\boldsymbol{x}^{[m]})$ as follows.

Lemma 8 *For each $m \in \{0, 1, 2\}$, $\alpha_k^{[m]}(\boldsymbol{x}^{[m]})$ is a nonincreasing function of $\boldsymbol{x}^{[m]} \in \mathcal{S}^{[m]}$.*

Proof. Here we describe only an outline of the proof. Consider the Markov chains $\{{}^1\hat{\boldsymbol{X}}^{[m]}(t)\}$ and $\{{}^2\hat{\boldsymbol{X}}^{[m]}(t)\}$ given in the previous subsection, and assume that ${}^1\hat{\boldsymbol{X}}^{[m]}(0) = {}^1\boldsymbol{x}^{[m]} \in \mathcal{S}^{[m]}$, ${}^2\hat{\boldsymbol{X}}^{[m]}(0) = {}^2\boldsymbol{x}^{[m]} \in \mathcal{S}^{[m]}$ and ${}^1\boldsymbol{x}^{[m]} \le {}^2\boldsymbol{x}^{[m]}$. Then, from Lemma 3, we obtain that ${}^1\hat{\boldsymbol{X}}^{[m]}(t) \le {}^2\hat{\boldsymbol{X}}^{[m]}(t)$ for all $t \ge 0$ w.p. 1. For the Markov chains, tagged calls ${}^1\hat{U}_0^{[m]}$ and ${}^2\hat{U}_0^{[m]}$ are defined like $\hat{U}_0^{[m]}$ was defined, and processes $\{{}^1\hat{W}^{[m]}(t)\}$ and $\{{}^2\hat{W}^{[m]}(t)\}$ are constructed in the same manner as used for constructing $\{\hat{W}^{[m]}(t)\}$. Studying the behaviors of ${}^1\hat{U}_0^{[m]}$ and ${}^2\hat{U}_0^{[m]}$, we obtain

$$ {}^2\hat{W}^{[m]}(t) = w \ (w \in \mathcal{A}^{[m]}) \Rightarrow {}^1\hat{W}^{[m]}(t) = w, \tag{14}$$

$$ {}^2\hat{W}^{[m]}(t) = 0 \Rightarrow {}^1\hat{W}^{[m]}(t) = 0, \tag{15}$$

and this corresponds to Lemma 6. From statement (15), we obtain

$$\alpha_k^{[m]}({}^1\boldsymbol{x}^{[m]}) \ge \alpha_k^{[m]}({}^2\boldsymbol{x}^{[m]}).$$

This leads us to the assertion of the lemma. □

3.4. Proof of Theorem 1

Proof of Theorem 1. For $m \in \{0, 1, 2\}$, first we obtain some relations between $\zeta_k^{[m]}$ and $\xi_{(k,l)}^{[m]}$. Let $\boldsymbol{e}_k$ be a vector with a suitable dimension whose kth element is one and whose other elements are all zero (i.e., kth unit vector). From PASTA [13], calls generated in area $k \in \mathcal{A}_0$ see time averages and $\zeta_k^{[m]}$ is given by

$$\zeta_k^{[m]} = \sum_{\boldsymbol{x} \in \mathcal{S}^{[m]}} 1_{\{x_k < c_k - g_k\}} \alpha_k^{[m]}(\boldsymbol{x} + \boldsymbol{e}_k) P^{[m]}(\boldsymbol{x}) = E\Big[1_{\{X_k^{[m]} < c_k - g_k\}} \alpha_k^{[m]}(\boldsymbol{X}^{[m]} + \boldsymbol{e}_k)\Big],$$

where $\mathcal{S}^{[0]} = \mathcal{S}$ and $\mathcal{S}^{[1]} = \mathcal{S}^{[2]} = \mathcal{S}_0$. For $k \in \mathcal{A}_0$, $\boldsymbol{x} \in \mathcal{S}$, Lemma 7 claims that $\alpha_k^{[0]}(\boldsymbol{x}+\boldsymbol{e}_k) \ge \alpha_k^{[1]}(\boldsymbol{x}_{\mathcal{A}_0} + (\boldsymbol{e}_k)_{\mathcal{A}_0})$, where $(\boldsymbol{e}_k)_{\mathcal{A}_0}$ is the restriction of $\boldsymbol{e}_k$ to $\mathcal{A}_0$, and Lemma 8 implies that $1_{\{x_k < c_k - g_k\}} \alpha_k^{[1]}(\boldsymbol{x}_{\mathcal{A}_0} + (\boldsymbol{e}_k)_{\mathcal{A}_0})$ is nonincreasing and bounded with respect to $\boldsymbol{x}_{\mathcal{A}_0}$. Hence, we obtain

$$\begin{aligned}
\zeta_k^{[0]} &= E\Big[1_{\{X_k^{[0]} < c_k - g_k\}} \alpha_k^{[0]}(\boldsymbol{X}^{[0]} + \boldsymbol{e}_k)\Big] \\
&\ge E\Big[1_{\{X_k^{[0]} < c_k - g_k\}} \alpha_k^{[1]}(\boldsymbol{X}_{\mathcal{A}_0}^{[0]} + (\boldsymbol{e}_k)_{\mathcal{A}_0})\Big] \\
&\ge E\Big[1_{\{X_k^{[1]} < c_k - g_k\}} \alpha_k^{[1]}(\boldsymbol{X}^{[1]} + (\boldsymbol{e}_k)_{\mathcal{A}_0})\Big] = \zeta_k^{[1]}.
\end{aligned} \tag{16}$$

To derive the second inequality in (16), we use Lemma 4. In the same way, we obtain, for $k \in \mathcal{A}_0$,

$$\zeta_k^{[0]} \le \zeta_k^{[2]}. \tag{17}$$

For Model 0, let $\zeta_{(k,l)}^{[0]}$ be the probability that an arbitrary call generated in area (k,l) starts its talk (holding time) at base station k and successfully completes the talk at some base station. $\zeta_{(k,l)}^{[0]}$ is represented in terms of $\zeta_k^{[0]}$ and $\xi_{(k,l)}^{[0]}$ as follows.

$$\begin{aligned}\zeta_{(k,l)}^{[0]} &= \frac{1}{2}E\Big[1_{\{X_k^{[0]}<c_k-g_k\}}\alpha_k^{[0]}(\boldsymbol{X}^{[0]}+\boldsymbol{e}_k)\Big] \\ &\quad + \frac{1}{2}E\Big[1_{\{X_k^{[0]}<c_k-g_k\}}\,1_{\{X_l^{[0]}\ge c_l-g_l\}}\alpha_k^{[0]}(\boldsymbol{X}^{[0]}+\boldsymbol{e}_k)\Big] \\ &= \frac{1}{2}\zeta_k^{[0]} + \frac{1}{2}E\Big[\Big(1_{\{X_k^{[0]}<c_k-g_k\}} - 1_{\{X_k^{[0]}<c_k-g_k\}}\,1_{\{X_l^{[0]}<c_l-g_l\}}\Big)\,\alpha_k^{[0]}(\boldsymbol{X}^{[0]}+\boldsymbol{e}_k)\Big] \\ &= \zeta_k^{[0]} - \xi_{(k,l)}^{[0]},\end{aligned} \tag{18}$$

where $\xi_{(k,l)}^{[0]}$ is given by

$$\xi_{(k,l)}^{[0]} = \frac{1}{2}E\Big[1_{\{X_k^{[0]}<c_k-g_k\}}\,1_{\{X_l^{[0]}<c_l-g_l\}}\alpha_k^{[0]}(\boldsymbol{X}^{[0]}+\boldsymbol{e}_k)\Big].$$

For $m \in \{1,2\}$, for $k \in \mathcal{A}_0$, $l \in \mathcal{C}_0^{(k)}$, $\xi_{(k,l)}^{[m]}$ is also given by

$$\xi_{(k,l)}^{[m]} = \frac{1}{2}E\Big[1_{\{X_k^{[m]}<c_k-g_k\}}\,1_{\{X_l^{[m]}<c_l-g_l\}}\alpha_k^{[m]}(\boldsymbol{X}^{[m]}+\boldsymbol{e}_k)\Big].$$

Since $1_{\{x_k<c_k-g_k\}}\,1_{\{x_l<c_l-g_l\}}\alpha_k^{[m]}(\boldsymbol{x})$ is nonincreasing and bounded with respect to $\boldsymbol{x} \in \mathcal{S}^{[m]}$, in the same way as that used for $\zeta_k^{[m]}$, we obtain the following inequality of $\xi_{(k,l)}^{[m]}$ for $k \in \mathcal{A}_0$, $l \in \mathcal{C}_0^{(k)}$.

$$\xi_{(k,l)}^{[1]} \le \xi_{(k,l)}^{[0]} \le \xi_{(k,l)}^{[2]} \tag{19}$$

In the case of $k \in \mathcal{A}_0$ and $l \in \overline{\mathcal{C}}_0$, the next inequality can be used.

$$0 \le \xi_{(k,l)}^{[0]} \le \frac{1}{2}E\Big[1_{\{X_k^{[0]}<c_k-g_k\}}\alpha_k^{[0]}(\boldsymbol{X}^{[0]}+\boldsymbol{e}_k)\Big] = \frac{1}{2}\zeta_k^{[0]} \le \frac{1}{2}\zeta_k^{[2]} \tag{20}$$

Assume that the system is in steady state, and let U_0 denote an arbitrary call generated in zone k in Model 0. Remind that zone k consists of area k and areas (k,l), $l \in \mathcal{C}^{(k)}$. In order to obtain inequality (3), we define the following events for Model 0.

- A: U_0 successfully completes its talk (holding time) at some base station.
- B_k: U_0 starts its talk (holding time) at base station k.
- C_k: U_0 is generated in area k.
- $C_{(k,l)}$: U_0 is generated in area (k,l).

Then, $\zeta_k^{[0]}$ and $\zeta_{(k,l)}^{[0]}$ are given by $\zeta_k^{[0]} = \Pr(A, B_k \,|\, C_k)$ and $\zeta_{(k,l)}^{[0]} = \Pr(A, B_k \,|\, C_{(k,l)})$, and θ_k is represented as

$$\theta_k = \Pr(A \,|\, B_k) = \frac{\Pr(A, B_k, C_k) + \sum_{l\in\mathcal{C}^{(k)}} \Pr(A, B_k, C_{(k,l)})}{\Pr(B_k, C_k) + \sum_{l\in\mathcal{C}^{(k)}} \Pr(B_k, C_{(k,l)})}. \tag{21}$$

From (18), the numerator of (21) is given by

$$\Pr(A, B_k, C_k)+\sum_{l\in\mathcal{C}^{(k)}} \Pr(A, B_k, C_{(k,l)}) = \frac{\lambda_k}{\Lambda_k}\zeta_k^{[0]}+\sum_{l\in\mathcal{C}^{(k)}} \frac{\lambda_{(k,l)}}{\Lambda_k}\zeta_{(k,l)}^{[0]} = \zeta_k^{[0]}-\sum_{l\in\mathcal{C}^{(k)}} \frac{\lambda_{(k,l)}}{\Lambda_k}\xi_{(k,l)}^{[0]}. \quad (22)$$

Applying (16), (17), (19) and (20) to this formula, we obtain

$$\zeta_k^{[1]} - \sum_{l\in\mathcal{C}_0^{(k)}} \frac{\lambda_{(k,l)}}{\Lambda_k}\xi_{(k,l)}^{[2]} - \frac{1}{2}\sum_{l\in\overline{\mathcal{C}}_0^{(k)}} \frac{\lambda_{(k,l)}}{\Lambda_k}\zeta_k^{[2]}$$
$$\leq \Pr(A, B_k, C_k) + \sum_{l\in\mathcal{C}^{(k)}} \Pr(A, B_k, C_{(k,l)}) \leq \zeta_k^{[2]} - \sum_{l\in\mathcal{C}_0^{(k)}} \frac{\lambda_{(k,l)}}{\Lambda_k}\xi_{(k,l)}^{[1]}. \quad (23)$$

The denominator of (21) is given by

$$\begin{aligned}
&\Pr(B_k, C_k) + \sum_{l\in\mathcal{C}^{(k)}} \Pr(B_k, C_{(k,l)}) \\
&= \frac{\lambda_k}{\Lambda_k}\Pr(X_k^{[0]} < c_k - g_k) \\
&\quad + \sum_{l\in\mathcal{C}^{(k)}} \frac{\lambda_{(k,l)}}{\Lambda_k}\left(\frac{1}{2}\Pr(X_k^{[0]} < c_k - g_k) + \frac{1}{2}\Pr(X_k^{[0]} < c_k - g_k, X_l^{[0]} \geq c_l - g_l)\right) \\
&= \left(\frac{\lambda_k}{\Lambda_k} + \frac{1}{2}\sum_{l\in\mathcal{C}^{(k)}} \frac{\lambda_{(k,l)}}{\Lambda_k}\right)\left(1-\phi_k^{[0]}\right) + \frac{1}{2}\sum_{l\in\mathcal{C}^{(k)}} \frac{\lambda_{(k,l)}}{\Lambda_k}\Pr(X_k^{[0]} < c_k - g_k, X_l^{[0]} \geq c_l - g_l), \quad (24)
\end{aligned}$$

where $\Pr(X_k^{[0]} < c_k - g_k, X_l^{[0]} \geq c_l - g_l)$ can be represented as

$$\Pr(X_k^{[0]} < c_k - g_k, X_l^{[0]} \geq c_l - g_l) = \bar{P}_l^{[0]}(c_l - g_l) - \bar{P}_{k,l}^{[0]}(c_k - g_k, c_l - g_l) = \phi_l^{[0]} - \phi_{(k,l)}^{[0]}$$

and it also satisfies

$$0 \leq \Pr(X_k^{[0]} < c_k - g_k, X_l^{[0]} \geq c_l - g_l) \leq \Pr(X_k^{[0]} < c_k - g_k) = 1 - \phi_k^{[0]}.$$

Corollary 5 implies that

$$\phi_l^{[2]} \leq \phi_l^{[0]} \leq \phi_l^{[1]} \quad \text{and} \quad \phi_{(k,l)}^{[2]} \leq \phi_{(k,l)}^{[0]} \leq \phi_{(k,l)}^{[1]},$$

and applying these inequalities to (24), we get

$$\begin{aligned}
&\left(\frac{\lambda_k}{\Lambda_k} + \frac{1}{2}\sum_{l\in\mathcal{C}^{(k)}} \frac{\lambda_{(k,l)}}{\Lambda_k}\right)\left(1-\phi_k^{[1]}\right) + \frac{1}{2}\sum_{l\in\mathcal{C}_0^{(k)}} \frac{\lambda_{(k,l)}}{\Lambda_k}\left(\phi_l^{[2]} - \phi_{(k,l)}^{[1]}\right) \\
&\leq \Pr(B_k, C_k) + \sum_{l\in\mathcal{C}^{(k)}} \Pr(B_k, C_{(k,l)}) \\
&\leq \left(\frac{\lambda_k}{\Lambda_k} + \frac{1}{2}\sum_{l\in\mathcal{C}^{(k)}} \frac{\lambda_{(k,l)}}{\Lambda_k}\right)\left(1-\phi_k^{[2]}\right) + \frac{1}{2}\sum_{l\in\mathcal{C}_0^{(k)}} \frac{\lambda_{(k,l)}}{\Lambda_k}\left(\phi_l^{[1]} - \phi_{(k,l)}^{[2]}\right) + \frac{1}{2}\sum_{l\in\overline{\mathcal{C}}_0^{(k)}} \frac{\lambda_{(k,l)}}{\Lambda_k}\left(1-\phi_k^{[2]}\right).
\end{aligned} \quad (25)$$

Applying (23) and (25) to (21), inequality (3) is derived. □

4. Numerical Examples

In this section, we show some numerical results for a hexagon model, where zones are placed on a plane and each zone except boundary zones neighbors six others like the one shown in Figure 2. In this figure, we assume that the call generation rates in the shaded zones are higher than those in unshaded zones. Hence we use new notations $\tilde{\lambda}_1$, $\tilde{\lambda}_2$, $\tilde{\lambda}'_1$ and $\tilde{\lambda}'_2$ in the following meanings.

$$\lambda_k = \tilde{\lambda}_1,\ 1 \le k \le 7, \quad \lambda_{(k,l)} = \tilde{\lambda}_2,\ 1 \le k \le 7,\ l \in \mathcal{C}^{(k)},$$
$$\lambda_k = \tilde{\lambda}'_1,\ k > 7, \quad \lambda_{(k,l)} = \tilde{\lambda}'_2,\ k > 7,\ l \in \mathcal{C}^{(k)} \setminus \{1, 2, \ldots, 7\}.$$

The number of base stations in the model is not specified but is assumed to be large so that direct calculations of call completion probabilities are difficult. For analyzing the model, we apply our method to the model and evaluate bounds of a call completion probability by simulation. In the table below, *ratio* indicates the accuracy ratio that is defined as the ratio of the upper bound to the corresponding lower bound. This accuracy ratio measures tightness of the bounds.

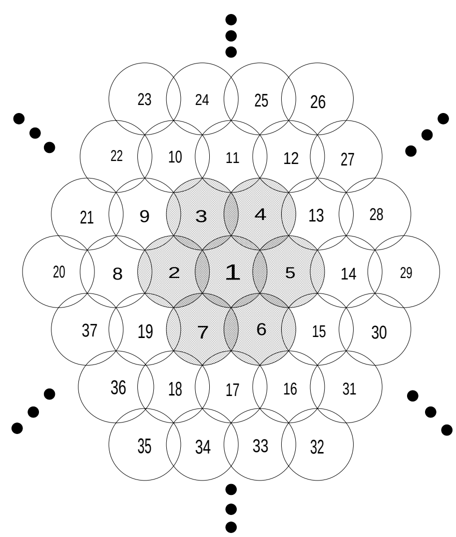

Figure 2: Hexagon model

It is expected that the larger the number of focused base stations is the tighter the bounds of a call completion probability is. To see this, we show some simulation results for the hexagon model in Table 1, where *size* indicates the number of focused base stations. The simulation results represent point estimates and 95% confidence intervals of the bounds. The model parameters are set as

$$\tilde{\lambda}_2 = 0.2\,\tilde{\lambda}_1, \quad \tilde{\lambda}'_1 = 0.5\,\tilde{\lambda}_1, \quad \tilde{\lambda}'_2 = 0.2\,\tilde{\lambda}'_1,$$
$$\mu = 20, \quad c_k = 10,\ k \ge 1, \quad g_k = 0,\ k \ge 1, \quad \gamma_{k,l} = 2,\ k \ge 1,\ l \in \mathcal{C}^{(k)},$$

where the value of $\tilde{\lambda}_1$ is set so that $(\tilde{\lambda}_1 + 3\tilde{\lambda}_2)/(c_1\mu)$ becomes one. The table shows upper and lower bounds of θ_1. In the simulation experiments, we have generated $20 \times N_0$ thousands calls for each run, where N_0 is the number of focused base stations, and used 100 replicas for computing one point estimate and its interval estimate. Hence, $2 \times N_0$ million calls have been generated for one simulation result. In the case of $N_0 = 61$, it has taken about a couple of ten minutes to obtain a simulation result by our personal computer, which have a single processor of 2.4 GHz and 512 MB memories. From the table, it can be seen that as the number of the focused base stations becomes larger, the bounds become tighter.

Table 1: Upper and lower bounds of θ_1 in a hexagon model with different numbers of focused base stations

Size	upper bound	lower bound	ratio
7	1.09872 ± 0.00277	0.69137 ± 0.00228	1.59
19	0.94281 ± 0.00175	0.86754 ± 0.00217	1.09
37	0.92450 ± 0.00174	0.91306 ± 0.00149	1.01
61	0.91959 ± 0.00165	0.91921 ± 0.00151	1.00

5. Conclusions

In this paper, we have proposed a method to obtain tight upper and lower bounds of call completion probabilities taking account of the whole network. The call completion probability is one of the most important measures since, in mobile communication networks, calls having started their talks may forcibly be terminated because of unsuccessful handovers and such call terminations are very unpleasant for users. Using our method, we can evaluate the bounds by standard methods such as simulation with less computational burden, where the tightness of the bounds can be controlled by choosing a set of focused base stations with appropriate size.

From our method and that proposed in Ref. [11], it becomes possible to examine exact relations between model parameters and various performance measures for some standard models. Moreover, it becomes also possible to examine accuracy of typical approximation techniques such as the decomposition method. Results of such an experiment will be presented elsewhere. It would be also possible to generalize our method. Mobile communication networks with other control schemes such as dynamic channel assignment are candidates for them. These remain as future works.

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