

LOCATIONS AND SERVICE START TIME OF FLOW-COVERING FACILITIES WITH MULTIPLE COVERAGE LEVELS

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Abstract This paper develops an extended version of MFCLSTP (Maximum Flow-Covering Location and service Start Time Problem, Tanaka 2011) by introducing multiple coverage levels based on the arrival time to a destination. The original MFCLSTP determines the locations of facilities and the start time of services of fixed duration to maximize coverage for flows on the way back home from work. In MFCLSTP, each flow is either fully covered if commuters can be back home by a given time (after consuming service from start to end at a facility), or not covered at all otherwise. In many situations, a service which ensures that commuters can be back home early is more desirable. To describe this situation, we introduce different levels of coverage and allow the value of coverage to vary depending on the arrival time to a destination (home). The model is applied to the railway network of Chukyo area in Japan by using commuter flow data for railway users in this area. By employing a model with two coverage levels, we obtain the optimal set of stations to site facilities and their service start times. The results show that the optimal time to start a service tends to be earlier when more importance is placed on covering flows that can return home early.

Keywords: Facility planning, dynamic location model, service start time, flow-covering, railway network, commuter traffic flow, coverage level

1. Introduction

Location researchers have developed many types of facility location models that can assist public and private sector decision makers. Daskin [9] provides an excellent taxonomy to classify location models. Facility location is a generally long-term issue, and, in the process of solving the issue, the environment may change considerably. Focusing on this aspect, researchers have attempted to make dynamic location models by incorporating the temporal dimension into facility location problems. In this regard, various important models have been proposed which describe time as a long-term planning horizon (see the review article of Owen and Daskin [15]).

The demands for services are strongly dependent on people's spatiotemporal activities in daily life. Hence, an alternative approach to incorporating dynamic aspects into location models involves focusing on hourly variations in demands and considering the provision of services at facilities in spatiotemporal dimensions. However, this approach has not been widely employed in existing dynamic location models. This paper focuses on a model which takes this modeling approach, called MFCLSTP [19] (Maximum Flow-Covering Location and service Start Time Problem), and presents an extension of this model.

MFCLSTP assumes flows on the way back home from work as potential demands for services, where each flow is designated not only by a pair of origin-destination nodes, but also the departure time from the origin node. Thus, temporal variations in travel demands are explicitly modeled in the input data. It is assumed that each facility provides a service for a fixed number of hours (e.g., after-work lectures at graduate schools, movies at movie

theaters, and sports games at stadiums), and commuters consume the service from the start to end if the commuters can arrive at the facility by the start time of the service and return home by the time they want to. Under these assumptions, the model attempts to find the most suitable locations of p facilities simultaneously, as well as the optimal time to start each service at each facility, in order to maximize the flow coverage. MFCLSTP explores an interesting trade-off of a service start time between being sufficiently late to allow many commuters to be in time for the service, and being sufficiently early for commuters to be able to return home by the time they want to.

As is mentioned, in MFCLSTP, each flow is fully covered if commuters can be back home by a given time after consuming a service at a facility; otherwise it is not covered at all. In many situations, however, the earlier one can be back home, the more desirable a service is. For example, on Monday night, many commuters may want to be back home early enough to have good rest for the next day. Thus, commuters who can access a service at a facility and can be back home by 22:00 (10:00 p.m.) are more likely to become actual users of a facility than commuters who can be back home by 23:00 but not by 22:00.

To describe this gradual-coverage structure, this paper extends MFCLSTP by introducing different levels of coverage, where the coverage varies depending on the arrival time to a destination (home). The objective function then is the sum of the covered flows weighted by the corresponding coverage value. The proposed model determines the locations of service facilities and their service start times that maximize the objective function.

The proposed model has potential applications when planning services of various types over a target area, such as railway networks and road networks. For example, it is common for graduate schools which provide after-work lectures to be located near railway stations. In such situations, identifying a suitable set of stations to site schools and determining the start time of lectures for each school are important. When trips back home are regarded as demands for services, the abovementioned gradual-coverage structure is especially important; the time to arrive back home affects the likelihood of commuters to become actual users of facilities.

This paper applies the model to locational analysis over the railway network of Chukyo area, the third largest metropolitan area in Japan after Tokyo and Osaka. We use commuter flow data for railway users in this area and analyze the optimal set of railway stations to site facilities and their service start times. To examine the effects of changing coverage values on the optimal solutions, we employ a model with two coverage levels and consider two scenarios of different coverage values. Here, coverage levels are based on the arrival time at the destination, and flows for higher-level coverage are defined with an earlier time to be back home than flows for lower-level coverage.

In both scenarios, we fix the value of the higher-level coverage as unity and give a small value to the lower-level coverage in one scenario and larger value in the other scenario (in both cases the value is less than 1). The former scenario places an emphasis on covering flows in which commuters can be back home early. The results show that in the former scenario, facilities tend to start service earlier than in the latter scenario to cover as much flow as possible with higher-level coverage.

The remainder of this paper is organized as follows. In Section 2, we review some related research. In Section 3, we introduce general provisions and basic assumptions, and present an integer programming formulation the proposed model. In Section 4, the railway network of Chukyo area and flow data are presented. Then, in Section 5, optimal solutions are obtained and analyzed. Finally, in Section 6, we conclude the paper and discuss future research directions.

2. Related Research

Location problems we encounter in many practical situations are dynamic since the input parameters (e.g., demands, travel times and land acquisition costs) may change over time. Focusing on this aspect, researchers have developed various types of dynamic location models. Many of these models describe time as a long-term planning horizon. Typically, the locations of facilities as well as the timing to open and close them are determined by extending known static models. Examples include research by Schilling [17], Gunawardane [12], and Drezner and Wesolowsky [10]. For detailed reviews of these types of dynamic location problems, see Current et al. [8], Owen and Daskin [15], and Snyder [18].

An alternative approach to devising dynamic location models involves describing hourly variations in demands and determining locations of facilities as well as their service start times. Compared to models assuming the long-term planning horizon, this approach has not been widely employed. Research by Bloxham and Church [5] appears to be the first to focus on the service operation schedules of facilities. Their model, termed the p -median scheduling and location problem (PMSLP), places p facilities on a network, and simultaneously allocates operating resources by determining the number of hours that each facility should operate. The objective of PMSLP is to minimize the weighted distance traveled by all users to their closest open facility. This model simplifies the situation by assuming that the demand generated at a node in a given time period can be served by the closest open facility during that period, irrespective of the access time to the facility.

Recently, Tong et al. [20] proposed an interesting model structurally similar to PMSLP. The model, motivated by service provision planning of farmers' markets, determines the locations and the associated service schedules for a fixed number of facilities (farmers' markets). The model classifies customers into two groups, workers and non-workers. Trips made by non-workers originate from home on both weekdays and weekends. On weekdays, workers have regular work hours, and hence access to a farmers' market is more temporally constrained than that of non-workers. Thus, on weekdays, customers may stop by a markets during their lunch break, or make a stop on their way back home if it is convenient. The model of Tong et al. was used in a case study to determine the sites of farmers' markets in Tucson, Arizona. Since the basic motivation of that model seems close to that of our model, it is helpful to point out some differences and characteristics of the model of Tong et al. One of the major differences is that the model employs a median objective, while our model aims to cover as much flow as possible (covering objective). Another important aspect of our model is that it focuses on the times of departure from the origin and arrival at the destination for each flow. In particular, the varying coverage value is introduced depending on the arrival time at the destination node.

Some ambulance deployment problems also assume hourly variations in demands and consider the operation of ambulances in a dynamic setting (e.g., [16]). The potential demand for emergency services and the travel times between points in a city can change throughout the day. By focusing on this issue, models that allow ambulance vehicles to change their locations at any point in time have been explored. In such studies, time-dependent point-based demands are assumed, and the objective is to cover as many potential demands as possible. Thus, the definition of coverage is a straightforward extension of MCLP. On the other hand, our model assumes flow-based demands with the additional information of the departure time from the origin node. In addition, the coverage considered here is not the standard definition of coverage (i.e., existence of a facility within a maximum service distance/time), but is based on service accessibility in spatiotemporal dimensions.

Church and Robert [6] first proposed the notion of varying coverage values in the weighted benefit maximal covering problem (WBMC) by extending the maximal covering location problem (MCLP) [7]. MCLP attempts to maximally cover demands serviced within a given distance from a fixed number of facilities (Church and ReVelle [7]). In contrast with the assumption of MCLP that coverage value is constant, coverage value in WBMC is given by a function of distance to the nearest facility providing a service, and the objective function is given by the sum of the covered demands weighted by the corresponding coverage value. In the proposed version of MFCLSTP, multiple threshold values of arrival times to a destination play an analogous role to multiple coverage radii in WBMC. Although, the proposed model shares the abovementioned similarities with WBMC for varying coverage, the proposed model focuses on the arrival time to a destination and is thus unique to the spatiotemporal situation considered here.

A typical situation WBMC assumes is that the coverage value decreases as the distance to the nearest facility increases. In a retail situation, this may represent a situation where the shopping frequency for customers declines with the increase in distance to a shop because of the increasing inconvenience of traveling further. A network and a discrete model with a general non-increasing coverage function was analyzed in Berman and Krass [3]. Planar versions are also proposed in Drezner et al. [11]. Models of this type are called gradual-coverage models. Recent developments can be seen in the review article by Berman et al. [2]. The proposed model can be seen as a gradual-coverage version of MFCLSTP in the sense that the model incorporates multiple coverage levels.

MFCLSTP and its extension considered in this paper assume flow-based demands, and can thus be seen as dynamic variants of the flow-capturing (or interception) location problems (FCLP), which are independently introduced by Hodgson [14] and Berman et al. [4]. In this class of models, demands for services are represented as flows traveling along paths between pairs of origin-destination nodes across a network. FCLP attempts to place a given number of facilities in the network in order to maximize the total number of flows having a facility along their pre-planned route. The flow capturing approach is suitable for determining the optimal service locations of various types, such as automatic teller machines, convenience stores, advertising billboards, and vehicle inspection stations. Details on FCLP variants can be found in a recent article by Zeng et al. [22]. Although various types of generalized models have been proposed, the existing models are static models. MFCLSTP can be seen as a dynamic version of FCLP where each flow is identified not only by a pair of origin-destination nodes, but also by the departure time from the origin node. In addition, MFCLSTP determines the service start times of facilities, as well as their locations.

3. Model Description and Formulation

First, we introduce the original version of MFCLSTP [19], after which we extend it to a model with multiple coverage levels.

3.1. The original MFCLSTP

MFCLSTP considers a situation shown in Figure 1 (a), where a temporal dimension is introduced into a network. A two-dimensional version (one-dimensional city, plus a temporal dimension) is also shown in Figure 1 (b) to facilitate explanation. A decision maker introduces p identical facilities that provide a service for a fixed number of hours c , as illustrated by the bold line segments in Figure 1. The service provided at a facility must be consumed from start to end. Let us use N to denote a set of nodes used for both origins and destinations of trips, and T to denote a set of times of departure from origin nodes. To represent

services at facilities, K is a set of candidate locations, and S is a set of service start times. We assume that T and S are finite.

MFCLSTP assumes that a service is provided to after-work commuter flows, where each flow is identified by the triplet (i, j, t) of an origin node $i \in N$, a destination node $j \in N$, and a time of departure from the origin node, $t \in T$. The volume of each flow is given by f_{ijt} . The service at each facility is denoted by the pair (k, s) of a location $k \in K$ and a service start time $s \in S$. A commuter flow is said to be *covered* when commuters can access a facility before s , stay there until $s + c$ (meaning consumption of a service of c hours at the facility), and then return home by a given time t_h (here, the subscript “h” represents home). This condition can be written as $t + u_{ik} \leq s$ and $s + c + u_{kj} \leq t_h$. Here, u_{ij} is the travel time between node i and node j . In formulating MFCLSTP, the following a_{ks}^{ijt} is given by:

$$a_{ks}^{ijt} = \begin{cases} 1 & \text{if flow } (i, j, t) \text{ can be covered by a service } (k, s), \\ 0 & \text{otherwise.} \end{cases}$$

Two types of binary variables are introduced:

$$x_{ks} = \begin{cases} 1 & \text{if a facility is located at node } k \text{ and starts its service at time } s, \\ 0 & \text{otherwise,} \end{cases}$$

$$y_{ijt} = \begin{cases} 1 & \text{if commuter flow } (i, j, t) \text{ is covered,} \\ 0 & \text{otherwise.} \end{cases}$$

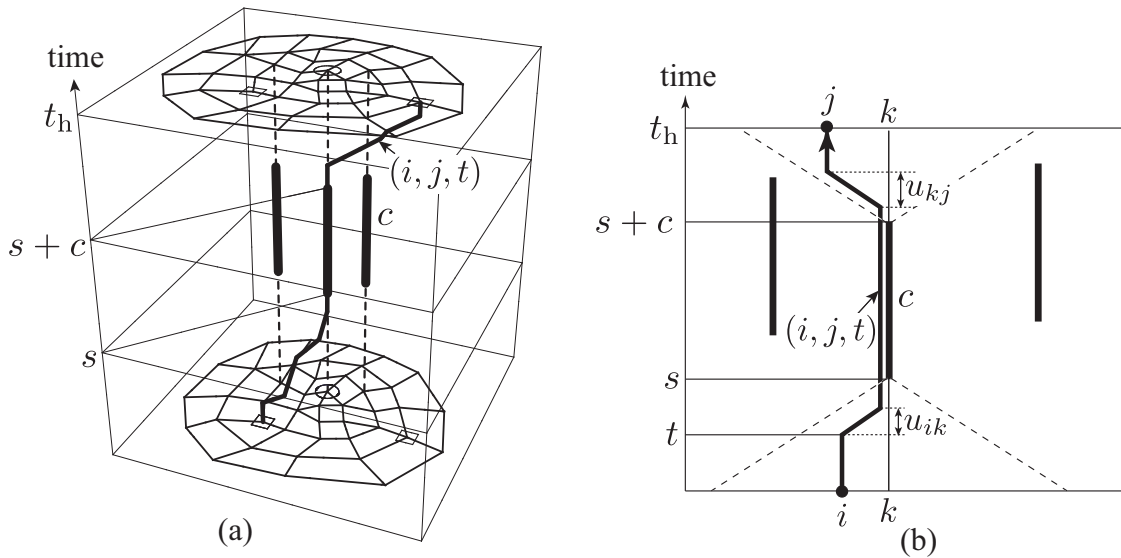


Figure 1: Total flow coverage in spatiotemporal dimensions in the case of (a) a general network and (b) a linear city

In MFCLSTP, two different situations can be considered. The first situation assumes that the service start time of each facility can be independently determined. On the other hand, the second situation assumes that all facilities have the same service start time.

Using the above definitions, the independent service start time model (I-model) is formulated as follows:

Independent start time model (I-model)

$$\text{maximize } \sum_{i \in N} \sum_{j \in N} \sum_{t \in T} f_{ijt} y_{ijt} \quad (3.1)$$

$$\text{subject to } \sum_{k \in K} \sum_{s \in S} x_{ks} = p, \quad (3.2)$$

$$y_{ijt} \leq \sum_{k \in K} \sum_{s \in S} a_{ks}^{ijt} x_{ks}, \quad \forall i \in N, \forall j \in N, \forall t \in T, \quad (3.3)$$

$$x_{ks} \in \{0, 1\}, \quad \forall k \in K, \forall s \in S, \quad (3.4)$$

$$y_{ijt} \in \{0, 1\}, \quad \forall i \in N, \forall j \in N, \forall t \in T. \quad (3.5)$$

The objective function (3.1) is the total number of covered flows, that is, the number of commuters with access to at least one of the p facilities. Constraint (3.2) stipulates that exactly p facility services are provided. Notice that this constraint does not prohibit placing multiple facilities at the same node, and two or more colocated facilities with different service start times may cover different flows. Constraints (3.3) require that at least one facility service, (k, s) , be accessible from commuter flow (i, j, t) for this flow to be covered. Finally, constraints (3.4) and (3.5) are standard binary constraints on the decision variables.

Next, consider a situation in which the service start times of all facilities must be the same (common start time model, or C-model). To formulate this model, the variables z_s that determine the service start time are introduced as follows:

$$z_s = \begin{cases} 1 & \text{if all facilities start the service at time } s, \\ 0 & \text{otherwise.} \end{cases}$$

With this definition of z_s , the C-model can be formulated similarly to the I-model by introducing additional constraints:

Common start time model (C-model)

$$\text{maximize } (3.1)$$

$$\text{subject to } (3.2), (3.3), (3.4), (3.5),$$

$$p z_s = \sum_{k \in K} x_{ks}, \quad \forall s \in S, \quad (3.6)$$

$$\sum_{s \in S} z_s = 1, \quad (3.7)$$

$$z_s \in \{0, 1\}, \quad \forall s \in S. \quad (3.8)$$

Note that constraint (3.7) is not needed in the integer programming formulation of this problem since constraint (3.6) ensures that exactly one start time is selected for all facilities [19]. However, when using a mathematical programming solver to find an optimal solution, the inclusion of constraint (3.7) strengthens the linear programming relaxation considerably, and hence its inclusion is desirable.

3.2. MFCLSTP with multiple coverage levels

We extend the original MFCLSTP by introducing coverage levels. It is assumed that an earlier arrival time at home is more desirable for commuters. We formulate a model with an

arbitrary number of coverage levels, while we use a model with two coverage levels, as shown in Figure 2, for ease of the explanation of its concept. Let us denote the lower coverage level by *partial* coverage and the higher level by *full* coverage.

As shown in Figure 2, there are two threshold values for the limit to the arrival time at home: one is t_h^f for full coverage and the other is t_h^p for partial coverage. We denote the full and partial coverage values as w_f and w_p respectively. On the basis of assumption, we set $w_f > w_p$. The importance for a decision maker to fully cover the flows increase as w_f becomes large relative to w_p . In the case of a retailer, a larger value of w_f compared to w_p may mean that fully covered flows are much more likely to become the customers of the retailer than partially covered flows.

In Figure 2, two flows (i, j, t) and (i', j', t') and three access patterns P1, P2, and P' for these two flows are shown. In this case, flow (i, j, t) has two possible access patterns P1 and P2 to the services at the center and on the left, respectively. Since this flow can be back home by t_h^f using pattern P1, the flow is fully covered. As this case indicates, due to the maximization of the objective, each flow is naturally assigned to the highest possible level. On the other hand, flow (i', j', t') is covered only partially. This is because the only accessible service is the one on the right, with access pattern P', and this flow can be back home by t_h^p , but not by t_h^f .

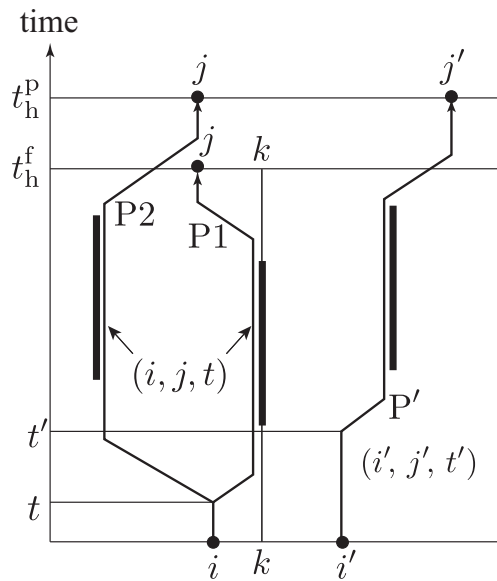


Figure 2: Definition of full coverage and partial coverage with two coverage levels

Based on the above discussion, we formulate a problem allowing for an arbitrary number of coverage levels. Let L be the set of coverage levels. For each coverage level $l \in L$, we denote the coverage value by w_l , where $w_{l_1} < w_{l_2} < \dots < w_{l_{|L|}}$. The coverage index is written as follows:

$$a_{ks}^{ijtl} = \begin{cases} 1 & \text{if flow } (i, j, t) \text{ can arrive at a facility at node } k \text{ which starts a service at } s \text{ and} \\ & \text{consume } c \text{ hours of service, and then can get to a destination node by } t_h^l, \\ 0 & \text{otherwise.} \end{cases}$$

Flow coverage variables now also have an additional subscript l :

$$y_{ijtl} = \begin{cases} 1 & \text{if commuter flow } (i, j, t) \text{ is covered at service level } l \in L, \\ 0 & \text{otherwise.} \end{cases}$$

Using the above definitions, the independent start time model is formulated as follows:

I-model

$$\text{maximize } \sum_{i \in N} \sum_{j \in N} \sum_{t \in T} \sum_{l \in L} w_l f_{ijt} y_{ijtl} \quad (3.9)$$

$$\text{subject to } \sum_{k \in K} \sum_{s \in S} x_{ks} = p, \quad (3.10)$$

$$y_{ijtl} \leq \sum_{k \in K} \sum_{s \in S} a_{ks}^{ijtl} x_{ks}, \quad \forall i \in N, \forall j \in N, \forall t \in T, \forall l \in L, \quad (3.11)$$

$$\sum_{l \in L} y_{ijtl} \leq 1, \quad \forall i \in N, \forall j \in N, \forall t \in T, \quad (3.12)$$

$$x_{ks} \in \{0, 1\}, \quad \forall k \in K, \forall s \in S, \quad (3.13)$$

$$y_{ijtl} \in \{0, 1\}, \quad \forall i \in N, \forall j \in N, \forall t \in T, \forall l \in L. \quad (3.14)$$

The objective function (3.9) is the sum of the covered flows for each level weighted by the corresponding coverage value. Constraint (3.11) requires that at least one facility service (k, s) be accessible by level l for commuter flow (i, j, t) for this flow to be covered by service level l . Constraint (3.12) stipulates that each flow is either covered by exactly one level or not covered at all. Due to the form of the objective function, each flow is naturally assigned to the coverage with the highest possible level. Finally, constraints (3.13) and (3.14) are standard binary constraints on the decision variables.

We omit a formulation of the C-model as it is identical to the formulation of the I-model with the addition of constraints (3.6), (3.7), and (3.8).

4. Railway Network of Chukyo Area and Flow Data

The proposed model is applied to the railway network of Chukyo area. First, we explain how network data is constructed, and then introduce dynamic flow data extracted from census data.

4.1. Chukyo railway network

We construct the railway network covered by 2005 census data for Chukyo area [23]. Chukyo metropolitan area is Japan's third largest metropolitan region after Tokyo and Osaka. Nagoya, the capital of Aichi Prefecture, is the center of this area. The entire network is shown in Figure 3 (a), and the central region is shown Figure 3 (b). Each unit section on the axes represents a distance of 1 km, and the origin of the coordinates is set at Nagoya station, the largest station in this area. Stations denoted by larger dots with their names are analyzed in the next section. These are stations which appear at least once in some optimal solution. The network is composed of 747 stations along 54 lines, from a total of 751 stations along 55 lines in the census data. Four Shinkansen (bullet train) stations are excluded from the set of target stations. This is because Shinkansen users generally travel much larger distances than assumed in this application, and thus they may behave differently from other railway users.

Stations with the same name or very close to each other are treated as a single candidate location for siting a facility. Such aggregation of stations is conducted using the Station Database [24], which contains information on groups of stations between which transfers are possible. Among the targeted 747 stations, 172 stations have at least one transfer station, and these are aggregated into a total of 73 stations. Thus, the number of target stations becomes 648 ($= 747 - 172 + 73$) after aggregation. All 648 stations are used as candidate locations for facilities.

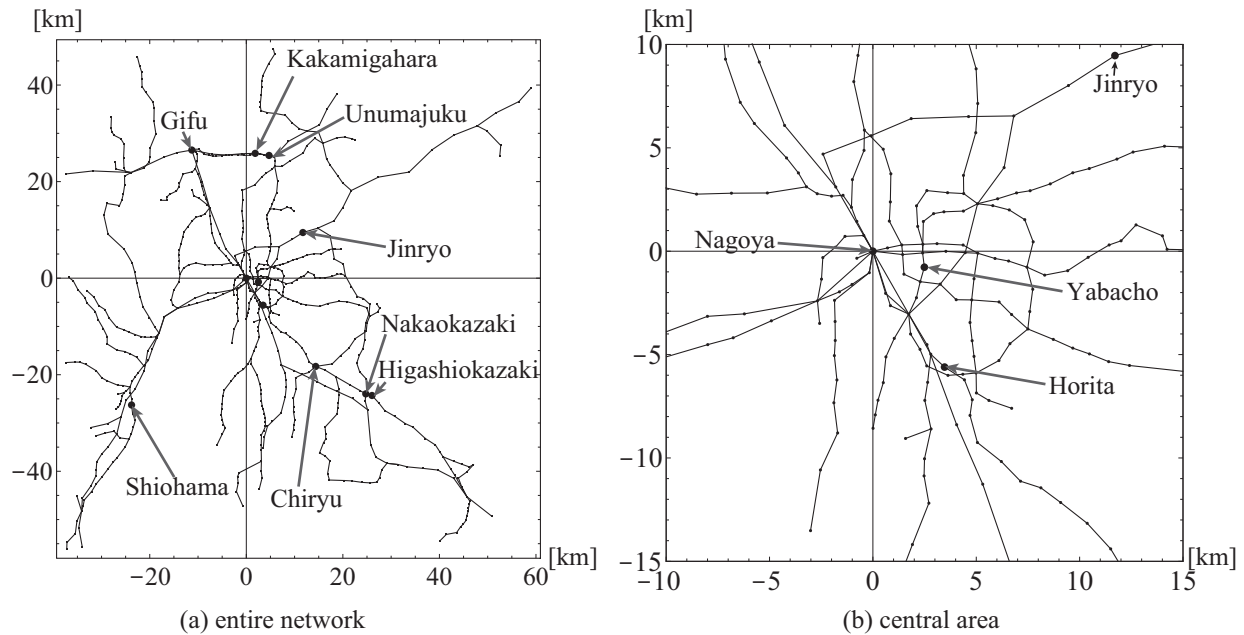


Figure 3: Chukyo metropolitan railway network

4.2. Flow data

In order to construct dynamic flow data, we first make an origin-destination (OD) traffic flow matrix from census data and determine the distribution of departure times from origin station for each OD pair. Here, we present origin-destination (OD) data which gives the number of commuters traveling from an origin station i to a destination station j , and see how origins and destinations of trips are distributed over the network. In total, 7,720 out of 419,904 (648 by 648) entries in the OD matrix have non-zero flow volumes, and the total flow volume is 684,935.

Figure 4 (a) and (b) show the total number of origins and destinations for all flows at each station. Trip origins (workplace locations) are strongly concentrated at the central area of Nagoya, while destinations (home locations) are more spread out and not concentrated at particular locations. Roughly speaking, after-work flows are generated from the center of Nagoya city in various directions.

5. Application to Chukyo Railway Network

We focus on a model with two coverage levels (i.e., $|L| = 2$) and look for optimal solutions for both the I-model and the C-model. Basic parameters values are set to $c = 3$ h, $t_h^f = 22:00$, $t_h^p = 23:00$.

The travel time u_{ij} for each station pair (i, j) is constructed as follows: First, the cost (time) between adjacent stations is set as the length of the link (Euclidean distance) between

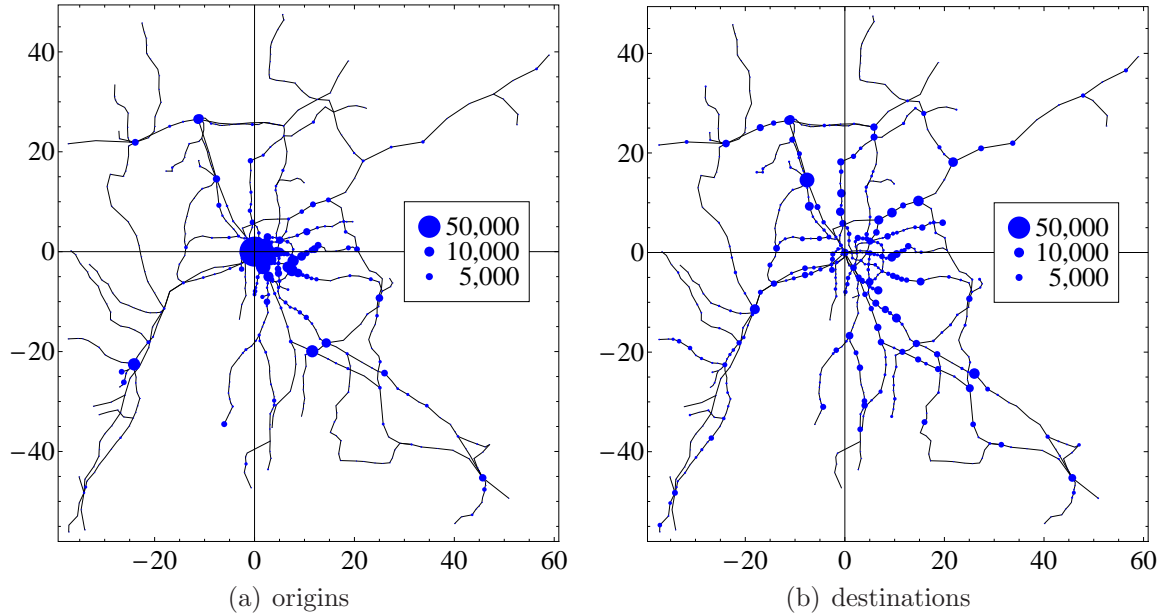


Figure 4: Number of origins (a) and destinations (b) for each station

the stations divided by the speed of the trains. We use the same speed of 50 km/h for all trains when calculating link costs. Then, the travel time matrix is obtained by calculating the shortest paths for all station pairs by using the Dijkstra algorithm. Values u_{ij} , together with t_h^f and t_h^p , are used to generate the coverage index.

To create dynamic flow data f_{ijt} , we assume that, for each OD flow pair, 20% of its flow volume departs from an origin node at 17:00, 30% at 18:00, 30% at 19:00 and the remaining 20% at 20:00. (The value in the first decimal place is rounded to the nearest integer value, resulting in a total flow volume of 685,616.)

To examine the effects of different coverage values on the optimal solutions, we consider two scenarios for both the I-model and the C-model and compare the following four cases:

Case 1: I-model, Monday Night Scenario ($w_f = 1.0$ and $w_p = 0.2$)

Case 2: I-model, Friday Night Scenario ($w_f = 1.0$ and $w_p = 0.8$)

Case 3: C-model, Monday Night Scenario ($w_f = 1.0$ and $w_p = 0.2$)

Case 4: C-model, Friday Night Scenario ($w_f = 1.0$ and $w_p = 0.8$)

We call the first scenario ($w_f = 1.0$ and $w_p = 0.2$) the Monday Night Scenario (MNS) and the second one ($w_f = 1.0$ and $w_p = 0.8$) the Friday Night Scenario (FNS). These names are because on Monday night commuters often want to go back home early enough to have a good rest for the next day, while on Friday night they do not worry much about the time of arriving home, in contrast to Monday night. In MNS, covering flows that can arrive home by $t_h^f = 22:00$ is much more important (five times more important in this case) than covering flows that can arrive home by $t_h^p = 23:00$. In FNS, partial coverage is not so drastically different from full coverage. Thus, on Friday night, it is advisable for a decision maker to start the service later than on Monday night in order to allow commuters arrive in time for the start of the service while ensuring that they can be back home by 23:00 (partial coverage). Based on the above settings, we obtained exact optimal solutions for 10 instances ($p = 1$ through $p = 10$) for each of the above four cases using a mathematical programming solver, IBM ILOG CPLEX 12.4, which took several minutes to several tens of minutes to solve each instance.

5.1. Objective values and coverage volume

Let us focus on the optimal objective value and coverage volume for each optimal solution. Figure 5 shows the optimal values for the two scenarios (MNS and FNS). The vertical distance between the two curves corresponds to the advantage of the I-model over the C-model. When comparing the results of these two models, we should also consider the operational advantage of the C-model over the I-model due to the common service hours.

The difference in the objective values is much greater for FNS than MNS. In MNS, the objective value for $p = 2$ in the I-model is approximately the same as that for $p = 3$ in the C-model, while in FNS, the objective value for $p = 2$ in the I-model is roughly the same as that for $p = 10$ in the C-model. As we analyze in more detail in Figure 6, for the C-model in FNS, all covered flows are partially covered. This is due to the late start of services at facilities. In contrast, for the I-model in FNS, about half of the covered flows are fully covered, while the total number of covered flows (the sum of partially and fully covered flows) is almost the same as in the C-model. This difference in full coverage volume gives rise to the abovementioned difference in objective values between the I-model over the C-model in this case.

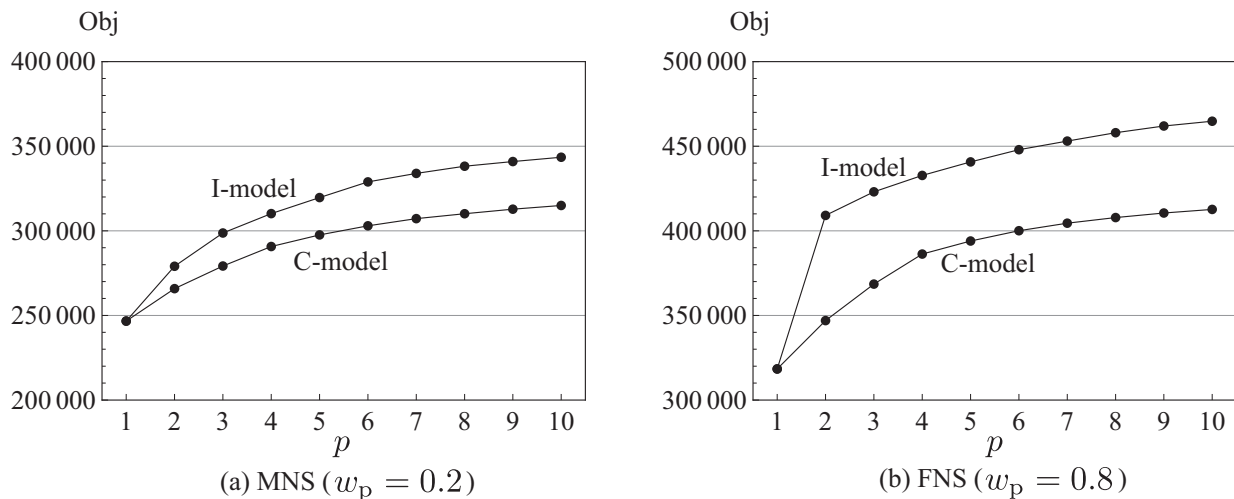


Figure 5: Objective values as the number of facilities for MNS and FNS

Next, to analyze the effects of different coverage values on the optimal solutions, we focus on the total flow volume for full coverage and partial coverage. These values are shown as percentages of the total flow volume of 685,616 in each optimal solution. Figure 6 (a) through (d) show the results of the four cases, indicated by white bars for full coverage and black and gray bars for partial coverage. Fully and partially covered flows are further classified according to the departure time from the origin node. In these instances, the latest possible departure time to be fully covered is 18:00, and the departure time to be partially covered is 19:00, due to the parameter values of $c = 3$ hours, $t_h^f = 22:00$ and $t_h^p = 23:00$. This arises from the fact that in order for flows departing from an origin node at 18:00 (19:00) to be fully (partially) covered, they require 3 h at a facility added to the time to go to the facility and return home from the facility. This makes the earliest possible time of arrival home later than 22:00 (23:00). The solid horizontal line represents the maximum possible value of full coverage with respect to the total flow volume when we can site any number of facilities; the value in this case is 48.13%. Similarly, the dashed horizontal line means the maximum possible value of partial or full coverage with respect to the total flow volume,

and the value in this case is 78.16%. These values indicate what can be achieved for each p .

First, let us focus on the results of the I-model as shown in Figure 6 (a) and (b). Different coverage patterns can be seen for MNS and FNS. The percentage of fully covered flows in MNS is larger than that in FNS at all p but $p = 2$, in which case both scenarios have the same optimal solution. This difference is due to the relative importance of full coverage to partial coverage. The percentage of covered flows (the sum of full and partial coverage) in FNS is slightly larger than that in MNS, except for the case of $p = 2$. This is because partially covering a given flow is easier than fully covering the same flow, and a flow that can be back home by $t_h^f = 22:00$ can also be back home by $t_h^p = 23:00$. In the case of $p = 10$, the full coverage volume in MNS is rather close to the maximum possible value, while the total (full and partial) coverage volume has not reached its maximum value. The opposite trend can be observed in FNS.

Let us focus on the results of the C-model as shown in Figure 6 (c) and (d). Similar tendencies can be observed as in the case of the I-model, but the difference is much more apparent. The C-model does not allow for much flexibility in determining the start time. The choice is between all facilities starting the service early or late. In these instances, all facilities start the service early in MNS and late in FNS. As shown in Figure 6 (d), all flows are partially covered in FNS. This is due to the late start (19:20) of the service at all facilities, as we discuss in more detail below. In that case, the earliest possible time to arrive home (when the time to go to a facility and return home from the facility is negligibly small) is 22:20 after staying for three hours at a facility.

5.2. Analysis of optimal solutions

Next, we analyze the optimal solutions. Figures 7 through 10 show optimal facility locations over the network for $p = 1$ through $p = 6$ for the four cases. Tables 1 through 4 list the names of stations and start times for all optimal solutions. In some optimal solutions of the I-model, two or three facilities are located at the same station with different service start times. Gray disks in Figures 7 and 8 indicate such stations. Such collocation of facilities is observed only at Nagoya and Yabacho, two stations located in the central area of Nagoya City. On the other hand, in the case of the C-model, there is no advantage in siting two or more facilities at the same station because of the same start time. Colocated facilities cover the same set of flows, which does not bring any additional benefit in terms of the objective value. Thus, collocation does not occur in the C-model. Below, we analyze each of the four cases.

Case 1: I-model, MNS

Figure 7 and Table 1 show optimal solutions for Case 1: I-model and MNS ($w_p = 0.2$). The optimal single facility location is Yabacho, and its start time is 18:20. Yabacho is located close to Nagoya station as well as two other important stations: Sakae and Kamimaezu, at both of which two or more subway lines intersect. Thus, access to and from other stations is quite good. A service start time of 18:20 is early enough to allow many commuters to be fully covered (by allowing them to go back home before $t_h^f = 22:00$ after consuming the service at the facility). As can be seen from Figure 6 (a), a single facility can fully cover a large number of flows.

In the case of $p = 2$, both facilities are located at Yabacho, and the service start times are one hour apart: 18:20 and 19:20. When temporal factors are ignored, collocation of facilities is not observed since they cover the same set of flows. This result indicates that the introduction of a temporal dimension is highly important when determining the optimal service provision at a facility. As this result shows, Yabacho is an extremely important

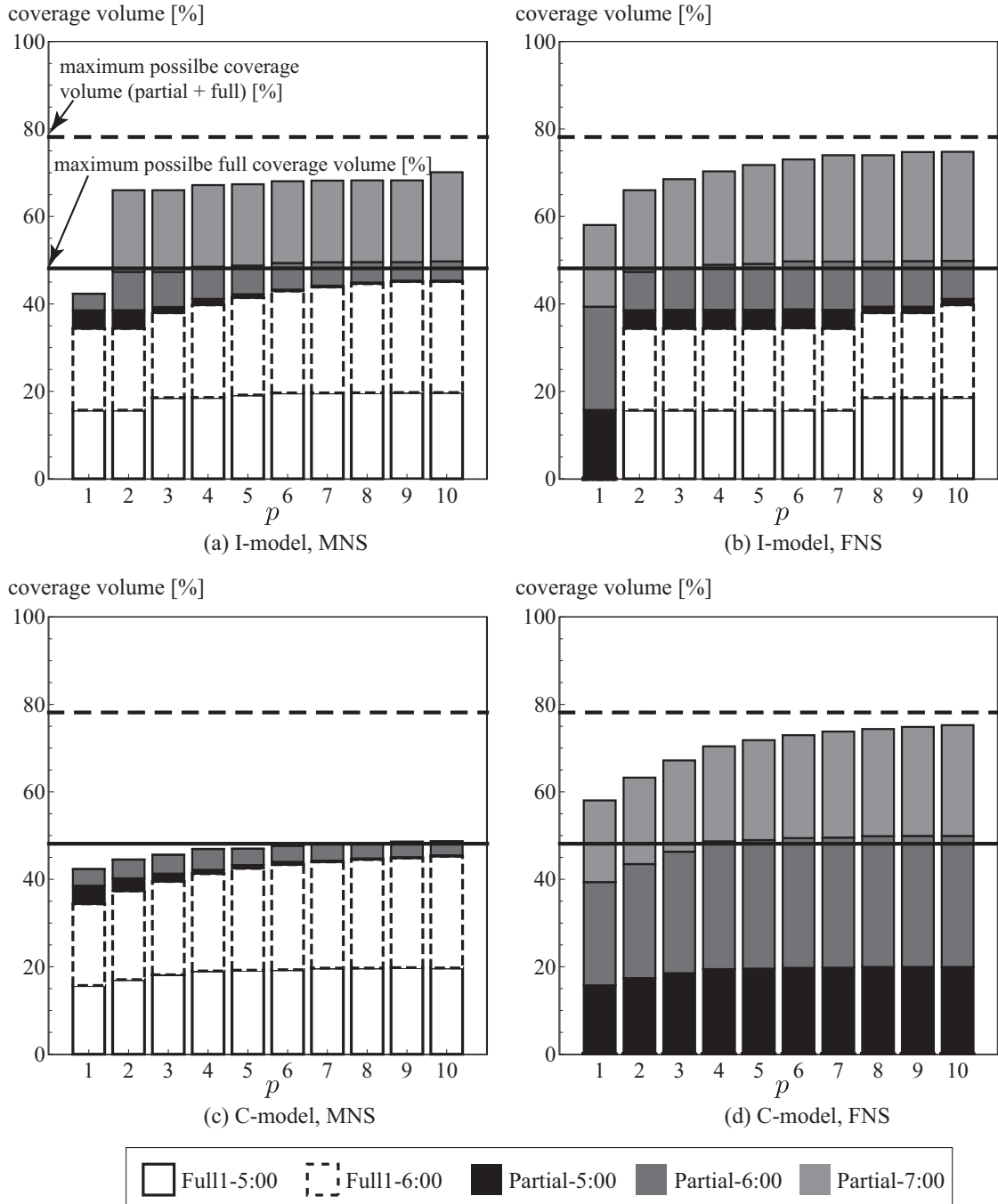


Figure 6: Percentage of full and partial coverage volume for each p

location for covering after-work flows in this area. Due to the presence of a second facility at Yabacho which starts service at 19:20, the number of commuters partially covered by the second facility is greatly increased, as can be observed from Figure 6 (a).

The optimal solution for $p = 3$ includes a facility at Nagoya starting service at 18:00 in addition to two facilities providing the same service at Yabacho with start times as in the case of $p = 2$. The facility at Nagoya further increases the full coverage volume. In the case of $p = 4$, three facilities are located at Yabacho with different service start times in addition to one at Nagoya.

In the case of $p = 5$, one facility is located at Nakaokazaki southeast of Nagoya, in addition to the four closely located facilities at the central area. This is due to the large number of destinations at Nakaokazaki and the presence of some large destinations between Nagoya and Nakaokazaki. For $p = 6$, another facility in the suburban area is located at Shiohama. The service start time of these facilities is 18:20, which contributes to the increase in full coverage.

Case 2: I-model, FNS

Figure 8 and Table 2 show the optimal solutions for Case 2: I-model and FNS ($w_p = 0.8$). Case 1 and Case 2 share a similar location pattern, but one important difference is that in contrast to Case 1, in Case 2 facilities tend to be located in suburban areas. Regarding the optimal service start times, they are generally later than those in the corresponding optimal solution in Case 1. This is due to the fact that the value for partial coverage is not vastly different from that for full coverage in this case. By starting the service late, more commuters can be in time for the start of the service. At the same time, this start time is early enough to allow commuters to return home by $t_h^p = 23:00$.

As for $p = 1$, exactly the same location, Yabacho, is selected, but the service start time is 19:20, one hour later than that in MNS. This difference results in greater partial coverage and lower (zero in this case) full coverage. The solution for $p = 2$ is exactly the same as that for MNS. In contrast to Case 1, one facility (Chiryu) for $p = 3$ and two facilities (Chiryu and Shiohama) for $p = 4$ are located in suburban areas, and their service start time is late (19:20). These late services contribute to the increase in partial coverage compared to the single late service at the center in the case of $p = 3$ and $p = 4$ in Case 1. For $p = 5$ and $p = 6$, three facilities at Yabacho are selected with different start times. Services provided in suburban areas start late (19:20), in contrast to the early start times in Case 1.

Case 3: C-model, MNS

Figure 9 and Table 3 show optimal solutions for Case 3: C-model and MNS ($w_p = 0.2$). As intuitively expected, the selected stations are more dispersed than those in the I-model. In all cases, the service starts at 18:20. For $p = 2$, Chiryu is selected as the location for a facility together with Nagoya. In the case of $p = 3$, Yokkaichi is added to the solution for the case of $p = 2$. Then, for $p = 4$, Gifu, which has a large population and is located northwest of the center, also has a facility in addition to the ones in the case of $p = 3$. It is interesting to note that all facilities in suburban areas are located in different directions from the center of Nagoya. For $p = 5$ and $p = 6$, two facilities are located near the center, and the other facilities are located similarly to those in the case of $p = 4$.

Case 4: C-model, FNS

Figure 10 and Table 4 show optimal solutions for Case 4: C-model and FNS ($w_p = 0.8$). The location patterns are similar to those in Figure 9. However, the service start time of all facilities is 19:20, one hour later than the solution in Figure 10. In the C-model, the choice is between all facilities starting the service early or late. Due to the late start in FNS, no

flows are fully covered, as shown in Figure 6. However, the percentage of flows with partial coverage is large.

Summary of the four cases

The characteristics of the optimal solutions of the four cases above (MNS or FNS and I-model or C-model) can be summarized as follows. Depending on the relative importance of the partial coverage value w_p to the full coverage value w_f different optimal solutions are obtained in terms of both locations and service start times. For the I-model, facilities tend to be sited in suburban areas more often in FNS than in MNS. For MNS, in which the full coverage value is much more important than the partial coverage value, service start times are generally earlier than those for FNS. Thus, it is important to determine desirable coverage values according to the types of services provided and the corresponding behavior of people accessing the services.

We also see that siting pattern in the I-model is drastically different from that in the C-model. In the former, two or more facilities tend to be located at the central area, while in the latter, selected facility locations are more spatially dispersed. In particular, in some solutions of the I-model, colocated facilities with different service start times are observed.

6. Conclusion

This paper presented an extended version of MFCLSTP (Maximum Flow-Covering Location and service Start Time Problem) [19] developed by introducing multiple coverage levels which are dependent on the arrival time at the destination node. MFCLSTP determines the siting of facilities and the corresponding start times of services of fixed duration at selected facility locations to maximize coverage for flows on the way back home from work. In MFCLSTP, each flow is either fully covered if it can be back home by a given time (after consuming a service at a facility), or not covered at all otherwise. In many situations, a service start time which ensures an earlier time to be back home is more desirable. To describe this situation, we introduced different levels of coverage and allowed the value of coverage to vary depending on the arrival time to a destination node (home).

The model was applied to the railway network of Chukyo area, the third largest metropolitan area in Japan after Tokyo and Osaka metropolitan area. Candidate locations for facilities were set for all 648 stations in the network. Using commuter flow data for railway users in this area, optimal solutions were obtained and analyzed in detail. The model with two levels of coverage was applied, and two scenarios were considered: a Monday night scenario and a Friday night scenario. The former assumed that the relative importance of partial coverage to full coverage is low, while that of the latter is high. We also considered two models with different strategies for the start time of services: I-model (*independent* start time model) and C-model (*common* start time model), and examined how these differences affect the optimal solutions. The former model assumed that the service start time can be set independently for each facility, while the latter assumed that all facilities start the service at the same time.

The results showed that the relative importance of the partial coverage value w_p to the full coverage value w_f has a great impact on the optimal solutions. For MNS, in which the full coverage value is much larger than the partial coverage value, service start times are generally earlier than those for FNS. Thus, it is important to determine desirable coverage values according to the types of services under study and the corresponding behavior of people accessing the services. We also demonstrated that the optimal location pattern in

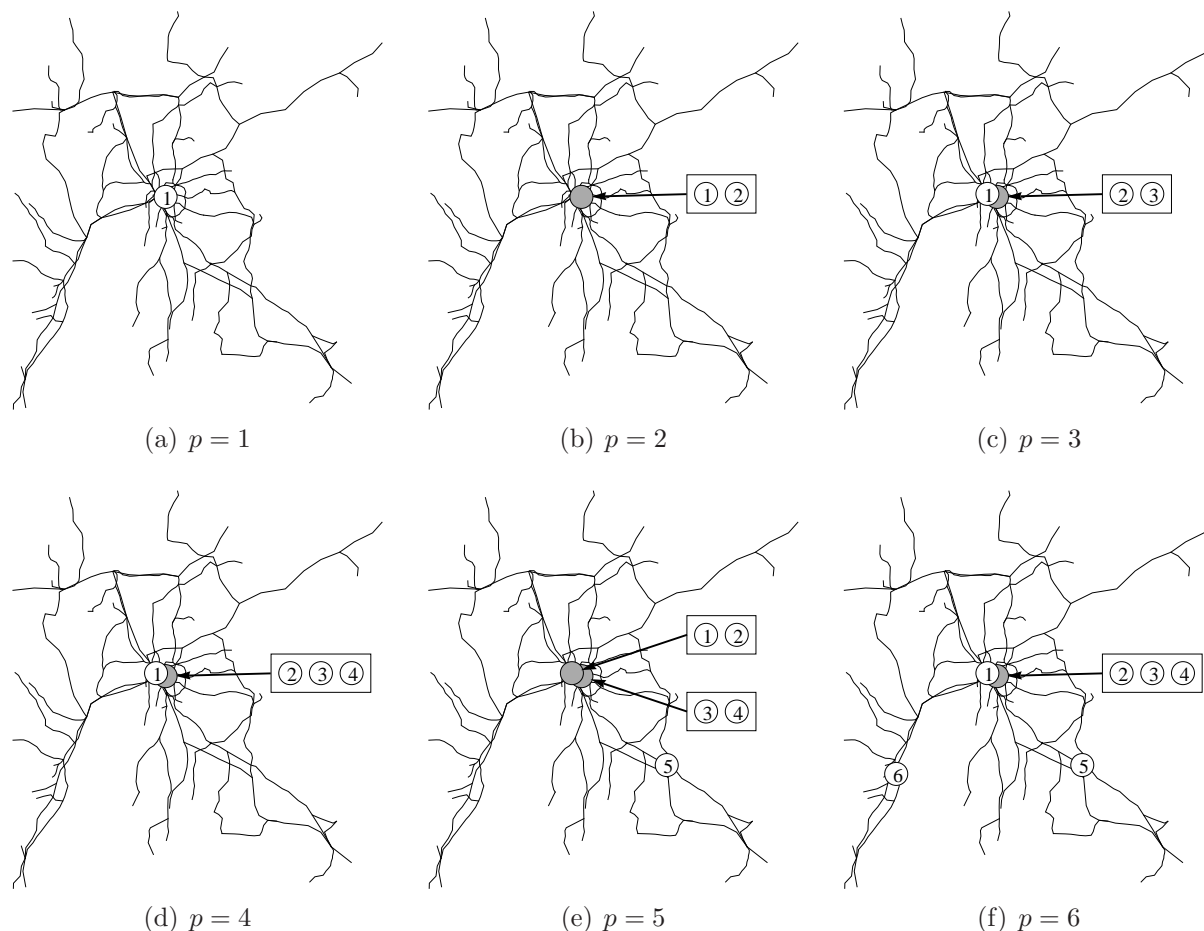


Figure 7: Optimal facility locations for the I-model in MNS ($w_p = 0.2$)

Table 1: Optimal solutions for the I-model in MNS ($w_p = 0.2$)

	$p = 1$	$p = 2$	$p = 3$	$p = 4$	$p = 5$	$p = 6$
1	Yabacho 6:20	Yabacho 6:20	Nagoya 6:00	Nagoya 6:00	Nagoya 6:00	Nagoya 6:00
2		Yabacho 7:20	Yabacho 6:20	Yabacho 6:20	Nagoya 7:20	Yabacho 6:20
3			Yabacho 7:20	Yabacho 6:40	Yabacho 6:20	Yabacho 6:40
4				Yabacho 7:20	Yabacho 6:40	Yabacho 7:20
5					Nakaokazaki 6:20	Nakaokazaki 6:20
6						Shiohama 6:20

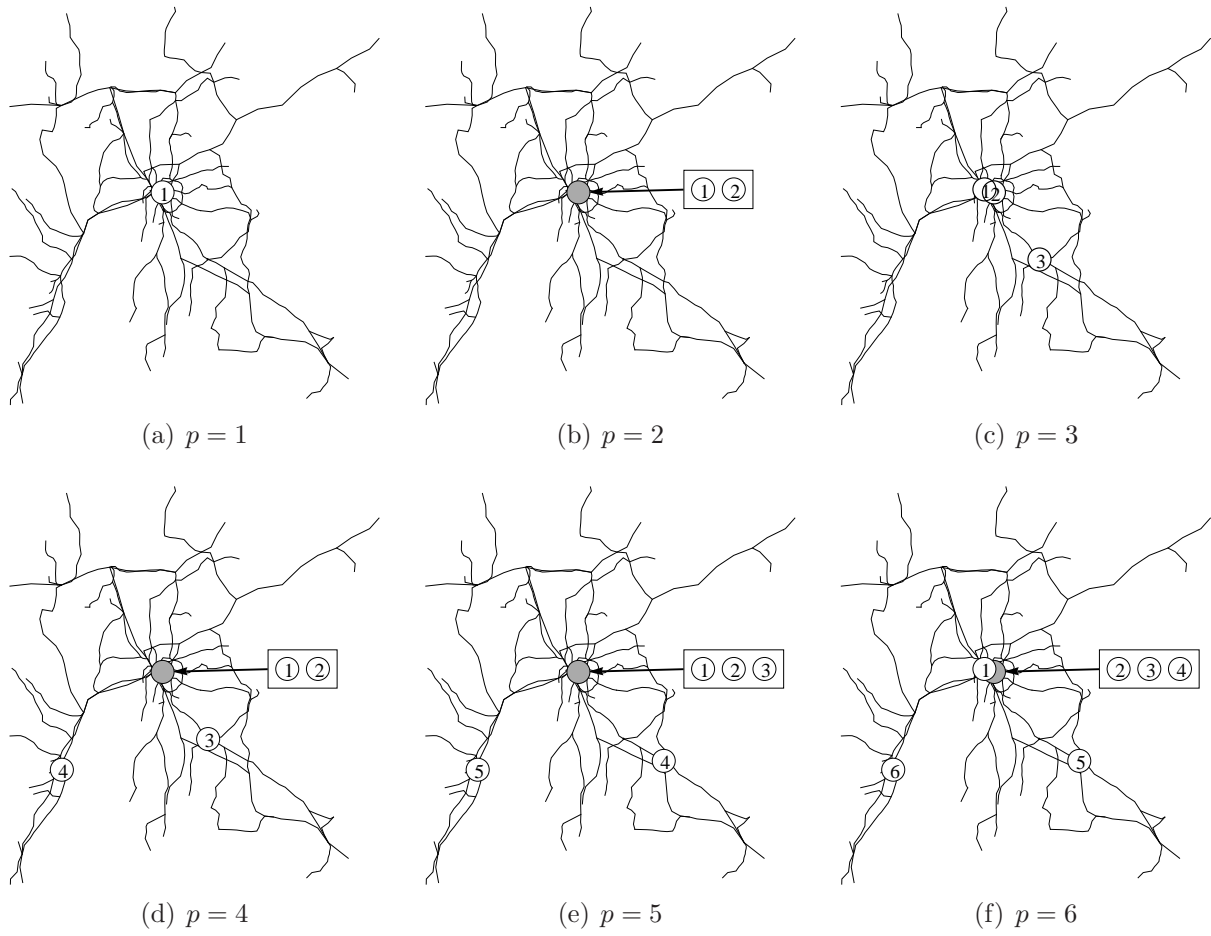


Figure 8: Optimal facility locations for the I-model in FNS ($w_p = 0.8$)

Table 2: Optimal solutions for the I-model in FNS ($w_p = 0.8$)

	$p = 1$	$p = 2$	$p = 3$	$p = 4$	$p = 5$	$p = 6$
1	Yabacho 7:20	Yabacho 6:20	Nagoya 7:20	Yabacho 6:20	Yabacho 6:20	Nagoya 7:00
2		Yabacho 7:20	Yabacho 6:20	Yabacho 7:20	Yabacho 7:20	Yabacho 6:20
3			Chiryu 7:20	Chiryu 7:20	Yabacho 7:40	Yabacho 7:20
4				Shiohama 7:20	Nakaokazaki 7:20	Yabacho 7:40
5					Shiohama 7:20	Nakaokazaki 7:20
6						Shiohama 7:20

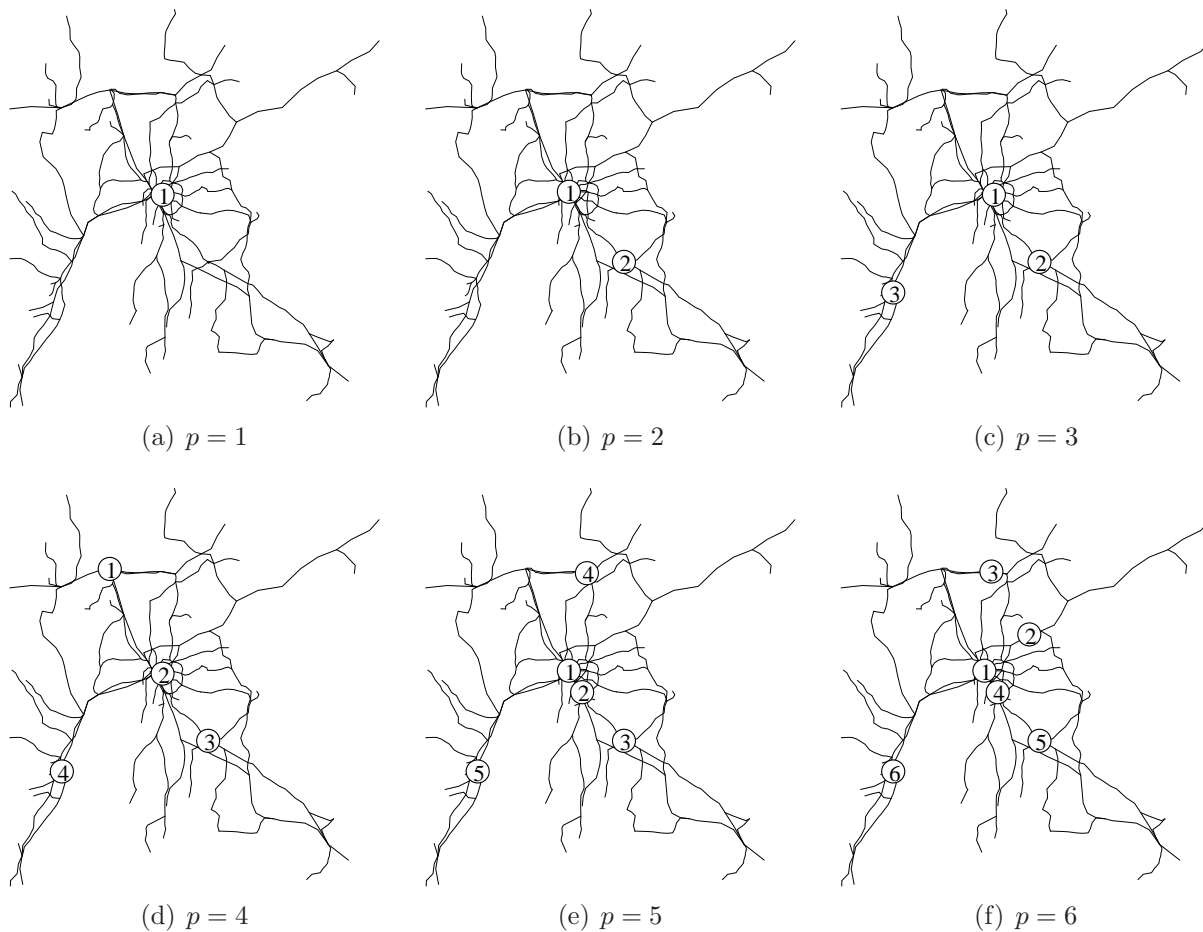


Figure 9: Optimal facility locations for the C-model in MNS ($w_p = 0.2$)

Table 3: Optimal solutions for the C-model in MNS ($w_p = 0.2$)

	$p = 1$	$p = 2$	$p = 3$	$p = 4$	$p = 5$	$p = 6$
1	Yabacho	Nagoya	Yabacho	Gifu	Nagoya	Nagoya
2		Chiryu	Chiryu	Yabacho	Horita	Jinryo
3			Shiohama	Chiryu	Chiryu	Kakamigahara
4				Shiohama	Unumajuku	Horita
5					Shiohama	Chiryu
6						Shiohama
	6:20	6:20	6:20	6:20	6:20	6:20

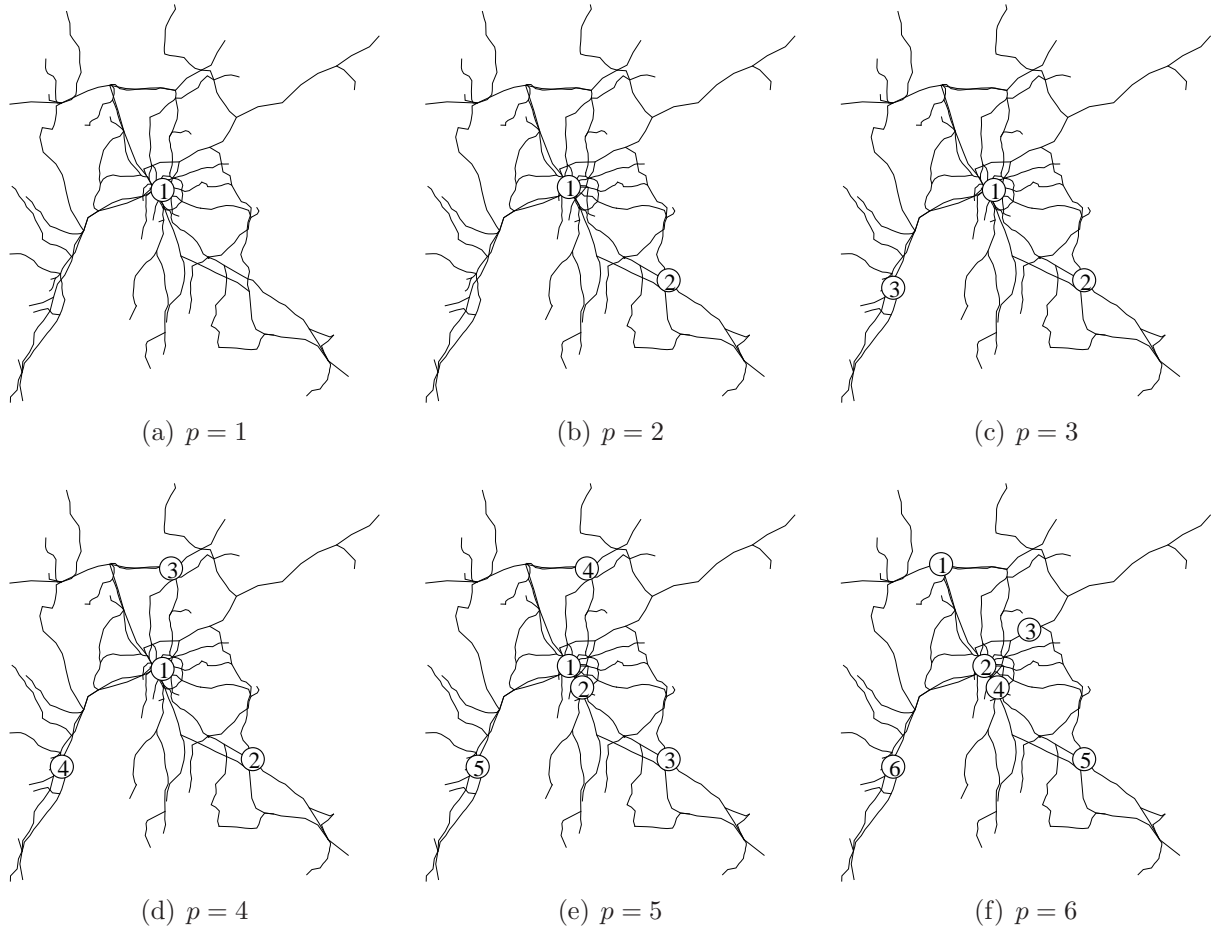


Figure 10: Optimal facility locations for the C-model in FNS ($w_p = 0.8$)

Table 4: Optimal solutions for the C-model in FNS ($w_p = 0.8$)

	$p = 1$	$p = 2$	$p = 3$	$p = 4$	$p = 5$	$p = 6$
1	Yabacho	Nagoya	Yabacho	Yabacho	Nagoya	Gifu
2		Higashiokazaki	Higashiokazaki	Higashiokazaki	Horita	Nagoya
3			Shiohama	Unumajuku	Higashiokazaki	Jinryo
4				Shiohama	Unumajuku	Horita
5					Shiohama	Higashiokazaki
6						Shiohama
	7:20	7:20	7:20	7:20	7:20	7:20

the I-model is spatially more clustered than that in the C-model. In particular, colocated facilities with different service start times were observed in some solutions of the I-model.

There are several possibilities for future research directions. One is to consider other types of services which may be consumed while people stop at a facility for a fixed length of time during the opening hours of the facility, as opposed to the complete consumption we considered in this paper. Examples of such service facilities include fitness gymnasiums, restaurants and others. In such situations, the opening hours of the facilities may be one of the decision variables rather than an input parameter.

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