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PRICE COMPETITION AND SOCIAL WELFARE COMPARISONS BETWEEN LARGE-SCALE AND SMALL-SCALE RETAILERS*

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Abstract In some localities, a large-scale chain retailer competes against a small-scale local independent retailer that specializes in, for instance, vegetables, fruits, and flowers produced locally for local consumption. The former usually attracts consumers by emphasizing its width and depth of products variety, whereas the latter seeks to overcome its limited products assortment by offering lower prices for them than the chain store. This is possible for the local store partly because of lower labor costs and for various other reasons.

This study employs the Hotelling unit interval to examine price competition in a duopoly featuring one large-scale chain retailer and one local retailer. To express differences in their product assortments, we assume that the large-scale retailer denoted by $\mathscr{A}$ sells two types of product, G_1 and G_2, whereas the local retailer denoted by $\mathscr{B}$ sells only G_1. Moreover, we assume that all the consumers purchase G_1 at $\mathscr{A}$ or $\mathscr{B}$ after comparing prices and buy G_2 at $\mathscr{A}$ on an as-needed basis. We examine both Nash and Stackelberg equilibrium to indicate that the local retailer can survive competition with the large-scale chain retailer even if all the consumers purchase both G_1 and G_2. We also reveal that a monopolistic market structure, not duopoly, can optimize the social welfare if consumers always purchase both G_1 and G_2.

Keywords: Game theory, chain store, local independent retailer, duopoly, Hotelling model, price competition

1. Introduction

A large-scale chain retailer store customarily offers a wide and deep product variety to attract more consumers over a wider area. Product width is a term signifying the different types of products a retailer offers; product depth refers to the variety of a product offered. It is a tenet in business economics that social welfare gains and revenue gains from product width and depth must be balanced against the prospect of lower consumer prices and lessened product choice.

Lancaster [13] has surveyed the issue of product variety from an economist's point of view. The term product variety in his study corresponds broadly to the number of "brands" as that term appears in the marketing literature.

Kök, Fischer, and Vaidyanathan [12] have extensively reviewed literature on assortment planning. Kök and Fischer [11] have proposed a method by which retailers could optimize product assortment and estimate consumer demand. Cachon and Kök [3] have examined the determination of product assortment among multiple merchandise categories and basket shopping consumers. Toporowski and Lademann [18] have reviewed the literature that examines assortment, price, and location in food retailing.

Some chain retailers set prices according to local markets (Dobson and Waterson [4])

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to compete against local independent retailers. Focusing on grocery retailers, Leszczyc, Shinha, and Timmermans [15] have estimated a dynamic hazard model to understand factors influencing timing of consumer purchases, store choices, and the competitive dynamics of retail competition. Swoboda, Berg, Schramm-Klein, and Foscht [17] have discussed the relative importance of retail brand equity and store accessibility for determining store loyalty in different local competitive contexts.

In contrast, local independent stores are usually small-scale, for example, discount supermarkets offering everyday-low-pricing and minimal service. They offer fewer product groups and variants within groups, but they acquire neighborhood consumers who want to reduce their dependence on automobile (Handy and Cliftono [8]) by offering lower prices than competitors on targeted groups of products (Kahn and McAlister [10]; Guptill and Wilkins [7]). Lal and Rao [16] have investigated the factors underlying the success of every-day-low-pricing. Bell and Lattin [1] have linked consumer preference for every-day-low-price retailers versus high as well as low-price retailers to the expected dollar cost of a household's shopping basket.

In some areas of Japan, local independent retailers sell vegetables, fruits, flowers, and fish produced locally for local consumption. They sell their limited assortments of product groups at lower prices than those offered by large-scale chain retailer and cater to customers for whom freshness and prices are essential. They can do so because they buy directly at lower prices from local growers, farms, and cooperatives and their labor cost are often below those of chain retailers.

This study uses the Hotelling unit interval model to address price competition in a duopoly featuring a large-scale chain retailer and a small-scale local independent retailer. We first discuss Nash equilibrium and then investigate Stackelberg equilibrium to indicate that the local independent retailer can survive competition with the chain retailer. We examine social welfare and its optimality at Nash and Stackelberg equilibrium. All the propositions in this study are straightforwardly proven and therefore omitted.

2. Model

2.1. Notation and assumptions

Our notations and assumptions are as follows:

(1) Consumers, who are homogeneous except locations, are uniformly distributed along the Hotelling unit interval $[0, 1]$ (Hotelling [9], Biscaia and Mota [2]).

(2) Large-scale retailer $\mathscr{A}$ is located at 0 and a small-scale local independent retailer $\mathscr{B}$ is at 1 on the horizontal unit interval $[0, 1]$.

(3) To express the differences in the product breadth, we assume that $\mathscr{A}$ sells two types of products (G_1 and G_2), whereas $\mathscr{B}$ sells only G_1.

(4) Consumers purchase only G_1 with probability α and purchase G_1 and G_2 simultaneously with probability $1 - \alpha$.

(5) $\mathscr{A}$ sells G_1 and G_2 at prices $p_1^{(A)}$ and p_2, respectively, and $\mathscr{B}$ sells G_1 at price $p_1^{(B)}$.

(6) The raw prices of G_1 and G_2 are a and b, respectively, where $p_1^{(A)},\ p_1^{(B)} \geq a$ and $p_2 \geq b$. Since we concentrate upon the price competition between $\mathscr{A}$ and $\mathscr{B}$, p_2 is assumed to be determined by some suitable criterion and thus p_2 is given.

(7) Consumer willingness to pay (WTP in short) for G_1 and G_2 are u_1 and u_2, respectively.

(8) Traveling cost for each consumer is c per unit of distance.

(9) $u_2 - p_2 > 0$.

(10) $u_1 - a > \frac{9c}{2}$.

In assumption (6), we assume that the raw price of G_1 at $\mathscr{A}$ is identical to that at $\mathscr{B}$. In the real circumstances, however, the latter could be smaller than the former. If $\mathscr{B}$ can survive the competition under assumption (6), $\mathscr{B}$ can survive more easily in the actual environment. Besides, assumption (9) ensures consumers an incentive to purchase G_2, while assumption (10) is introduced to assure that all the consumers over [0, 1] have positive utility in purchasing G_1 occasionally with G_2 at equilibrium prices, which will be discussed more precisely later.

2.2. Indexes

2.2.1. Boundaries

When a consumer at location $x \in [0, 1]$ purchases G_1 only at $\mathscr{A}$, her utility U_{1A} is given by

$$U_{1A} = u_1 - p_1^{(A)} - 2cx, \tag{1}$$

while if she visits $\mathscr{B}$ to buy G_1, her utility U_{1B} is given by

$$U_{1B} = u_1 - p_1^{(B)} - 2c(1 - x). \tag{2}$$

Hence, when a consumer purchases G_1 only, the boundary by which consumers are categorized delineates

$$\tilde{x}_1 = \begin{cases} \max\left(0, \frac{1}{2} - \frac{p_1^{(A)} - p_1^{(B)}}{4c}\right), & p_1^{(A)} \geq p_1^{(B)} \\ \min\left(1, \frac{1}{2} - \frac{p_1^{(A)} - p_1^{(B)}}{4c},\right), & p_1^{(A)} < p_1^{(B)} \end{cases}, \tag{3}$$

where consumers with $x \in [0, \tilde{x}_1]$ travel to retailer $\mathscr{A}$, and consumers having $x \in (\tilde{x}_1, 1]$ visit $\mathscr{B}$.

Note above that we take into account the consumers' round-trip travel costs from their home to the store unlike the usual Hotelling models (see, e.g., [2, 5, 6, 14]). This is to shed light upon the difference in the traveling costs between the two types of consumers specifically; consumers visiting $\mathscr{A}$ only and those visiting both $\mathscr{A}$ and $\mathscr{B}$.

If a consumer at $x \in [0, 1]$ purchases G_1 and G_2 at $\mathscr{A}$, her utility U_{2A} is given by

$$U_{2A} = u_1 + u_2 - p_1^{(A)} - p_2 - 2cx, \tag{4}$$

while if she visits $\mathscr{B}$ to buy G_1 and then $\mathscr{A}$ to obtain G_2, her utility U_{2B} becomes

$$U_{2B} = u_1 + u_2 - p_1^{(B)} - p_2 - 2c. \tag{5}$$

Then, the boundary $\tilde{x}_2$ in the case of purchasing both G_1 and G_2 is given by

$$\tilde{x}_2 = \begin{cases} \max\left(0, 1 - \frac{p_1^{(A)} - p_1^{(B)}}{2c}\right), & p_1^{(A)} \geq p_1^{(B)} \\ 1, & p_1^{(A)} < p_1^{(B)} \end{cases}, \tag{6}$$

where consumers with $x \in [0, \tilde{x}_2]$ travel to retailer $\mathscr{A}$ to buy both G_1 and G_2, and consumers with $x \in (\tilde{x}_2, 1]$ visit $\mathscr{B}$ to obtain G_1 and then $\mathscr{A}$ to purchase G_2.

In the following, we concentrate upon the case of $1-\frac{p_1^{(A)}-p_1^{(B)}}{2c}\in[0,1]$ for simplicity, and hence we additionally introduce assumption (11).

(11) $0\le p_1^{(A)}-p_1^{(B)}\le 2c$.

Note that assumption (11) simplifies $\tilde{x}_1$ and $\tilde{x}_2$;

$$\tilde{x}_1 = \frac{1}{2}-\frac{p_1^{(A)}-p_1^{(B)}}{4c}, \qquad (7)$$

$$\tilde{x}_2 = 1-\frac{p_1^{(A)}-p_1^{(B)}}{2c}. \qquad (8)$$

Equations (7) and (8) engender Proposition 2.1.

Proposition 2.1. $\tilde{x}_2=2\tilde{x}_1$.

2.2.2. Expected profits

When consumers behave as observed above, expected profit to $\mathscr{A}$ is given by

$$\begin{aligned}\Pi_A\left(p_1^{(A)},p_1^{(B)}\right) &= \alpha\tilde{x}_1\left(p_1^{(A)}-a\right)+(1-\alpha)\left[\tilde{x}_2\left(p_1^{(A)}-a\right)+(p_2-b)\right]\\ &= [\alpha\tilde{x}_1+(1-\alpha)\tilde{x}_2]\left(p_1^{(A)}-a\right)+(1-\alpha)(p_2-b), \qquad (9)\end{aligned}$$

and $\mathscr{B}$ earns his expected profit given by

$$\Pi_B\left(p_1^{(A)},p_1^{(B)}\right)=[\alpha(1-\tilde{x}_1)+(1-\alpha)(1-\tilde{x}_2)]\left(p_1^{(B)}-a\right). \qquad (10)$$

2.2.3. Best responses

The derivative of $\Pi_A\left(p_1^{(A)},p_1^{(B)}\right)$ with respect to $p_1^{(A)}$ is

$$\frac{\partial\Pi_A\left(p_1^{(A)},p_1^{(B)}\right)}{\partial p_1^{(A)}} = \frac{(2-\alpha)\left(a+2c-2p_1^{(A)}+p_1^{(B)}\right)}{4c},$$

while that of $\Pi_B\left(p_1^{(A)},p_1^{(B)}\right)$ in reference to $p_1^{(B)}$ is

$$\frac{\partial\Pi_B\left(p_1^{(A)},p_1^{(B)}\right)}{\partial p_1^{(B)}} = \frac{\alpha}{2}+\frac{(2-\alpha)\left(a+p_1^{(A)}-2p_1^{(B)}\right)}{4c}.$$

By solving $\frac{\partial\Pi_A\left(p_1^{(A)},p_1^{(B)}\right)}{\partial p_1^{(A)}}=0$ with respect to $p_1^{(A)}$, we obtain the best response of $\mathscr{A}$ against $\mathscr{B}$ which is written as

$$p_1^{(A)}=\frac{p_1^{(B)}+a}{2}+c. \qquad (11)$$

The best response of $\mathscr{B}$ against $\mathscr{A}$, which is a solution to $\frac{\partial\Pi_A\left(p_1^{(A)},p_1^{(B)}\right)}{\partial p_1^{(B)}}=0$ with respect to $p_1^{(B)}$, is

$$p_1^{(B)}=\frac{p_1^{(A)}+a}{2}+\frac{c\alpha}{2-\alpha}. \qquad (12)$$

2.2.4. Consumer surplus

Consumer surplus is an essential index for evaluating the equilibrium. Its intricate structure is given by

$$\begin{aligned} & CS\left(p_1^{(A)}, p_1^{(B)}\right) \\ = \;& \alpha\left\{\int_0^{\tilde{x}_1}(u_1 - p_1^{(A)} - 2cx)dx + \int_{\tilde{x}_1}^1 [u_1 - p_1^{(B)} - 2c(1-x)]dx\right\} \\ & +(1-\alpha)\left[\int_0^{\tilde{x}_2}(u_1 + u_2 - p_1^{(A)} - p_2 - 2cx)dx + \int_{\tilde{x}_2}^1 (u_1 + u_2 - p_1^{(B)} - p_2 - 2c)dx\right] \\ = \;& [\alpha\tilde{x}_1 + (1-\alpha)\tilde{x}_2]\left[2c - \left(p_1^{(A)} - p_1^{(B)}\right)\right] \\ & + \left(u_1 - p_1^{(B)} - 2c\right) + (1-\alpha)(u_2 - p_2) + c\alpha(1 - 2\tilde{x}_1^2) - c(1-\alpha)\tilde{x}_2^2. \end{aligned} \tag{13}$$

3. Equilibrium

3.1. Nash equilibrium

Solving Eqs. (11) and (12) with respect to $p_1^{(A)}$ and $p_1^{(B)}$ simultaneously, we obtain the Nash equilibrium with regard to price as follows:

$$p_1^{(A)*} = a + \frac{2c}{3} + \frac{4c}{3(2-\alpha)}, \tag{14}$$

$$p_1^{(B)*} = a - \frac{2c}{3} + \frac{8c}{3(2-\alpha)}. \tag{15}$$

By substituting Eqs. (14) and (15) into (7) and (8), the boundaries $\tilde{x}_1$ and $\tilde{x}_2$ become

$$\tilde{x}_1^* = \frac{1}{6} + \frac{1}{3(2-\alpha)}, \qquad \tilde{x}_2^* = \frac{1}{3} + \frac{2}{3(2-\alpha)}.$$

At present, however, these solutions are, actually, candidates for the Nash equilibrium because we have not yet verified that all the consumers over [0, 1] can have positive utility through the behaviors described by these solutions.

Let us examine the case where consumers purchase G_1 only. A consumer at location $x \in [0, \tilde{x}_1^*]$ has the minimum utility U_{1A}^* when $x = \tilde{x}_1^*$, whereas the utility of a consumer at $x \in (\tilde{x}_1^*, 1]$ shows its infimum U_{1B}^* when $x \to \tilde{x}_1^* + 0$. They are given by

$$U_{1A}^* = U_{1B}^* = u_1 - a - \frac{c(4-\alpha)}{2-\alpha}.$$

Assumption (10) assures $U_{1A}^* = U_{1B}^* > 0$ due to $0 \le \alpha \le 1$.

In case consumers purchase G_1 and G_2, a consumer at $x \in [0, \tilde{x}_2^*]$ encounters her minimum utility when $x = \tilde{x}_2$, and a consumer at $x \in (\tilde{x}_2^*, 1]$ has a constant utility as long as $x \in (\tilde{x}_2, 1]$. They are given by

$$U_{2A}^* = U_{2B}^* = u_1 + u_2 - a - p_2 - \frac{4c(4-\alpha)}{3(2-\alpha)}.$$

Assumptions (9) and (10) assure that $U^*_{2A} = U^*_{2B} > 0$.

We have confirmed that all the consumers over [0, 1] have an incentive to behave according to the solutions.

At the Nash equilibrium, the shares S^*_A and S^*_B of G_1 held by $\mathscr{A}$ and $\mathscr{B}$, respectively, become

$$\begin{aligned} S^*_A &= \alpha\tilde{x}^*_1 + (1-\alpha)\tilde{x}^*_2 = \frac{2}{3} - \frac{\alpha}{6}, \\ S^*_B &= \alpha\left(1-\tilde{x}^*_1\right) + (1-\alpha)\left(1-\tilde{x}^*_2\right) = \frac{1}{3} + \frac{\alpha}{6}. \end{aligned}$$

Expected profits to $\mathscr{A}$ and $\mathscr{B}$ at the Nash equilibrium are

$$\begin{aligned} \Pi^*_A &:= \Pi_A\left(p_1^{(A)*}, p_1^{(B)*}\right) = (1-\alpha)(p_2 - b) + c\left(\frac{2}{3} - \frac{\alpha}{9}\right) + \frac{4c}{9(2-\alpha)}, \\ \Pi^*_B &:= \Pi_B\left(p_1^{(A)*}, p_1^{(B)*}\right) = \frac{c(2+\alpha)^2}{9(2-\alpha)}, \end{aligned}$$

and the consumer surplus is given by

$$CS^* := CS\left(p_1^{(A)*}, p_1^{(B)*}\right) = u_1 - a + (1-\alpha)(u_2 - p_2) - c\left[1 - \frac{17\alpha}{18} + \frac{22}{9(2-\alpha)}\right].$$

The indexes derived above initiate Proposition 3.1.

Proposition 3.1.

(i) $p_1^{(A)*} \geq p_1^{(B)*}$, *where the equality holds only when* $\alpha = 1$.

(ii) *Both* $p_1^{(A)*}$ *and* $p_1^{(B)*}$ *are increasing in* α, *whereas their difference* $\left(p_1^{(A)*} - p_1^{(B)*}\right)$ *decreases with increasing* α.

(iii) *The profit* $\Pi^*_A - (1-\alpha)(p_2 - b)$ *to* $\mathscr{A}$ *gained from* G_1 *increases with* α, *whereas its share* S^*_A *decreases with* α. *In addition, the profit* Π^*_B *to* $\mathscr{B}$ *and its corresponding share* S^*_B *increases with* α.

(iv) *Boundary* $\tilde{x}^*_1$ *increases from* $\frac{1}{3}$ *to* $\frac{1}{2}$ *as* α *increases, whereas boundary* $\tilde{x}^*_2$ *is increasing in* α *from* $\frac{4}{6}$ *to* 1.

Proposition 3.1–(ii) implies that $\mathscr{B}$ should refrain its price low to compete with $\mathscr{A}$ when more consumers purchase both G_1 and G_2, and consequently $\mathscr{A}$ also makes its price lower in accordance with $\mathscr{B}$. It also suggests that if more consumers purchase G_1 only, $\mathscr{B}$ can raise its price along with $\mathscr{A}$ to a certain degree because $\mathscr{A}$ becomes less advantageous. Further, their prices become closer to each other as α increases.

Proposition 3.1–(iii) reveals that when more consumers purchase G_1 only, $\mathscr{A}$ can increase his profit gained from G_1 by raising its price against its decreasing share. Proposition 3.1–(iii) also indicates that $\mathscr{B}$ can earn more profit and share by raising its price when more consumers buy G_1 only.

Proposition 3.1–(iv) shows the range of $\tilde{x}^*_1$ and $\tilde{x}^*_2$.

3.2. Stackelberg equilibrium

In the real circumstances, the large-scale chain retailer can be considered a price leader and the small-scale local retailer a price follower within a Stackelberg game framework.

Substituting Eq. (12) into Eqs. (7), (8), and (9) gives

$$\Pi_A\left(p_1^{(A)}\right) = (1-\alpha)(p_2-b) - \frac{\left(p_1^{(A)}-a\right)\left[(2-\alpha)\left(p_1^{(A)}-2c-a\right)-4c\right]}{8c}. \qquad (16)$$

By differentiating $\Pi_A\left(p_1^{(A)}\right)$ with respect to $p_1^{(A)}$, we have

$$\frac{\partial \Pi_A\left(p_1^{(A)}\right)}{\partial p_1^{(A)}} = 1 - \frac{\alpha}{4} - \frac{\left(p_1^{(A)}-a\right)(2-\alpha)}{4c}.$$

Hence, the solution to $\frac{\partial \Pi_A\left(p_1^{(A)}\right)}{\partial p_1^{(A)}} = 0$ is

$$p_1^{(A)**} = a + c + \frac{2c}{2-\alpha}, \qquad (17)$$

which is the optimal price of G_1 for $\mathscr{A}$ against the best response of $\mathscr{B}$.

When $\mathscr{A}$ adopts the selling price in Eq. (17), the optimal price of G_1 for $\mathscr{B}$ becomes

$$p_1^{(B)**} = a - \frac{c}{2} + \frac{3c}{2-\alpha}. \qquad (18)$$

The boundaries $\tilde{x}_1$ and $\tilde{x}_2$ in Eqs. (7) and (8) become

$$\tilde{x}_1^{**} = \frac{1}{8} + \frac{1}{4(2-\alpha)}, \qquad \tilde{x}_2^{**} = \frac{1}{4} + \frac{1}{2(2-\alpha)}.$$

In the same manner as we observed in 3.1., let us examine here whether or not all the consumers can accept the behaviors suggested by the above solution.

Firstly, if consumers purchase G_1 only, a consumer's minimum utility U_{1A}^{**} at location $x = \tilde{x}_1^{**}$ and her infimum utility U_{1B}^{**} when $x \to \tilde{x}_1^{**} + 0$ are expressed by

$$U_{1A}^{**} = U_{1B}^{**} = u_1 - a - \frac{5c(4-\alpha)}{4(2-\alpha)}.$$

From assumption (10), we have $U_{1A}^{**} = U_{1B}^{**} > 0$.

Secondly, if consumers buy G_1 and G_2, a consumer's minimum utility U_{2A}^{**} at location $x = \tilde{x}_2^{**}$ and her constant utility U_{2B}^{**} at $x \to \tilde{x}_2 + 0$ are given by

$$U_{2A}^{**} = U_{2B}^{**} = u_1 + u_2 - a - p_2 - \frac{3c(4-\alpha)}{2(2-\alpha)}.$$

Assumptions (9) and (10) assure that $U_{2A}^{**} = U_{2B}^{**} > 0$.

We have verified that all the consumers have an incentive to behave as suggested by the above solutions.

These observations reveal the shares S_A^{**} and S_B^{**} of G_1 held by $\mathscr{A}$ and $\mathscr{B}$, respectively, are given by

$$\begin{aligned} S_A^{**} &= \frac{1}{2} - \frac{\alpha}{8}, \\ S_B^{**} &= \frac{1}{2} + \frac{\alpha}{8}. \end{aligned}$$

Expected profits to $\mathscr{A}$ and $\mathscr{B}$ are

$$\begin{aligned} \Pi_A^{**} &:= \Pi_A\left(p_1^{(A)**}, p_1^{(B)**}\right) = (1-\alpha)(p_2 - b) + c\left(\frac{3}{4} - \frac{\alpha}{8}\right) + \frac{c}{2(2-\alpha)}, \\ \Pi_B^{**} &:= \Pi_B\left(p_1^{(A)**}, p_1^{(B)**}\right) = \frac{c(4+\alpha)^2}{16(2-\alpha)}. \end{aligned}$$

Finally, the consumer surplus at the Stackelberg equilibrium is

$$CS^{**} := CS\left(p_1^{(A)**}, p_1^{(B)**}\right) = u_1 - a + (1-\alpha)(u_2 - p_2) - c\left[\frac{(42-31\alpha)}{32} + \frac{23}{8(2-\alpha)}\right].$$

The indexes in this subsection suggest Proposition 3.2.

Proposition 3.2.
(i) $p_1^{(A)**} > p_1^{(B)**}$.
(ii) *Both* $p_1^{(A)**}$ *and* $p_1^{(B)**}$ *are increasing in* α*, whereas their difference* $\left(p_1^{(A)**} - p_1^{(B)**}\right)$ *decreases with increasing* α*.*
(iii) *The profit* $\Pi_A^{**} - (1-\alpha)(p_2 - b)$ *to* $\mathscr{A}$ *from* G_1 *increases with* α*, its corresponding share* S_A^{**} *decreases with* α *though. The profit* Π_B^{**} *to* $\mathscr{B}$ *and his share* S_B^{**} *increases with* α*.*
(iv) *When* α *increases from* 0 *to* 1*,* $\tilde{x}_1^{**}$ *increases from* $\frac{1}{4}$ *to* $\frac{3}{8}$*, and* $\tilde{x}_2^{**}$ *increases from* $\frac{1}{2}$ *to* $\frac{3}{4}$*.*

Proposition 3.2-(ii) and (iii) can be discussed in the same manner as those for Proposition 3.1-(ii) and (iii), respectively. Proposition 3.2-(iv) provides the range of $\tilde{x}_1^{**}$ and $\tilde{x}_2^{**}$.

3.3. Comparison

This subsection compares the Nash and Stackelberg equilibrium. Table 1 summarizes the indexes derived above. Comparison between the Nash equilibrium with the Stackelberg equilibrium presents Proposition 3.3.

Proposition 3.3.
(i) $p_1^{(A)**} > p_1^{(A)*}$ *and* $p_1^{(B)**} > p_1^{(B)*}$*,*
(ii) $\tilde{x}_1^{**} < \tilde{x}_1^{*}$ *and* $\tilde{x}_2^{**} < \tilde{x}_2^{*}$*,*
(iii) $S_A^{**} > S_A^{*}$ *and* $S_B^{*} \geq S_B^{**}$*,*
(iv) $\Pi_A^{**} > \Pi_A^{*} > 0$ *and* $\Pi_B^{**} > \Pi_B^{*} > 0$*,*
(v) $CS^{**} < CS^{*}$*.*
(vi) $(CS^{*} - CS^{**})$ *decreases with increasing* α*.*

Table 1: Comparison

Indexes	Nash	Stackelberg
Boundary	$\tilde{x}_1^* = \frac{1}{6} + \frac{1}{3(2-\alpha)}$ $\tilde{x}_2^* = \frac{1}{3} + \frac{2}{3(2-\alpha)}$	$\tilde{x}_1^{**} = \frac{1}{8} + \frac{1}{4(2-\alpha)}$ $\tilde{x}_2^{**} = \frac{1}{4} + \frac{1}{2(2-\alpha)}$
Prices	$p_1^{(A)*} = a + \frac{2c}{3} + \frac{4c}{3(2-\alpha)}$ $p_1^{(B)*} = a - \frac{2c}{3} + \frac{8c}{3(2-\alpha)}$	$p_1^{(A)**} = a + c + \frac{2c}{2-\alpha}$ $p_1^{(B)**} = a - \frac{c}{2} + \frac{3c}{2-\alpha}$
Profits	$\Pi_A^* = (1-\alpha)(p_2 - b) + c\left(\frac{2}{3} - \frac{\alpha}{9}\right) + \frac{4c}{9(2-\alpha)}$ $\Pi_B^* = \frac{c(2+\alpha)^2}{9(2-\alpha)}$	$\Pi_A^{**} = (1-\alpha)(p_2 - b) + c\left(\frac{3}{4} - \frac{\alpha}{8}\right) + \frac{c}{2(2-\alpha)}$ $\Pi_B^{**} = \frac{c(4+\alpha)^2}{16(2-\alpha)}$
Consumer surplus	$CS^* = u_1 - a + (1-\alpha)(u_2 - p_2)$ $-c\left[1 - \frac{17\alpha}{18} + \frac{22}{9(2-\alpha)}\right]$	$CS^{**} = u_1 - a + (1-\alpha)(u_2 - p_2)$ $-c\left[\frac{(42-31\alpha)}{32} + \frac{23}{8(2-\alpha)}\right]$

Proposition 3.3-(iv) reveals that the local independent retailer $\mathscr{B}$ can survive competition against the large-scale chain retailer $\mathscr{A}$ because it earns a profit both at the Nash and Stackelberg equilibrium. In addition, $\mathscr{A}$ can earn more at the Stackelberg equilibrium than at the Nash equilibrium because $\mathscr{A}$ offers a higher price with a larger share of G_1 at the Stackelberg equilibrium. On the other hand, $\mathscr{B}$ can also earn more at the Stackelberg equilibrium than at the Nash equilibrium although its share at the Stackelberg equilibrium is smaller than that at the Nash equilibrium. This is because $\mathscr{B}$ offers a higher price which can absorb the influence by its decreasing share.

Proposition 3.3-(v) is directly led by (i). Moreover, the difference in consumer surplus $(CS^* - CS^{**})$ between the Nash and Stackelberg equilibrium is decreasing in α.

4. Social Welfare

This section confines itself to social welfare to explore socially optimal prices of the product G_1. Social welfare is given by

$$\begin{aligned} W\left(p_1^{(A)},\ p_1^{(B)}\right) &= \Pi_A\left(p_1^{(A)},\ p_1^{(B)}\right) + \Pi_B\left(p_1^{(A)},\ p_1^{(B)}\right) + CS\left(p_1^{(A)},\ p_1^{(B)}\right) \\ &= u_1 - a + (1-\alpha)(u_2 - b) - \frac{(2-\alpha)c}{2} - \frac{(2-\alpha)\left(p_1^{(A)} - p_1^{(B)}\right)^2}{8c}. \end{aligned} \tag{19}$$

At the Nash equilibrium, it becomes

$$\begin{aligned} W^* &:= W\left(p_1^{(A)*},\ p_1^{(B)*}\right) \\ &= (u_1 - a) + (1-\alpha)(u_2 - b) - c\left[1 - \frac{13\alpha}{18} + \frac{2}{9(2-\alpha)}\right], \end{aligned} \tag{20}$$

and at the Stackelberg equilibrium, it is given by

$$\begin{aligned} W^{**} &:= W\left(p_1^{(A)**},\ p_1^{(B)**}\right) \\ &= (u_1 - a) + (1-\alpha)(u_2 - b) - c\left[\frac{19}{16} - \frac{25\alpha}{32} + \frac{1}{8(2-\alpha)}\right]. \end{aligned} \tag{21}$$

Hence, we have $W^* > W^{**}$, which is a common understanding among economists.

Now, let us maximize the social welfare with respect to $p_1^{(A)}$ and $p_1^{(B)}$. By differentiating $W\left(p_1^{(A)},\ p_1^{(B)}\right)$ with respect to $p_1^{(A)}$ and $p_1^{(B)}$, we have

$$\frac{\partial W\left(p_1^{(A)}, p_1^{(B)}\right)}{\partial p_1^{(A)}} = -\frac{(2-\alpha)\left(p_1^{(A)} - p_1^{(B)}\right)}{4c}, \tag{22}$$

$$\frac{\partial W\left(p_1^{(A)}, p_1^{(B)}\right)}{\partial p_1^{(B)}} = \frac{(2-\alpha)\left(p_1^{(A)} - p_1^{(B)}\right)}{4c}. \tag{23}$$

By letting $\frac{\partial W\left(p_1^{(A)}, p_1^{(B)}\right)}{\partial p_1^{(A)}} = \frac{\partial W\left(p_1^{(A)}, p_1^{(B)}\right)}{\partial p_1^{(B)}} = 0$, the optimal prices $\left(p_1^{(A)},\ p_1^{(B)}\right) = \left(p_1^{(A)***},\ p_1^{(B)***}\right)$ maximizing $W\left(p_1^{(A)},\ p_1^{(B)}\right)$ satisfies

$$p_1^{(A)***} = p_1^{(B)***}, \tag{24}$$

along with

$$W^{***} := W\left(p_1^{(A)***},\ p_1^{(B)***}\right) = (u_1 - a) + (1-\alpha)(u_2 - b) - \frac{(2-\alpha)c}{2}.$$

Consequently, we reach Proposition 4.1.

Proposition 4.1. *If we have $p_1^{(A)} = p_1^{(B)}$ to maximize social welfare, consumers buying both G_1 and G_2 simultaneously visit only $\mathscr{A}$. Accordingly, social welfare is maximized not in a duopoly but in a monopoly dominated by $\mathscr{A}$ when $\alpha = 0$.*

Proposition 4.1 indicates that when maximizing the social welfare, it becomes more difficult for $\mathscr{B}$ to enter the market as α decreases. However, monopoly is generally considered undesirable for other economic reasons, which suggests the maximization of social welfare would be meaningless.

Table 2 compares the social welfare at Nash and Stackelberg equilibrium along with socially optimum.

Table 2: Comparison of social welfare

	Social welfare
Nash equilibrium	$W^{*} = (u_1 - a) + (1-\alpha)(u_2 - b) - c\left[1 - \frac{13\alpha}{18} + \frac{2}{9(2-\alpha)}\right]$
Stackelberg equilibrium	$W^{**} = (u_1 - a) + (1-\alpha)(u_2 - b) - c\left[\frac{19}{16} - \frac{25\alpha}{32} + \frac{1}{8(2-\alpha)}\right]$
Socially optimum	$W^{***} = (u_1 - a) + (1-\alpha)(u_2 - b) - \frac{(2-\alpha)c}{2}$

5. Conclusion

This study employed the Hotelling unit interval model to examine price competition between a large-scale chain retailer $\mathscr{A}$ and a small-scale local independent retailer $\mathscr{B}$. To represent the difference in product assortment between $\mathscr{A}$ and $\mathscr{B}$, we assumed that $\mathscr{A}$ sells product G_1 and G_2 and that $\mathscr{B}$ deals in only product G_1. We also assumed homogeneous consumers purchase G_1 from $\mathscr{A}$ or $\mathscr{B}$ and buy G_2 at $\mathscr{A}$. Then we focused on price competition over G_1. Moreover, we assumed that each individual consumer purchased G_1 only with probability α and purchased both G_1 and G_2 with probability $1-\alpha$.

Nash and Stackelberg equilibrium were examined, and the main results in this study are as follows:

(1) The local retailer can earn profits both at the Nash and Stackelberg equilibrium and survive competition with a large-scale chain retailer even when all the consumers purchase both G_1 and G_2.

(2) Both $\mathscr{A}$ and $\mathscr{B}$ earn more at the Stackelberg equilibrium than at the Nash equilibrium. However, consumer surplus diminishes more at the Stackelberg equilibrium than at the Nash equilibrium.

(3) Maximization of social welfare suggests $\mathscr{A}$ and $\mathscr{B}$ adopt a same price for G_1, where consumers buying both G_1 and G_2 visit only $\mathscr{A}$. Particularly when all the consumers purchase both G_1 and G_2, social welfare is maximized not in a duopoly but in a monopoly dominated by $\mathscr{A}$.

In this study, we concentrated on the case where the second boundary $\tilde{x}_2$ when consumers purchase both G_1 and G_2 is given by an interior solution under assumption (11). One of useful extensions of our work is to relax the conditions in assumption (11). Furthermore, all the consumers over [0, 1] are assumed to have positive utility through behaviors according to the Nash and Stackelberg equilibrium in terms of assumption (10). Another useful extension is to relax this assumption so that we can encounter the case where some consumers in a specific area of [0, 1] have negative utility at the Stackelberg equilibrium, they have positive utility at the Nash equilibrium though. These extensions are currently under investigation.

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