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MODELING THE EFFECT OF ORDER TIMING REDUCTION ON COORDINATING SUPPLY CHAIN INVENTORIES USING A SELECTIVE DISCOUNT STRATEGY

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Abstract Prior studies show that coordinating supply chain inventories with common replenishment epochs (CRE) provides benefit for vendor and buyers. Coordination using selective discount (SD) strategy allows some buyers to participate in the coordination scheme while other buyers continue to order as earlier. This can reduce the vendor's total cost more than that of all buyers participating and no buyer coordination in many circumstances. This paper considers the situation in which SD buyers place orders before CRE due to demand timing uncertainty. According to vendor flexibility, each SD buyer with early ordering incurs various levels of penalty cost to compensate for possible capacity loss for the vendor. The proposed models also allow for a more practical situation by considering the cases of equal and unequal probabilities of early ordering for SD buyers. The results indicate that the vendor's cost savings can be improved substantially based on the vendor flexibility, probability of early ordering, and the number of SD buyers involved in early ordering.

Keywords: Inventory, supply chain management, coordination

1. Introduction

An increasing amount of research the past decade has focused on the importance of establishing a relationship between companies and suppliers in the supply chain. Reliable collaboration between them has emerged as a strategic issue, which plays a pivotal role in inventory policies and management to maintain competition in the market. Through the strong interaction between the related parties, information in satisfying customer demands can be better shared in the organizational chain.

However, managing the inventories through better coordination across the entire supply chain is difficult due to system complexity. Coordination among parties in a supply chain involves buyers dealing with their inventories using economic order quantity (EOQ) model. Without a knowledge of customer demand information, the vendor purchases each buyer's order individually in a lot-to-lot basis. Therefore, an effective mechanism is necessary to create a win-win situation in reducing costs among the parties.

In response to this need, the concept of common replenishment epochs (CRE) was introduced to refine the well-known EOQ method. The basic idea of CRE was developed by Viswanathan and Piplani [22], hereafter referred to as VP, to study how a vendor can implement the CRE mechanism to buyers in a one-vendor, multi-buyer supply chain for a single product. In VP's model, each buyer originally replenishes his inventory based on the EOQ policy. The vendor considers reducing the fixed costs incurred for processing the buyers' orders by coordinating a replenishment of buyers. The vendor specifies common replenishment epochs and requires buyers to replenish only at those time periods. Under the proposed strategy, the vendor offers a price discount to entice buyers to comply with this strategy. From the outcome of their model, VP concludes that the vendor with CRE strategy saves substantial inventory and transaction costs, in addition to moving toward a win-win situation. An implicit assumption in VP's model is that all the buyers participate in the proposed coordination scheme and hence the vendor gives a discount that is the maximum of the discount required by each buyer for participation. Mishra [14] extended VP's model by allowing the possibility of some buyers to participate in the coordination scheme where they get a discount for ordering a larger quantity while other buyers continue to order as earlier without going for the discount. The vendor offers only a selective discount (hereafter referred to as CRE-based SD or SD policy) to those buyers who agree to replenish at the time epochs suggested by the vendor. Mishra [14] observed that it might be beneficial in many scenarios for the vendor to coordinate with at least a few buyers.

In practice, buyers with CRE are not always able to predict downstream conditions and will often issue procurement orders ahead of the required replenishment epochs, especially in urgent situations. In a coordination relationship, the vendor will only accept these orders by imposing a penalty on the SD buyers for early ordering to compensate for the vendor's loss caused by the order timing reduction. The penalty varies for each SD buyer with early procurement, depending on the vendor's flexibility and the number of SD buyers that place orders ahead of CRE.

Based on VP [22] and Mishra [14], this study proposed a revised model considering individual penalty imposed on SD buyers for early ordering according to the vendor's flexibility. This is done by dividing the penalties of all SD buyers' ordering ahead of CRE to have a partial penalty of one buyer ordering early, two buyers ordering early, up to having all SD buyers ordering early. This investigation conducts computations to investigate the SD strategy with early ordering and studies the impact of early ordering on vendor's total cost.

The rest of the paper is organized as follows: Section 2 reviews the studies relating to the buyer-vendor coordination using the CRE. Section 3 describes the CRE models presented by VP [22] and Mishra [14]. Section 4 proposes an extension to the SD model that considers early ordering of CRE-SD buyers. Section 5 provides a numerical example to illustrate the proposed model and compares the impact of the demand timing reduction on the vendor's total cost. Conclusions and directions for future work are finally drawn in Section 6.

2. Literature Review

Cooperation of inventories in buyer-vendor supply chains can be modeled as a joint replenishment problem. Papers dealing with the joint replenishment problem included Jackson et al. [22], VanEijs [18], and Viswanathan [19-20; 23]. Other studies of coordination in a supply chain system discussed the use of quantity discount as a method of coordination. Papers in this area included Parlar and Wang [15], Weng [26], Corbett and deGroote [5],Wang and Wu [24], and Wang [25].

Some authors studied integrated vendor-buyer inventory models to find the joint optimal inventory policy that minimizes the total cost for the buyer and the vendor. Goyal [8] examined a joint total relevant cost model for a single-vendor-single-buyer production inventory system. The model assumed that the vendor's lot size was an integer multiple of the purchasers order size in order to achieve a better joint total relevant cost. Some other papers dealing with optimization of the integrated vendor-buyer inventory problem included Banerjee [1], Goyal [9], Lu [13], Viswanathan [21], and Hill [10-11].

Many papers discussed one vendor, multi-buyer inventory models. Banerjee and Burton [2] developed an integrated production model for a vendor and multiple buyers under deterministic conditions, with the objective of minimizing total system cost incurred by all parties. Woo et al. [27] considered an integrated inventory model where a single vendor delivered the finished items to multiple buyers at a common cycle. Viswanathan and Piplani [22] studied the benefit of coordinating inventories through the use of CRE in a one-vendor, multi-buyer supply chain for a single product. Mishra [14] generalized the CRE model proposed by VP [22] to allow for a selective discount policy that excluded some buyers to minimize the vendor's total cost. Yao and Chiou [28] studied the integrated single vendor, multi-buyer inventory problem. Zhanga et al. [29] developed an integrated vendor managed inventory (VMI) model for a single vendor and multiple buyers. Chan and Kingsman [3] proposed a coordinated single-vendor multi-buyer supply chain model by synchronizing delivery and production cycles. They concluded that the synchronized-cycles policy worked better than an independent optimization and restricted buyers to adopt a common order cycle. Chu and Leon [4] studied the problem of coordinating a single-vendor multi-buyer inventory system where restricted privacy information to solve the problem. The objective was to minimize the total average setup/ordering and inventory related cost. Darwish and Odah [6] considered a single-vendor multi-retailer supply chain operating under VMI mode to find the operating policies for the vendor and retailers with the objective of minimizing the total cost of the supply chain.

The above-mentioned literature has placed little attention on the coordination problem in a supply chain using CRE-based replenishment. The model presented in this paper extends the work of VP [22] and Mishra [14] to consider new aspects of the CRE-based SD buyers that order before the CRE. The proposed model in this paper differs from existing ones in the following perspectives. First, the model includes a condition where SD buyers who order early will incur a penalty cost to compensate for the vendor's loss in capacity to satisfy the urgent demands from SD buyers. Second, the penalty imposed on each of the SD buyers with early ordering depends on the vendor's flexibility to analyze the impact of early ordering on the vendor's total cost. Finally, to allow for a more practical situation, we consider respectively the cases of equal probability of early ordering and unequal probability of early ordering as well as various levels of possible savings gained for the vendor from riskier SD buyers.

3. The CRE Models

This section reviews two CRE-based models developed by Viswanathan and Piplani [22] and Mishra [14]. Consider a single vendor that supplies a single product to m buyers with demand D_i, order cost K_i, and holding cost rate h_i for buyer i, all known to the vendor. The purchase price P of the item is the same for all buyers.

According to the EOQ model, the optimal replenishment interval for each buyer i is given by $t_i^U = \sqrt{2K_iD_i/h_iP}/D_i = \sqrt{2K_i/h_iPD_i}$, and the corresponding inventory costs, which is the sum of order and holding cost, for buyer i are given by $g_i^U = K_i/t_i^U + (1/2)h_iPD_it_i^U$, i.e., $g_i^U = \sqrt{2K_ih_iPD_i}$.

Before coordination between the vendor and the buyers, the vendor will incur an order processing cost (denoted by A_i) and delivery cost (denoted by A_{S_i}) for buyer i for processing orders from a set of buyers m together. Therefore, the total vendor order processing costs are given by $g_S^U = \sum_{i=1}^{m} (A_{S_i} + A_i)/t_i^U$.

Let $|C| < m$ be the number of buyers that agree to accept CRE-based replenishment strategy to place orders. This was referred to as selective discount (SD) policy in Mishra's model [14]. Under the CRE strategy, the vendor specifies that the replenishment interval for each buyer $i \in C$, defined as t_i^C, can only be an integer multiple of the CRE T_0, that is, $t_i^C = n_i T_0$, where C is the set of CRE buyers and $n_i = 1, 2, \cdots$. To minimize the inventory cost, each buyer will set replenishment interval t_i^C once T_0 is set. After coordination, the corresponding inventory cost for buyer i is given by $K_i/(n_i T_0) + (1/2) h_i P D_i n_i T_0$. Note that g_i^C is convex with respect to n_i. The minimum $g_i^C(n_i)$ can be determined by selecting the optimal n_i, denoted as n_i^*, such that $g_i^C(n_i^*)$ is smaller than both $g_i^C(n_i - 1)$ and $g_i^C(n_i + 1)$, to satisfy $n_i^*(n_i^* - 1) \le 2K_i/h_i P D_i T_0^2 \le n_i^*(n_i^* + 1)$. Therefore, $n_i^* = \left\lfloor (1/2)(-1 + \sqrt{1 + (8K_i/h_i P D_i T_0^2)}) \right\rfloor$, where $\lfloor x \rfloor$ is the largest integer that is less than or equal to x. When each buyer deviates from EOQ to order, inventory cost increases. The buyer will accept the CRE only if the offered price discount, defined as d $(0 \le d < 1)$ is large enough to compensate for the increase in inventory costs and still provide a minimum savings of 100 S% over the initial cost g_i^U. Let ρ_i be the discount acceptable to buyer i, then, the total dollar discount acceptable to buyer i, $\rho_i P D_i$, should be given by

$$\rho_i P D_i \ge K_i/(n_i T_0) + (1/2) h_i P D_i n_i T_0 - (1 - S)\sqrt{2 K_i h_i P D_i}. \tag{3.1}$$

Buyer i will accept the coordination scheme only if the discount $d \ge \rho_i$.

Assume that the vendor incurs a joint order processing cost of A_S for CRE buyers, irrespective of the number of buyers. The vendor's total cost under CRE is the sum of the order processing cost and the cost of discounts, which is $g_s^C = \frac{A_S}{T_0} + \sum\limits_{i \in C} (dPD_i + \frac{A_i}{n_i T_0}) + \sum\limits_{j \in \bar{C}} \frac{(A_j + A_{S_j})}{t_j^U}$, where $\bar{C}$ represents the set of buyers who do not accept the selective discount and continue to place orders as earlier. The problem of determining T_0, d, and C for the vendor is given by

$$\begin{aligned} \text{Min} \quad & g_s^C = \tfrac{A_S}{T_0} + \sum_{i \in C} (dPD_i + \tfrac{A_i}{n_i T_0}) + \sum_{j \in \bar{C}} \tfrac{(A_j + A_{S_j})}{t_j^U} \\ \text{s.t.} \quad & T_0 \in X,\ d \ge \rho_i,\ i \in C, \text{ and } n_i = 1, 2, \cdots . \end{aligned} \tag{3.2}$$

where X is the set of CREs being considered. For example, $X = \{1/365, 1/52, 2/52, 1/12, 2/12, 1/4\}$. Method for solving the model can be summarized as follows:

1. Set $T_0 = x$, $x \in X$. Use (3.1) to determine $d = \text{Max}\{\rho_i\}$, $i = 1, 2, \cdots, m$, and by substituting for d and n_i, determine the objective function value g_s^C from (3.2).
2. Select the T_0 and d pair that minimizes the objective function g_s^C.

4. A Demand Timing Reduction Model

In practice, buyers participating in the coordination scheme may place orders ahead of the CRE due to demand timing uncertainty. In this case, the vendor will incur a partial loss in capacity to fulfill the urgent demands from SD buyers. To alleviate the impact of demand timing reduction on the vendor, it is reasonable to assume that the CRE-based SD buyers will incur a penalty associated with early ordering to compensate for the vendor's loss.

Assume that the penalty varies for each SD buyer with early ordering, depending on the vendor's flexibility with respect to early ordering and the number of SD buyers that fail. For example, if there is only one SD buyer that orders ahead of the CRE, the penalty is limited and there is a partial impact on vendor flexibility. The vendor makes up the urgent demand based on the vendor's flexibility. The impact on the vendor's flexibility increases as the number of SD buyers ordering early increases, and the alleviation ability to the demand timing reduction decreases for the vendor. Flexibility is not amenable to simple definition since it is dimensional [7,16].

Flexibility can be described in terms of the length of time that an organization to respond to environmental changes. The ability to change for an organization varies; the degrees of flexibility can be day-to-day, within a day, every quarter, or every few years or so [17]. To simplify the analysis, we assume that the flexibility would be a constant parameter over a certain period of time here.

Let f represent the flexibility parameter that relates to the ability of the vendor to increase output in the event of early ordering from SD buyers. The f values close to zero indicate that the vendor has significantly utilized its capacity and therefore cannot supply additional production to satisfy requirements on an emergency basis. As the number of SD buyers ordering ahead of CRE increases, the additional penalties are identical for each buyer. On the other hand, f values close to one indicate high output flexibility that the vendor can supply almost all of demand requirements before the CRE. As the number of SD buyers ordering early increases, so does the additional penalties for each SD buyer.

Based on vendor flexibility and the number of SD buyers that fail, the penalty level for each SD buyer i with early ordering , β_i, can be formulated by the following form, which is

$$\beta_i = \frac{i^f}{\sum\limits_{a=1,2,\cdots,|C|} a^f}, \ 0 < f \leq 1,$$

where $|C|$ represents the number of elements in the set C. The value of β_i varies and it represents vendor's partial alleviation ability to respond to demand timing reduction. The formulation used here for penalty level is the ratio of the individual impact on the vendor's flexibility for each SD buyer i with early ordering to the overall impact on the vendor's flexibility for all SD buyers ordering early.

Figure 1 illustrates the behavior of the β_i function with $|C| = 3$ with different levels of f. Note that at $f = 0$, where the vendor does not have flexibility to respond to early ordering. The additional penalties are identical for each buyer, and thus the cumulative penalties linearly increase as the number of SD buyers ordering before the CRE increases. The additional penalties increase as the number of SD buyers ordering before the CRE increases in all the cases of $f > 0$. As f increases, so does the ability of the vendor to alleviate the impact of early ordering from a SD buyer. However, this ability diminishes as more SD buyers order before the CRE. Therefore, partial penalties increase.

Based on the penalty level for the demand timing reduction, the partial penalty cost for each SD buyer with early ordering in the set C is given by

$$Y_{i,C} = P \times D_i \times d \times \beta_i.$$

The total penalty cost is the cumulative penalties of all the SD buyers in the set C order ahead of the CRE and is determined by

$$Y = \sum_{i=1,\cdots,|C|} P \times D_i \times d \times \beta_i.$$

To formulate the partial penalties of the individual SD buyer ordering ahead of the CRE, the probability of early ordering for SD buyers must be considered in the model. Let $P_{i,C}$ represent the probability that i of the SD buyers order ahead of the CRE, the expected

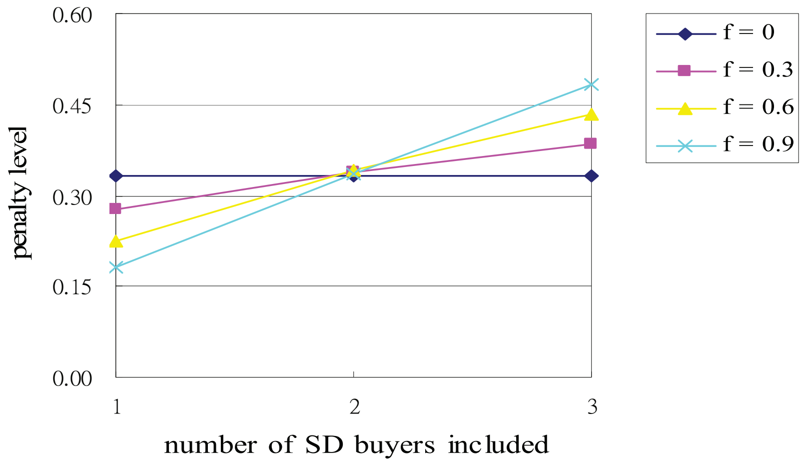

Figure 1: Penalty level and the number of SD buyers

demand timing reduction cost (ETRC) for the partial penalty per SD buyer ordering before CRE is:

$$ETRC = \sum_{i=1,\cdots,|C|} Y_{i,C} P_{i,C}.$$

The vendor's total cost is then reduced, given by

$$TC_S^C = g_s^C - ETRC.$$

Using the algorithm provided in VP [22] and Mishra [14], the optimal solution to the proposed model can be found.

Two cases are considered in the following two sections to determine the probabilities of i out of SD buyers that order before the CRE.

4.1. Case 1: Early ordering of equal probability

The probability of ordering before the CRE could be constant across all the SD buyers. In this case, suppose that all the SD buyers have the same probability of early ordering, E. Note that $P_{i,C}$ is a binomial variable, that is, the probability of early ordering for i out of the SD buyers is determined by $P_{i,C} = \binom{|C|}{i} E^i(1-E)^{|C|-i}$. For example, if $|C| = 2$, the probability for each outcome is $P_{0,2} = (1-E)(1-E)$, $P_{1,2} = E(1-E) + E(1-E)$, and $P_{2,2} = EE$, respectively. Using the $P_{i,C}$ formulation for the equal probability of early ordering, the vendor's total cost under the CRE-based SD policy with of the SD buyers with early ordering is calculated by

$$TC_S^C = \frac{A_S}{T_0} + \sum_{i\in C}[dPD_i + \frac{A_i}{n_i T_0}] - \sum_{i=1,\cdots,|C|} P_{i,C} Y_{i,C} + \sum_{j\in\bar{C}} \frac{(A_j + A_{S_j})}{t_j^U}.$$

4.2. Case 2: Early ordering of unequal probability

The probability of demand timing reduction for a set of SD buyers may vary significantly. The probability that there are i out of the SD buyers ordering before the CRE is the sum of the probabilities that specific SD buyers order before the CRE. That is, if $|C| = 2$, the probability of each of the three outcomes is

$$P_{0,2} = (1 - E_1)(1 - E_2)$$
$$P_{1,2} = E_1(1 - E_2) + E_2(1 - E_1)$$
$$P_{2,2} = E_1 E_2$$

where E_i represents the probability of demand timing reduction for SD buyer i.

Associated with the different probabilities of demand timing reduction for the SD buyers, the vendor may gain a savings related to the price discounts. For example, a riskier SD buyer may accompany a discount reduction. The savings gained for the vendor can be expressed based on a proportion of the discount offered to the SD buyers. Since the probability of early ordering for each SD buyer may not be the same for each one, the less reliable the SD buyer, the more discount reduction offered from the vendor, and thus the more cost savings for the vendor. In this case, the vendor's total cost can be further reduced.

Let v_i be the percentage of the discount reduction for an SD buyer i with a high possibility of early ordering. Using the formulation described in this section, the vendor's savings (VS) gained from the discount reduction for each SD buyer with unequal probability of early ordering can be given by

$$VS_{i,C} = Pv_i dD_i.$$

Using the similar formulation in the equal probability case, the vendor's cumulative savings for i of the SD buyers with unequal probability of early ordering is given by

$$VS = \sum_{i=1,\ldots,|C|} Pv_i dD_i.$$

The expected savings for demand timing reduction (ESTR) per SD buyer ordering before the CRE unequal probability case is defined as

$$ESTR = \sum_{i=1,\cdots,|C|} VS_{i,C} P_{i,C}.$$

Therefore, the vendor's total cost for the partial discount reduction per SD buyer with unequal probability of early ordering is

$$TC_S^C = g_s^C - ETRC - ESTR.$$

Note that for the unequal probability case the vendor's savings is determined using the $P_{i,C}$ described in this section.

5. Model Comparisons

This section conducts a numerical study to compare the vendor's cost savings of the CRE-based SD and the proposed formulations. The results of the SD policy are used here, since Mishra's study [14] showed that SD is a better policy than inclusive discount (ID) or no coordination (ND) strategy in cost savings. Assume that the parameters used in this paper are similar to those used in Mishra [14]. We consider respectively the cases of equal and unequal probabilities of early ordering for SD buyers to study the impact of demand timing reduction on the vendor's total cost.

5.1. Equal probability of early ordering

From the proposed formulation, three levels of f were considered, representing the scenarios relevant to vendor flexibility to alleviate the impact of demand timing reduction from a set of SD buyers. At the highest value ($f = 0.8$) used in this experiment, the impact of demand timing reduction from the partial set of SD buyers on the vendor can be plainly alleviated. The vendor's total cost can be simply reduced from the penalty of the partial set of SD buyers. However, the lowest value of f represents a significant impact on the vendor's total cost. More reduction in vendor's total cost is expected from the penalty of the partial set of SD buyers.

The baseline values used in this analysis are consistent with the values used in Mishra [14]. For the study, only some of the cases for the selective discount policy used in Mishra [14] were selected to compare with the proposed model. The ordering cost and the demand for the buyers are given in Table 1. The detailed results, including the discount offered to SD buyers and the vendor's cost savings using the SD and the proposed strategies are summarized in Table 2. The results described in Table 2 demonstrate the importance of considering early ordering of SD buyers, vendor's capacity loss, and the ability of the vendor to alleviate the impact of early ordering from a partial set of SD buyers when minimizing the vendor's total cost. The table demonstrates, that while the SD policy is better than no coordination (ND) in vendor's savings, the proposed model improves further on these values in all cases.

Similar to the behavior of Mishra's model [14], the vendor's cost savings over ND increases as E increases in the proposed model. However, the degree of this increase was stated by the value of f. For example, for $f = 0.8$, the vendor's savings increases more than the values obtained by Mishra's model [14] at any values of E in all cases. Generally, as the value of f decreases, the value of the vendor's savings over ND increases. Obviously, as E increases, the percentage of the vendor's savings increases at any values of f, also noted in Figure 2.

Table 1: Data for buyers, sorted by replenishment interval prior to any coordination (Taken from Mishra, 2004)

Buyer	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Order cost (in $)	50	50	150	50	150	100	500	500	1500	500	1500	1000	5000	3000	3000
Annual demand (in millions)	2	1	2	0.5	1	0.5	2	1	2	0.5	1	0.5	2	1	0.5
Replenishment interval (in weeks; price=1, holding cost rate=0.15)	0.9	1.3	1.6	1.9	2.3	2.7	3.0	4.2	5.2	6.0	7.4	8.5	9.5	10.4	14.7

Table 2: Vendor's savings for equal probability of early ordering with $P = 1$, $h_i = 0.15$, and $S = 0.1$

Serial No.	A_S	A_{S_i}	A_i	Discount (%)	CRE	# buyers included	Mishra's model over ND (%)	E	f = 0.2	f = 0.5	f = 0.8
1	10	10	100	0.07	2	3	0.83	0.05	0.95	0.93	0.91
								.2	1.34	1.28	1.24
								.2	2.18	2.16	2.15
2	10	10	200	0.11	2	5	6.20	0.05	6.30	6.27	6.25
								.2	6.62	6.54	6.47
								.5	7.35	7.24	7.14
3	100	100	10	0.11	2	6	31.73	0.05	31.91	31.86	31.82
								.2	32.49	32.32	32.17
								.5	33.78	33.55	33.34
4	200	200	10	0.12	2	8	48.58	0.05	48.69	48.66	48.84
								.2	49.04	48.93	48.84
								.5	49.83	49.67	49.53

The vendor's costs related to demand timing reduction decrease considerably at the low values of f. This can be attributed to the fact that the additional penalties are relatively high at low values of f regardless of the values of E. In other words, at low values of f, the ability to alleviate the demand timing reduction is small. More SD buyers involved in early ordering further decrease the vendor's ability and each SD buyer incurs a higher additional penalty, thus improving the vendor's cost reduction.

5.2. Unequal probability of early ordering

When the SD buyers have unequal probabilities of demand timing reduction, the minimum total cost for the vendor will be based on the set of SD buyers that provide the lowest cost. Three levels of E_i were characterized for the SD buyers. Two ranges of the savings were used to describe the partial discount reduction for SD buyers with early ordering. Table 3 lists the levels used for these experimental parameters. Table 4 presents a summary of the results by experimental variables. The results demonstrate that considering unequal probabilities of early ordering and discount reduction significantly impacts on the vendor's cost compared with Mishra's model [14]. As the probability of early ordering increases, the vendor's cost savings increases regardless of the values of v_i or f in all cases. Obviously the change is more notable for the higher levels of v_i and the lower values of f. Thus, in extreme conditions of unreliable SD buyers, high discount reduction per SD buyer, and a low ability to alleviate early ordering from a set of SD buyers, the proposed model is an effective strategy to reduce vendor's cost.

The results in Table 4 are consistent with those obtained for the equal probability model, where a lower reliability of SD buyers resulted in a further increase in vendor's savings. The results also indicate that the cost savings may justify the discount reduction.

Table 3: Characteristics of SD buyers for unequal probability of early ordering

# buyers	E_i			v_i	
included	Low	Medium	High	Low	High
1	0.05	0.35	0.65	0.01	0.1
2	0.1	0.4	0.70	0.05	0.2
3	0.15	0.45	0.75	0.1	0.3
4	0.2	0.5	0.80	0.15	0.4
5	0.25	0.55	0.85	0.2	0.5
6	0.3	0.6	0.90	0.25	0.65
7	0.35	0.65	0.95	0.3	0.85
8	0.4	0.7	0.99	0.35	1

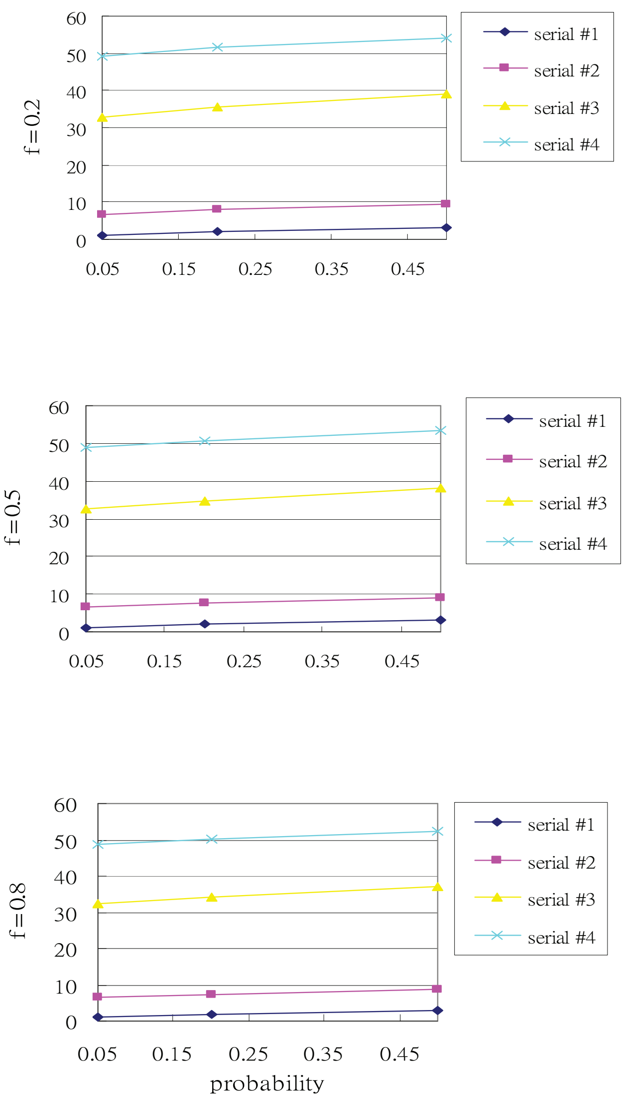

Figure 2: Vendor's cost savings with different flexibility and probability of early ordering

Table 4: Vendor's savings for unequal probability of early ordering with $P = 1$, $h_i = 0.15$, and $S = 0.1$

Serial No.	A_S	A_{S_i}	A_i	Discount (%)	CRE	# buyers included	Mishra's model over ND (%)	v_i	E	f 0.2	0.5	0.8
1	10	10	100	0.07	2	3	0.83	Low	low	1.15	1.11	1.08
									medium	2.11	2.06	2.02
									high	3.22	3.18	3.14
								High	low	1.30	1.26	1.22
									medium	2.64	2.59	2.55
									high	4.14	4.09	4.06
2	10	10	200	0.11	2	5	6.2	Low	low	6.81	6.74	6.67
									medium	7.93	7.81	7.70
									high	9.18	9.13	9.07
								High	low	6.95	6.87	6.81
									medium	9.02	8.90	8.79
									high	11.24	11.18	11.13
3	100	100	10	0.11	2	6	31.73	Low	low	33.49	33.30	33.16
									medium	35.36	35.09	34.86
									high	37.63	37.46	37.29
								High	low	34.13	33.94	33.79
									medium	38.39	38.12	37.89
									high	42.37	42.19	42.02
4	200	200	10	0.12	2	8	45.58	Low	low	50.27	50.14	50.05
									medium	52.16	52.00	51.86
									high	54.35	54.27	54.18
								High	low	52.36	52.23	52.13
									medium	56.25	56.09	55.95
									high	60.72	60.63	60.55

6. Conclusion

This study considers the situation in which SD buyers place orders before the CRE due to demand timing uncertainty in the coordination mechanism of a one-vendor, multi-buyer, and a single product supply chain. We present models that allow a more practical situation by considering the probability of early ordering that is equal and unequal respectively for each SD buyer to investigate the impact of demand timing reduction on the vendors total cost.

This paper shows the importance of the penalty incurred on a set of SD buyers with early ordering, as the vendor's cost reduction varies significantly for different values of f, particularly at the higher values of E. When SD buyers order ahead of the CRE, the vendor may not have the ability to easily deliver the requirements without production loss. Therefore, this study used different levels of flexibility for the vendor to characterize various aspects of partial loss. The proposed model indicates that the vendors cost savings can be improved substantially based on vendor flexibility, probability of early ordering, and the number of SD buyers involved in early ordering. The impact of early ordering on the vendors cost savings depends on the particular combination of these factors.

On the other hand, the analysis of the unequal probability model shows that considering the individual probabilities of early ordering for SD buyers and the possible benefits from reducing the discount offered to SD buyers with early ordering can significantly influence the vendors cost. The analysis shows that the probabilities of early ordering, the levels of savings, and vendor flexibility have an effect on vendors total cost, where less reliability, higher level of savings, and lower flexibility result in a higher percentage of cost savings for the vendor. However, the characteristics of the vendors cost savings depend on the particular combination of these factors. More SD buyers involved in early ordering also resulted in more vendors cost savings in both probability models.

This research extends the existing models by demonstrating that the proposed model behaves differently under a different consideration of early ordering. However, other factors may have to be considered. For example, developing a model that considers the impact of stochastic demand on the vendor's total cost under different CRE strategies would be interesting.

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